Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.2-Cosine/82-4.2.0-a-cos-^m-b-trg-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [294]. This is test number [82].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (294)	0.00 (0)
Mathematica	100.00 (294)	0.00 (0)
Fricas	67.01 (197)	32.99 (97)
Maple	66.67 (196)	33.33 (98)
Maxima	31.29 (92)	68.71 (202)
Mupad	27.21 (80)	72.79 (214)
Giac	13.27 (39)	86.73 (255)
Sympy	6.12 (18)	93.88 (276)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

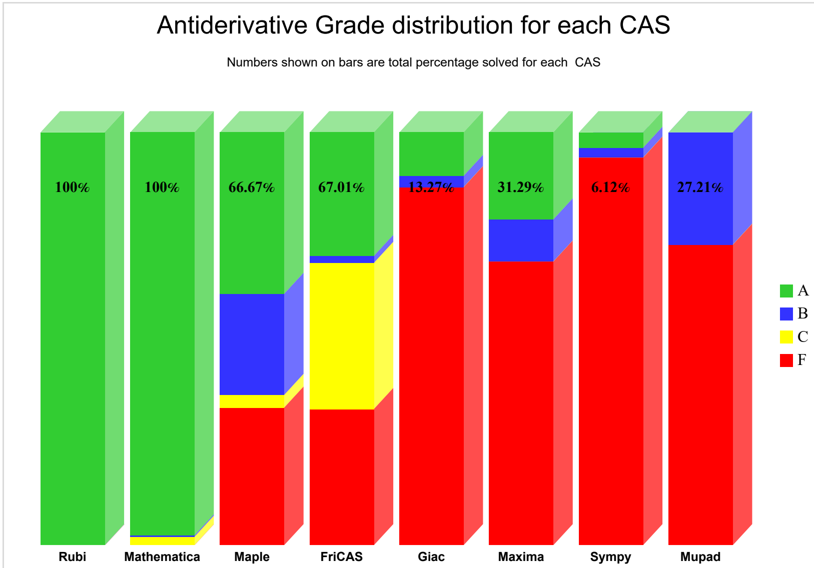
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

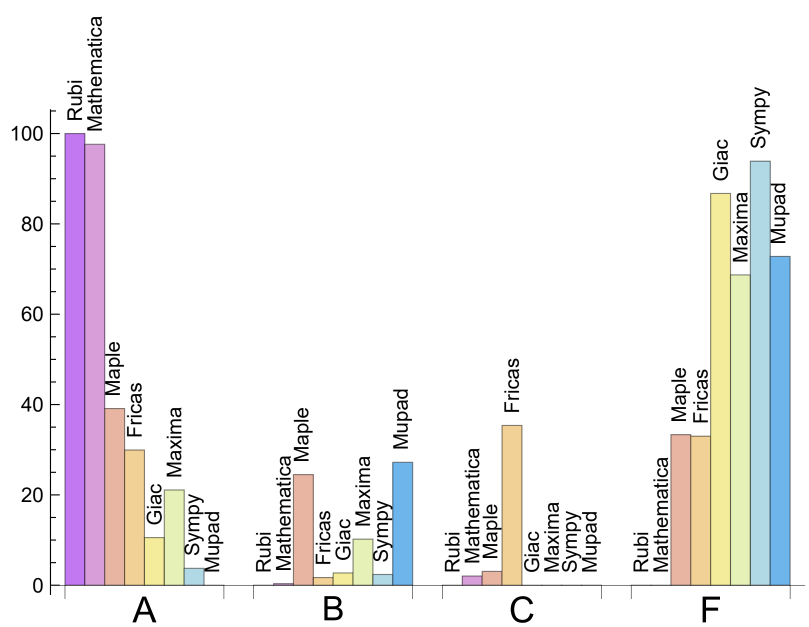
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	97.619	0.340	2.041	0.000
Maple	39.116	24.490	3.061	33.333
Fricas	29.932	1.701	35.374	32.993
Maxima	21.088	10.204	0.000	68.707
Giac	10.544	2.721	0.000	86.735
Sympy	3.741	2.381	0.000	93.878
Mupad	0.000	27.211	0.000	72.789

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	97	93.81	0.00	6.19
Maple	98	98.98	1.02	0.00
Maxima	202	99.50	0.00	0.50
Mupad	214	0.00	100.00	0.00
Giac	255	99.61	0.39	0.00
Sympy	276	47.10	52.90	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.20
Rubi	0.29
Maxima	0.38
Mathematica	0.75
Giac	1.24
Maple	3.29
Sympy	5.12
Mupad	11.39

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	54.98	1.09	43.50	0.87
Mathematica	59.26	0.90	55.00	0.91
Sympy	60.28	1.52	46.00	1.43
Rubi	66.71	0.98	61.00	1.00
Fricas	91.11	1.45	91.00	1.21
Maple	139.54	1.99	106.00	1.66
Maxima	221.09	2.81	48.50	0.86
Giac	5516.31	76.40	34.00	0.76

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

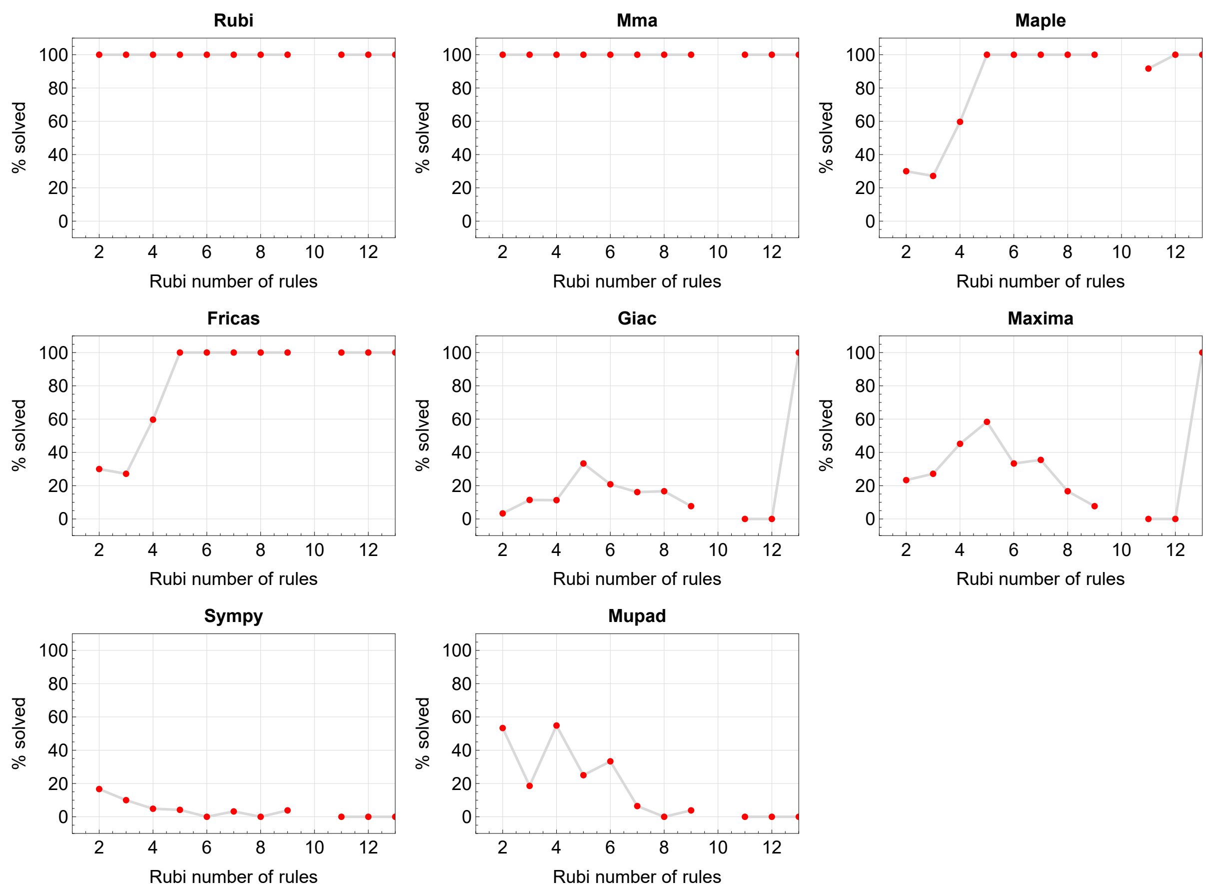


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

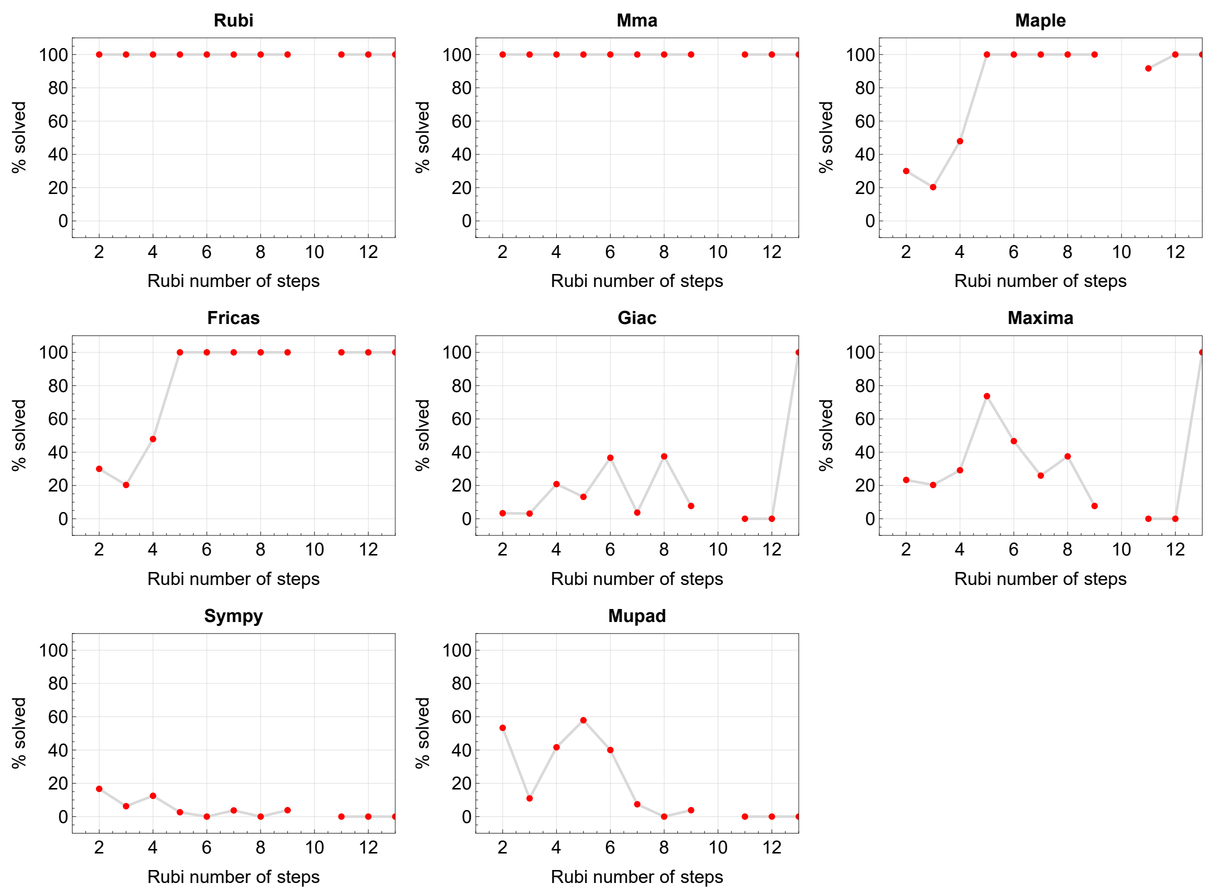


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

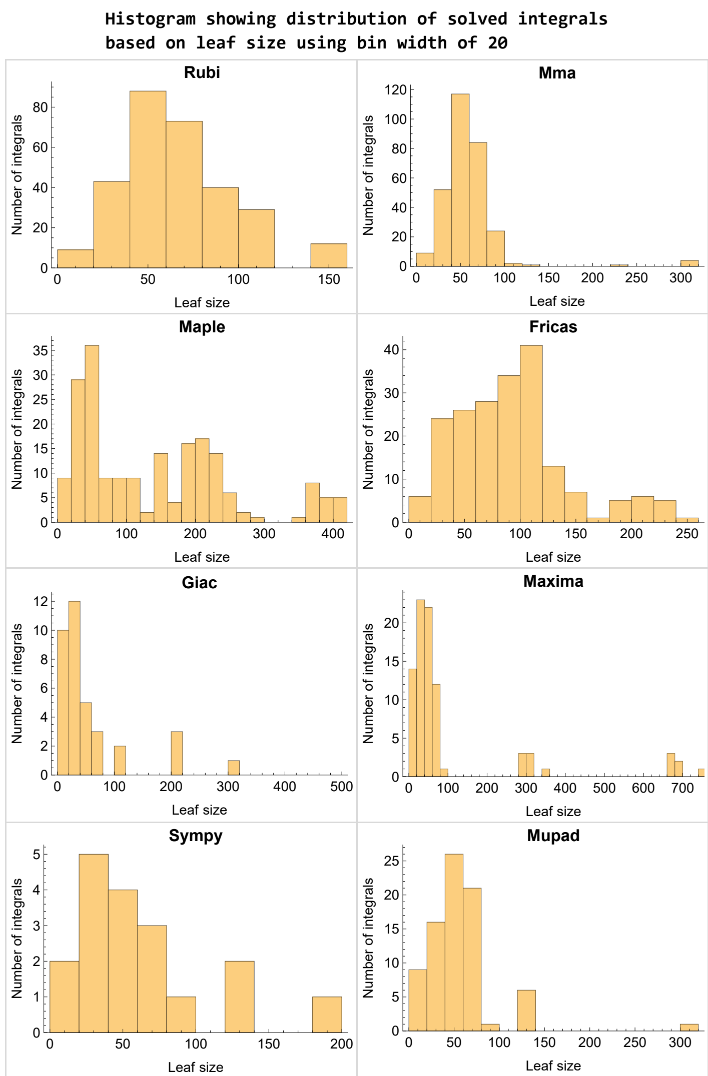


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

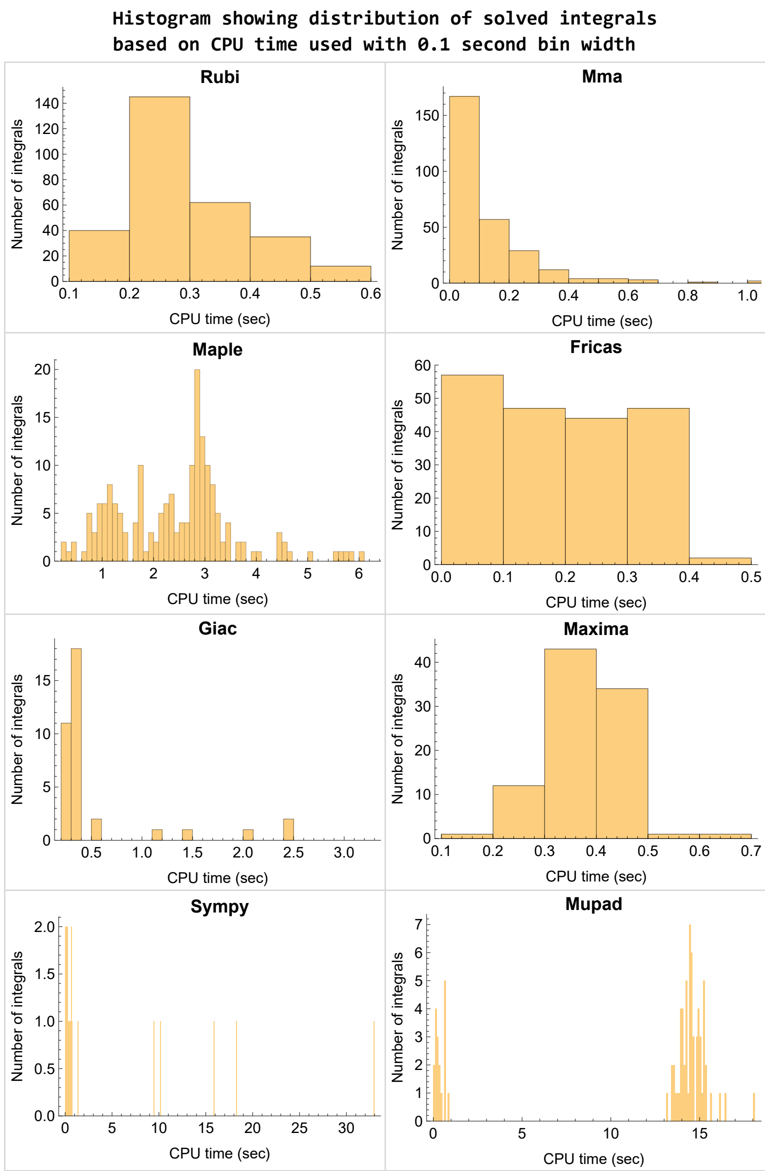


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

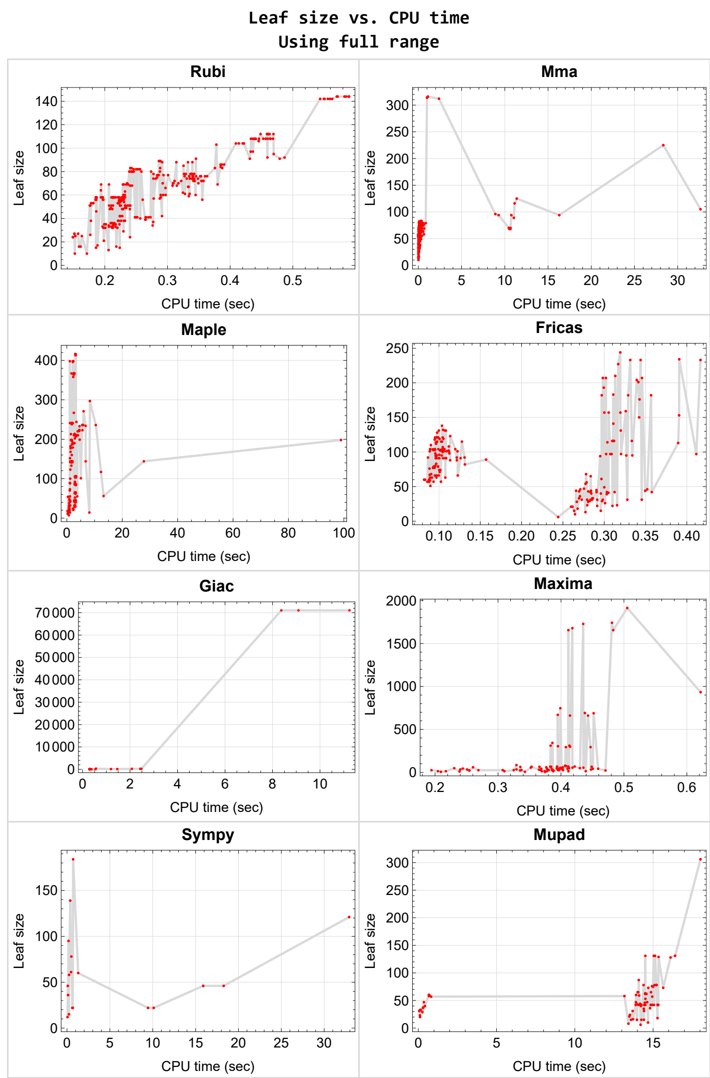


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{284, 285, 286, 287, 292\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

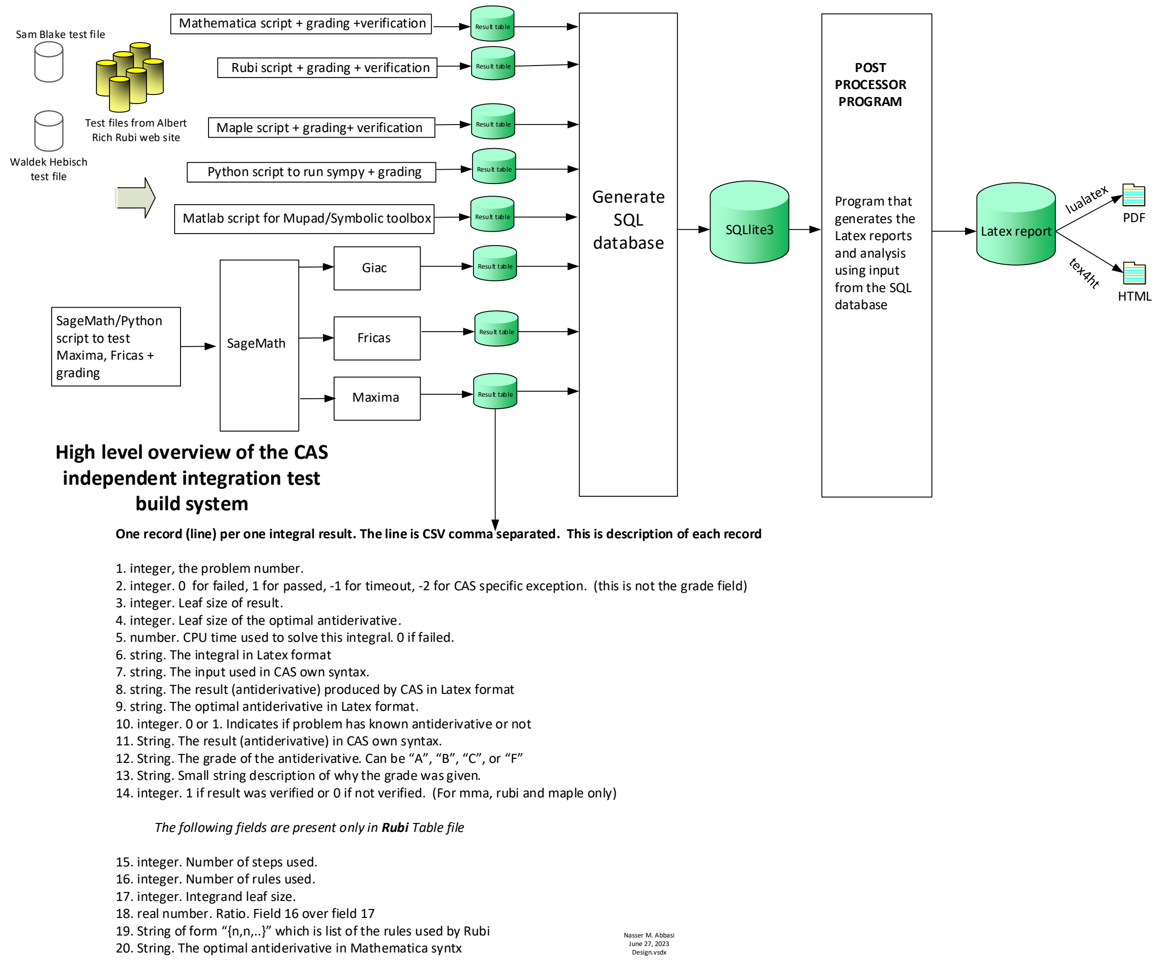
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 291, 293, 294 }

B grade { 1 }

C grade { 276, 284, 285, 286, 287, 292 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 17, 39, 40, 41, 44, 51, 52, 53, 54, 55, 56, 64, 66, 68, 78, 80, 90, 92, 102, 103, 104, 105, 115, 116, 117, 127, 128, 129, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278 }

B grade { 9, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 42, 43, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 93, 94, 95, 96, 97, 98, 99, 100, 106, 107, 108, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 274 }

C grade { 13, 21, 45, 46, 47, 48, 49, 50, 109 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 57, 58, 59, 60, 61, 62, 63, 65, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F(-1) timedout fail { 101 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 43, 44, 51, 52, 53, 54, 55, 56, 64, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 261, 262, 265, 266, 267, 270 }

B grade { 42, 271, 272, 275, 276 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 45, 46, 47, 48, 49, 50, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 263, 264, 268, 269, 273, 274, 277, 278 }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 57, 65, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F(-1) timedout fail { }

F(-2) exception fail { 58, 59, 60, 61, 62, 63 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 51, 52, 53, 54, 55, 56, 140, 141, 142, 143, 144, 146, 150, 151, 152, 153, 154, 156, 160, 161, 162, 163, 164, 165, 167, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 192, 193, 194, 195, 196, 261, 262, 265, 266, 267, 270, 271, 272, 275, 276 }

B grade { 42, 43, 44, 145, 147, 148, 149, 155, 157, 158, 159, 166, 168, 169, 170, 177, 178, 179, 180, 181, 187, 188, 189, 190, 191, 197, 198, 199, 200, 201 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F(-1) timedout fail { }

F(-2) exception fail { 59 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 43, 44, 51, 52, 53, 54, 55, 56, 143, 152, 261, 262, 265, 266, 267, 270, 271, 272, 275, 276 }

B grade { 64, 140, 141, 142, 150, 151, 161, 162 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 153, 154, 155, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F(-1) timedout fail { 160 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 25, 26, 27, 28, 29, 30, 37, 41, 54, 55, 56, 64, 107, 108, 109, 140, 141, 142, 143, 144, 146, 148, 150, 151, 152, 153, 154, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 178, 180, 182, 183, 184, 185, 186, 188, 190, 192, 193, 194, 195, 196, 198, 200, 261, 262, 265, 270 }

C grade { }

F normal fail { }

F(-1) timedout fail { 17, 18, 19, 20, 22, 23, 24, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 145, 147, 149, 155, 157, 159, 166, 168, 170, 177, 179, 181, 187, 189, 191, 197, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 3, 5, 7, 41, 144, 153, 154, 175, 176, 186 }

B grade { 2, 4, 6, 8, 64, 142, 143 }

C grade { }

F normal fail { 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 42, 43, 47, 48, 57, 59, 60, 61, 62, 63, 65, 69, 70, 71, 72, 73, 74, 75, 82, 107, 108, 109, 110, 111, 112, 113, 114, 121, 122, 123, 124, 125, 126, 135, 145, 146, 152, 177, 178, 187, 188, 189, 198, 202, 204, 205, 206, 207, 208, 209, 212, 213, 223, 224, 225, 226, 227, 228, 229, 230, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 262, 263, 264, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 284, 285, 286, 287, 291, 292, 293 }

F(-1) timedout fail { 9, 10, 16, 17, 18, 24, 25, 31, 39, 40, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 58, 66, 67, 68, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 115, 116, 117, 118, 119, 120, 127, 128, 129, 130, 131,

132, 133, 134, 136, 137, 138, 139, 140, 141, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 179, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 203, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 231, 238, 239, 252, 258, 259, 265, 266, 267, 270, 279, 283, 288, 289, 290, 294
}

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	10	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	1.00	1.00
time (sec)	N/A	0.152	0.012	0.268	0.239	0.265	0.060	0.294	14.677

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	22	22	46	18	18
N.S.	1	1.00	0.92	0.76	0.88	0.88	1.84	0.72	0.72
time (sec)	N/A	0.164	0.028	0.476	0.255	0.310	0.091	0.289	15.283

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	22	21	36	22	24
N.S.	1	1.00	1.00	0.85	0.85	0.81	1.38	0.85	0.92
time (sec)	N/A	0.176	0.010	0.911	0.254	0.262	0.124	0.293	13.520

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	51	33	31	33	36	95	32	31
N.S.	1	1.11	0.72	0.67	0.72	0.78	2.07	0.70	0.67
time (sec)	N/A	0.226	0.047	0.754	0.251	0.271	0.175	0.299	13.694

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	38	41	32	34	33	58	34	31
N.S.	1	0.93	1.00	0.78	0.83	0.80	1.41	0.83	0.76
time (sec)	N/A	0.184	0.016	0.996	0.240	0.286	0.238	0.296	0.054

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	77	43	42	48	46	139	46	42
N.S.	1	1.15	0.64	0.63	0.72	0.69	2.07	0.69	0.63
time (sec)	N/A	0.297	0.046	1.135	0.231	0.271	0.375	0.294	14.250

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	46	54	42	44	43	78	44	43
N.S.	1	0.85	1.00	0.78	0.81	0.80	1.44	0.81	0.80
time (sec)	N/A	0.183	0.015	1.276	0.244	0.270	0.508	0.319	13.988

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	103	55	55	59	56	184	60	53
N.S.	1	1.17	0.62	0.62	0.67	0.64	2.09	0.68	0.60
time (sec)	N/A	0.381	0.062	1.148	0.260	0.279	0.710	0.293	14.508

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	51	199	0	80	0	0	42
N.S.	1	1.08	0.78	3.06	0.00	1.23	0.00	0.00	0.65
time (sec)	N/A	0.289	0.124	4.632	0.000	0.096	0.000	0.000	14.502

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	40	202	0	74	0	0	42
N.S.	1	1.00	0.95	4.81	0.00	1.76	0.00	0.00	1.00
time (sec)	N/A	0.219	0.064	2.886	0.000	0.102	0.000	0.000	14.441

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	68	0	0	35
N.S.	1	1.00	0.86	4.26	0.00	1.62	0.00	0.00	0.83
time (sec)	N/A	0.222	0.052	2.154	0.000	0.091	0.000	0.000	14.407

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	57	0	0	15
N.S.	1	1.00	1.00	8.31	0.00	3.56	0.00	0.00	0.94
time (sec)	N/A	0.157	0.033	1.612	0.000	0.086	0.000	0.000	14.414

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	51	0	0	15
N.S.	1	1.00	1.00	1.12	0.00	3.19	0.00	0.00	0.94
time (sec)	N/A	0.158	0.038	0.246	0.000	0.090	0.000	0.000	13.594

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	182	0	93	0	0	42
N.S.	1	1.00	1.00	4.79	0.00	2.45	0.00	0.00	1.11
time (sec)	N/A	0.219	0.074	1.302	0.000	0.096	0.000	0.000	14.078

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	213	0	92	0	0	42
N.S.	1	1.00	0.86	5.07	0.00	2.19	0.00	0.00	1.00
time (sec)	N/A	0.221	0.072	1.456	0.000	0.087	0.000	0.000	14.248

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	66	59	358	0	110	0	0	42
N.S.	1	1.02	0.91	5.51	0.00	1.69	0.00	0.00	0.65
time (sec)	N/A	0.297	0.116	2.307	0.000	0.090	0.000	0.000	14.850

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	104	76	210	0	95	0	0	0
N.S.	1	1.06	0.78	2.14	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.417	0.126	4.013	0.000	0.096	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	213	0	91	0	0	0
N.S.	1	1.00	0.89	3.04	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.314	0.105	2.784	0.000	0.096	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	58	190	0	77	0	0	0
N.S.	1	1.00	0.83	2.71	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.311	0.065	2.322	0.000	0.093	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	142	0	63	0	0	0
N.S.	1	1.00	1.00	3.74	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.230	0.036	1.915	0.000	0.089	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	0	60	0	0	33
N.S.	1	1.00	1.00	1.42	0.00	1.58	0.00	0.00	0.87
time (sec)	N/A	0.224	0.042	0.388	0.000	0.082	0.000	0.000	0.157

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	198	0	104	0	0	0
N.S.	1	1.00	0.74	2.91	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.312	0.048	1.450	0.000	0.093	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	103	0	0	0
N.S.	1	1.00	0.71	3.35	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.310	0.082	1.646	0.000	0.093	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	68	366	0	121	0	0	0
N.S.	1	1.04	0.68	3.66	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.412	0.123	2.339	0.000	0.097	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.178	0.065	0.000	0.000	0.000	0.000	0.000	13.888

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.179	0.035	0.000	0.000	0.000	0.000	0.000	15.288

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.177	0.035	0.000	0.000	0.000	0.000	0.000	15.053

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.180	0.031	0.000	0.000	0.000	0.000	0.000	14.948

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.175	0.028	0.000	0.000	0.000	0.000	0.000	15.290

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.175	0.029	0.000	0.000	0.000	0.000	0.000	15.332

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	57
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.190	0.050	0.000	0.000	0.000	0.000	0.000	14.946

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	56	33	32	31	40	0	34	0
N.S.	1	1.06	0.62	0.60	0.58	0.75	0.00	0.64	0.00
time (sec)	N/A	0.359	0.015	1.229	0.365	0.296	0.000	0.347	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	23	24	17	26	0	17	0
N.S.	1	1.00	0.68	0.71	0.50	0.76	0.00	0.50	0.00
time (sec)	N/A	0.272	0.008	0.783	0.388	0.301	0.000	0.319	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	6	15	15	9	46
N.S.	1	1.00	1.00	1.15	0.46	1.15	1.15	0.69	3.54
time (sec)	N/A	0.205	0.005	0.819	0.375	0.296	0.230	0.378	14.477

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	48	38	65	0	0	0
N.S.	1	1.00	1.00	3.00	2.38	4.06	0.00	0.00	0.00
time (sec)	N/A	0.220	0.004	0.905	0.385	0.284	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	26	70	304	40	0	20	0
N.S.	1	1.00	0.62	1.67	7.24	0.95	0.00	0.48	0.00
time (sec)	N/A	0.293	0.013	1.045	0.395	0.284	0.000	0.355	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	69	36	89	933	49	0	29	0
N.S.	1	1.13	0.59	1.46	15.30	0.80	0.00	0.48	0.00
time (sec)	N/A	0.381	0.038	1.077	0.622	0.300	0.000	0.311	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	95	61	117	0	92	0	0	0
N.S.	1	0.81	0.52	1.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.475	0.152	12.326	0.000	0.122	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	59	50	271	0	66	0	0	0
N.S.	1	0.88	0.75	4.04	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.335	0.099	6.035	0.000	0.123	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	41	37	92	0	59	0	0	0
N.S.	1	0.93	0.84	2.09	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.274	0.046	2.910	0.000	0.089	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	37	31	244	0	71	0	0	0
N.S.	1	0.88	0.74	5.81	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.273	0.039	3.470	0.000	0.101	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	61	44	101	0	74	0	0	0
N.S.	1	0.86	0.62	1.42	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.334	0.086	5.052	0.000	0.100	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	91	57	297	0	92	0	0	0
N.S.	1	0.78	0.49	2.54	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.476	0.135	8.308	0.000	0.131	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	92	53	56	85	68	0	57	0
N.S.	1	0.70	0.40	0.42	0.64	0.52	0.00	0.43	0.00
time (sec)	N/A	0.477	0.151	13.264	0.329	0.278	0.000	0.331	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	60	38	38	55	42	0	25	0
N.S.	1	0.77	0.49	0.49	0.71	0.54	0.00	0.32	0.00
time (sec)	N/A	0.337	0.097	0.821	0.352	0.291	0.000	0.295	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	29	25	20	22	21	0	13	0
N.S.	1	0.81	0.69	0.56	0.61	0.58	0.00	0.36	0.00
time (sec)	N/A	0.224	0.034	1.013	0.471	0.260	0.000	0.314	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	6	18	0	6	6
N.S.	1	1.00	1.00	0.93	0.40	1.20	0.00	0.40	0.40
time (sec)	N/A	0.220	0.008	1.138	0.343	0.268	0.000	0.290	14.202

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	40	30	33	22	33	0	22	36
N.S.	1	0.60	0.45	0.49	0.33	0.49	0.00	0.33	0.54
time (sec)	N/A	0.249	0.038	0.855	0.335	0.270	0.000	0.286	14.808

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	56	47	46	34	45	0	34	306
N.S.	1	0.48	0.40	0.39	0.29	0.38	0.00	0.29	2.62
time (sec)	N/A	0.266	0.074	0.956	0.329	0.291	0.000	0.291	18.029

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	88	68	0	0	0	0	0	0
N.S.	1	1.19	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	72	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	44	0	15	61	300	40
N.S.	1	1.00	1.00	1.83	0.00	0.62	2.54	12.50	1.67
time (sec)	N/A	0.239	0.047	1.199	0.000	0.264	0.470	0.565	14.224

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	74	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	142	83	234	0	98	0	0	0
N.S.	1	1.15	0.67	1.90	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.562	0.355	6.764	0.000	0.113	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	108	75	221	0	101	0	0	0
N.S.	1	1.11	0.77	2.28	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.450	0.286	4.485	0.000	0.122	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	108	73	208	0	88	0	0	0
N.S.	1	1.14	0.77	2.19	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.435	0.263	3.465	0.000	0.114	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	62	211	0	88	0	0	0
N.S.	1	1.07	0.90	3.06	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.337	0.163	2.844	0.000	0.122	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	74	61	188	0	76	0	0	0
N.S.	1	1.10	0.91	2.81	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.332	0.079	2.211	0.000	0.088	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	142	0	63	0	0	0
N.S.	1	1.00	1.00	3.74	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.228	0.039	2.045	0.000	0.091	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	142	0	57	0	0	0
N.S.	1	1.00	1.00	3.64	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.262	0.048	1.437	0.000	0.088	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	72	48	196	0	101	0	0	0
N.S.	1	1.14	0.76	3.11	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.360	0.200	1.782	0.000	0.112	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	49	239	0	100	0	0	0
N.S.	1	1.09	0.70	3.41	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.352	0.224	1.837	0.000	0.105	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	108	69	364	0	118	0	0	0
N.S.	1	1.14	0.73	3.83	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.461	0.375	2.546	0.000	0.095	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	112	69	396	0	112	0	0	0
N.S.	1	1.14	0.70	4.04	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.467	0.336	2.135	0.000	0.097	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	144	79	414	0	128	0	0	0
N.S.	1	1.17	0.64	3.37	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.583	0.388	3.063	0.000	0.098	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	142	83	236	0	102	0	0	0
N.S.	1	1.13	0.66	1.87	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.553	0.264	5.704	0.000	0.102	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	108	75	223	0	103	0	0	0
N.S.	1	1.14	0.79	2.35	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.436	0.266	4.483	0.000	0.102	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	108	73	210	0	91	0	0	0
N.S.	1	1.10	0.74	2.14	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.432	0.202	3.411	0.000	0.100	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	74	65	213	0	89	0	0	0
N.S.	1	1.10	0.97	3.18	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.335	0.108	2.671	0.000	0.157	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	58	190	0	77	0	0	0
N.S.	1	1.00	0.83	2.71	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.314	0.045	2.162	0.000	0.095	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	144	0	63	0	0	0
N.S.	1	1.00	1.00	3.69	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.267	0.014	1.798	0.000	0.097	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	57	0	0	0
N.S.	1	1.00	1.00	3.51	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.268	0.025	1.269	0.000	0.102	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	72	50	198	0	102	0	0	0
N.S.	1	1.09	0.76	3.00	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.353	0.222	1.782	0.000	0.112	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	76	51	241	0	101	0	0	0
N.S.	1	1.06	0.71	3.35	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.355	0.177	1.781	0.000	0.089	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	108	69	365	0	119	0	0	0
N.S.	1	1.10	0.70	3.72	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.458	0.267	2.375	0.000	0.105	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	112	69	398	0	115	0	0	0
N.S.	1	1.12	0.69	3.98	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.453	0.374	2.136	0.000	0.104	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	144	79	416	0	132	0	0	0
N.S.	1	1.14	0.63	3.30	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.579	0.553	2.990	0.000	0.105	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	142	83	236	0	108	0	0	0
N.S.	1	1.14	0.66	1.89	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.548	0.364	10.455	0.000	0.112	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	108	75	223	0	107	0	0	0
N.S.	1	1.10	0.77	2.28	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.428	0.256	5.656	0.000	0.105	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	108	76	210	0	95	0	0	0
N.S.	1	1.11	0.78	2.16	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.433	0.145	3.607	0.000	0.097	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	213	0	91	0	0	0
N.S.	1	1.00	0.89	3.04	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.309	0.093	2.853	0.000	0.108	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	190	0	79	0	0	0
N.S.	1	1.00	0.82	2.64	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.344	0.048	2.771	0.000	0.099	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	63	0	0	0
N.S.	1	1.00	1.00	3.51	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.266	0.017	6.788	0.000	0.108	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	144	0	57	0	0	0
N.S.	1	1.00	0.93	3.51	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.261	0.208	27.801	0.000	0.093	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	72	50	198	0	104	0	0	0
N.S.	1	1.06	0.74	2.91	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.360	0.177	98.968	0.000	0.089	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	76	51	241	0	103	0	0	0
N.S.	1	1.06	0.71	3.35	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.356	0.251	1.135	0.000	0.105	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	69	367	0	123	0	0	0
N.S.	1	1.08	0.69	3.67	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.462	0.431	1.673	0.000	0.114	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	112	69	398	0	119	0	0	0
N.S.	1	1.12	0.69	3.98	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.457	0.682	1.065	0.000	0.101	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	144	79	0	0	138	0	0	0
N.S.	1	1.12	0.62	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.598	0.852	180.000	0.000	0.104	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	104	76	210	0	95	0	0	0
N.S.	1	1.06	0.78	2.14	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.408	0.086	3.615	0.000	0.097	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	142	73	233	0	101	0	0	0
N.S.	1	1.14	0.58	1.86	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.552	0.275	4.563	0.000	0.109	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	71	220	0	104	0	0	0
N.S.	1	1.08	0.71	2.20	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.436	0.227	3.905	0.000	0.120	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	108	63	207	0	91	0	0	0
N.S.	1	1.11	0.65	2.13	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.436	0.197	3.197	0.000	0.109	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	58	210	0	91	0	0	0
N.S.	1	1.03	0.81	2.92	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.341	0.190	2.570	0.000	0.096	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	51	187	0	79	0	0	58
N.S.	1	1.07	0.74	2.71	0.00	1.14	0.00	0.00	0.84
time (sec)	N/A	0.328	0.152	1.791	0.000	0.094	0.000	0.000	13.170

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	141	0	66	0	0	33
N.S.	1	1.00	1.00	3.44	0.00	1.61	0.00	0.00	0.80
time (sec)	N/A	0.248	0.016	1.725	0.000	0.095	0.000	0.000	0.137

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	0	60	0	0	33
N.S.	1	1.00	1.00	1.42	0.00	1.58	0.00	0.00	0.87
time (sec)	N/A	0.236	0.016	0.476	0.000	0.088	0.000	0.000	0.116

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	47	195	0	104	0	0	0
N.S.	1	1.08	0.72	3.00	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.353	0.063	1.904	0.000	0.095	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	76	48	238	0	103	0	0	0
N.S.	1	1.13	0.72	3.55	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.365	0.094	1.790	0.000	0.102	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	108	65	367	0	121	0	0	0
N.S.	1	1.11	0.67	3.78	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.467	0.231	2.418	0.000	0.099	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	112	63	267	0	115	0	0	0
N.S.	1	1.18	0.66	2.81	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.461	0.256	2.524	0.000	0.103	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	144	77	413	0	131	0	0	0
N.S.	1	1.15	0.62	3.30	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.580	0.447	3.185	0.000	0.100	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	142	76	236	0	101	0	0	0
N.S.	1	1.11	0.59	1.84	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.566	0.423	5.527	0.000	0.106	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	74	223	0	104	0	0	0
N.S.	1	1.08	0.74	2.23	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.443	0.279	4.592	0.000	0.107	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	66	210	0	91	0	0	0
N.S.	1	1.08	0.66	2.10	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.440	0.215	3.416	0.000	0.126	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	61	213	0	91	0	0	0
N.S.	1	1.03	0.85	2.96	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.338	0.188	2.853	0.000	0.105	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	54	190	0	79	0	0	0
N.S.	1	1.03	0.75	2.64	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.338	0.186	2.251	0.000	0.095	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	66	0	0	0
N.S.	1	1.00	1.00	3.51	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.246	0.018	2.027	0.000	0.097	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	60	0	0	0
N.S.	1	1.00	1.00	3.51	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.253	0.025	1.393	0.000	0.083	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	198	0	104	0	0	0
N.S.	1	1.00	0.74	2.91	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.320	0.032	1.742	0.000	0.099	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	51	241	0	103	0	0	0
N.S.	1	1.07	0.74	3.49	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.353	0.055	2.150	0.000	0.095	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	108	68	367	0	121	0	0	0
N.S.	1	1.10	0.69	3.74	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.467	0.133	2.653	0.000	0.098	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	112	66	398	0	115	0	0	0
N.S.	1	1.15	0.68	4.10	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.464	0.257	2.339	0.000	0.128	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	144	80	416	0	131	0	0	0
N.S.	1	1.14	0.63	3.30	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.581	0.346	3.118	0.000	0.107	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	142	76	236	0	101	0	0	0
N.S.	1	1.11	0.59	1.84	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.553	0.632	5.879	0.000	0.104	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	74	223	0	104	0	0	0
N.S.	1	1.08	0.74	2.23	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.436	0.515	4.423	0.000	0.104	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	66	210	0	91	0	0	0
N.S.	1	1.08	0.66	2.10	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.446	0.318	3.774	0.000	0.097	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	61	213	0	91	0	0	0
N.S.	1	1.03	0.85	2.96	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.341	0.229	2.813	0.000	0.100	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	54	190	0	79	0	0	0
N.S.	1	1.03	0.75	2.64	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.343	0.213	2.445	0.000	0.099	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	66	0	0	0
N.S.	1	1.00	1.00	3.51	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.256	0.018	1.914	0.000	0.100	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	144	0	60	0	0	0
N.S.	1	1.00	0.93	3.51	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.249	0.197	1.387	0.000	0.088	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	72	50	198	0	104	0	0	0
N.S.	1	1.06	0.74	2.91	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.340	0.040	1.742	0.000	0.089	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	103	0	0	0
N.S.	1	1.00	0.71	3.35	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.317	0.046	1.664	0.000	0.094	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	106	68	367	0	121	0	0	0
N.S.	1	1.09	0.70	3.78	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.441	0.100	2.785	0.000	0.095	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	112	66	398	0	115	0	0	0
N.S.	1	1.14	0.67	4.06	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.461	0.169	2.233	0.000	0.098	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	144	80	416	0	131	0	0	0
N.S.	1	1.15	0.64	3.33	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.590	0.394	3.171	0.000	0.108	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	68	367	0	121	0	0	0
N.S.	1	1.04	0.68	3.67	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.417	0.015	2.587	0.000	0.105	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	74	55	62	49	176	0	203	75
N.S.	1	0.76	0.56	0.63	0.50	1.80	0.00	2.07	0.77
time (sec)	N/A	0.285	0.244	3.099	0.393	0.343	0.000	2.076	14.994

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	48	45	40	42	39	0	71047	57
N.S.	1	0.69	0.64	0.57	0.60	0.56	0.00	1014.96	0.81
time (sec)	N/A	0.228	0.101	3.174	0.459	0.303	0.000	9.095	0.815

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	48	45	42	25	150	121	105	62
N.S.	1	0.76	0.71	0.67	0.40	2.38	1.92	1.67	0.98
time (sec)	N/A	0.213	0.153	3.069	0.388	0.342	32.912	1.465	14.574

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	13	28	60	31	44
N.S.	1	1.00	1.00	0.91	0.41	0.88	1.88	0.97	1.38
time (sec)	N/A	0.197	0.116	3.070	0.439	0.281	1.301	0.540	14.172

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	26	94	22	0	20
N.S.	1	1.00	1.00	0.88	1.08	3.92	0.92	0.00	0.83
time (sec)	N/A	0.148	0.030	2.269	0.337	0.295	0.615	0.000	0.109

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	40	65	113	0	0	0
N.S.	1	1.00	1.00	1.21	1.97	3.42	0.00	0.00	0.00
time (sec)	N/A	0.200	0.015	3.228	0.449	0.390	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	54	28	0	0	59
N.S.	1	1.00	1.00	0.91	1.69	0.88	0.00	0.00	1.84
time (sec)	N/A	0.218	0.024	2.915	0.436	0.267	0.000	0.000	14.109

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	57	52	84	661	201	0	0	0
N.S.	1	0.79	0.72	1.17	9.18	2.79	0.00	0.00	0.00
time (sec)	N/A	0.282	0.054	3.088	0.443	0.341	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	48	45	42	294	41	0	0	128
N.S.	1	0.69	0.64	0.60	4.20	0.59	0.00	0.00	1.83
time (sec)	N/A	0.245	0.098	2.960	0.447	0.305	0.000	0.000	16.116

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	83	66	103	1656	227	0	0	0
N.S.	1	0.78	0.62	0.96	15.48	2.12	0.00	0.00	0.00
time (sec)	N/A	0.380	0.126	2.977	0.483	0.317	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	75	55	63	53	183	0	204	76
N.S.	1	0.74	0.54	0.62	0.52	1.81	0.00	2.02	0.75
time (sec)	N/A	0.284	0.203	2.814	0.399	0.311	0.000	2.432	14.934

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	49	45	41	45	43	0	71048	58
N.S.	1	0.68	0.62	0.57	0.62	0.60	0.00	986.78	0.81
time (sec)	N/A	0.229	0.138	2.793	0.398	0.281	0.000	11.242	0.680

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	49	45	43	28	153	0	106	63
N.S.	1	0.75	0.69	0.66	0.43	2.35	0.00	1.63	0.97
time (sec)	N/A	0.212	0.147	3.043	0.395	0.391	0.000	1.198	14.497

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	13	29	46	0	29
N.S.	1	1.00	0.97	0.91	0.39	0.88	1.39	0.00	0.88
time (sec)	N/A	0.202	0.131	2.968	0.370	0.297	15.888	0.000	0.256

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	26	95	22	0	21
N.S.	1	1.00	1.00	0.88	1.04	3.80	0.88	0.00	0.84
time (sec)	N/A	0.154	0.042	2.402	0.368	0.327	9.452	0.000	13.482

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	41	68	114	0	0	0
N.S.	1	1.00	0.97	1.21	2.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.211	0.019	2.773	0.422	0.304	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	30	54	29	0	0	60
N.S.	1	1.00	0.97	0.91	1.64	0.88	0.00	0.00	1.82
time (sec)	N/A	0.221	0.024	2.845	0.403	0.282	0.000	0.000	13.914

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	58	52	85	691	204	0	0	0
N.S.	1	0.78	0.70	1.15	9.34	2.76	0.00	0.00	0.00
time (sec)	N/A	0.285	0.057	2.869	0.438	0.339	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	49	45	43	299	42	0	0	129
N.S.	1	0.68	0.62	0.60	4.15	0.58	0.00	0.00	1.79
time (sec)	N/A	0.232	0.071	2.771	0.414	0.283	0.000	0.000	15.349

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	84	67	104	1742	234	0	0	0
N.S.	1	0.76	0.61	0.95	15.84	2.13	0.00	0.00	0.00
time (sec)	N/A	0.367	0.103	3.033	0.481	0.391	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	63	57	55	77	61	0	0	73
N.S.	1	0.54	0.49	0.47	0.66	0.53	0.00	0.00	0.63
time (sec)	N/A	0.232	0.164	3.744	0.406	0.297	0.000	0.000	15.647

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	77	55	65	59	193	0	203	78
N.S.	1	0.72	0.51	0.61	0.55	1.80	0.00	1.90	0.73
time (sec)	N/A	0.285	0.257	2.881	0.403	0.299	0.000	2.478	15.097

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	51	45	43	49	47	0	71047	60
N.S.	1	0.67	0.59	0.57	0.64	0.62	0.00	934.83	0.79
time (sec)	N/A	0.224	0.152	2.898	0.430	0.276	0.000	8.371	0.669

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	51	45	45	32	159	0	0	40
N.S.	1	0.74	0.65	0.65	0.46	2.30	0.00	0.00	0.58
time (sec)	N/A	0.209	0.170	3.197	0.448	0.326	0.000	0.000	0.420

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	13	31	0	0	31
N.S.	1	1.00	0.91	0.91	0.37	0.89	0.00	0.00	0.89
time (sec)	N/A	0.197	0.145	2.743	0.412	0.345	0.000	0.000	14.423

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	24	26	97	0	0	23
N.S.	1	1.00	0.89	0.89	0.96	3.59	0.00	0.00	0.85
time (sec)	N/A	0.149	0.047	2.335	0.381	0.309	0.000	0.000	0.099

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	43	72	116	0	0	0
N.S.	1	1.00	0.92	1.19	2.00	3.22	0.00	0.00	0.00
time (sec)	N/A	0.213	0.026	2.652	0.408	0.310	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	54	31	0	0	62
N.S.	1	1.00	0.91	0.91	1.54	0.89	0.00	0.00	1.77
time (sec)	N/A	0.222	0.023	2.753	0.418	0.285	0.000	0.000	14.493

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	60	52	87	747	210	0	0	0
N.S.	1	0.77	0.67	1.12	9.58	2.69	0.00	0.00	0.00
time (sec)	N/A	0.291	0.065	2.814	0.399	0.313	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	51	45	45	311	46	0	0	131
N.S.	1	0.67	0.59	0.59	4.09	0.61	0.00	0.00	1.72
time (sec)	N/A	0.245	0.066	2.666	0.383	0.352	0.000	0.000	14.507

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	86	66	106	1914	244	0	0	0
N.S.	1	0.74	0.57	0.91	16.50	2.10	0.00	0.00	0.00
time (sec)	N/A	0.387	0.151	3.241	0.505	0.319	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	60	57	52	68	54	0	0	73
N.S.	1	0.56	0.53	0.49	0.64	0.50	0.00	0.00	0.68
time (sec)	N/A	0.226	0.118	2.912	0.412	0.277	0.000	0.000	14.695

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	74	55	62	49	182	0	0	78
N.S.	1	0.76	0.56	0.63	0.50	1.86	0.00	0.00	0.80
time (sec)	N/A	0.279	0.245	3.167	0.394	0.297	0.000	0.000	14.392

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	48	45	40	42	42	0	0	60
N.S.	1	0.69	0.64	0.57	0.60	0.60	0.00	0.00	0.86
time (sec)	N/A	0.226	0.087	2.980	0.401	0.303	0.000	0.000	0.691

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	48	45	42	25	157	0	0	65
N.S.	1	0.76	0.71	0.67	0.40	2.49	0.00	0.00	1.03
time (sec)	N/A	0.208	0.163	3.095	0.403	0.305	0.000	0.000	14.015

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	13	31	46	0	47
N.S.	1	1.00	1.00	0.91	0.41	0.97	1.44	0.00	1.47
time (sec)	N/A	0.200	0.123	2.837	0.378	0.284	18.278	0.000	14.623

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	26	97	22	0	37
N.S.	1	1.00	1.00	0.88	1.08	4.04	0.92	0.00	1.54
time (sec)	N/A	0.148	0.029	2.344	0.308	0.302	0.670	0.000	0.315

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	40	65	116	0	0	0
N.S.	1	1.00	1.00	1.21	1.97	3.52	0.00	0.00	0.00
time (sec)	N/A	0.208	0.017	2.879	0.394	0.311	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	59	31	0	0	62
N.S.	1	1.00	1.00	0.91	1.84	0.97	0.00	0.00	1.94
time (sec)	N/A	0.210	0.026	2.850	0.377	0.292	0.000	0.000	14.506

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	57	52	84	661	207	0	0	0
N.S.	1	0.79	0.72	1.17	9.18	2.88	0.00	0.00	0.00
time (sec)	N/A	0.280	0.040	3.315	0.414	0.298	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	48	45	42	294	44	0	0	131
N.S.	1	0.69	0.64	0.60	4.20	0.63	0.00	0.00	1.87
time (sec)	N/A	0.230	0.101	2.827	0.408	0.350	0.000	0.000	15.040

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	83	66	103	1656	233	0	0	0
N.S.	1	0.78	0.62	0.96	15.48	2.18	0.00	0.00	0.00
time (sec)	N/A	0.374	0.072	3.283	0.412	0.344	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	77	55	65	49	182	0	0	78
N.S.	1	0.72	0.51	0.61	0.46	1.70	0.00	0.00	0.73
time (sec)	N/A	0.282	0.411	2.847	0.413	0.329	0.000	0.000	15.224

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	51	45	43	42	42	0	0	60
N.S.	1	0.67	0.59	0.57	0.55	0.55	0.00	0.00	0.79
time (sec)	N/A	0.231	0.077	3.059	0.369	0.358	0.000	0.000	0.689

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	51	45	45	25	157	0	0	65
N.S.	1	0.74	0.65	0.65	0.36	2.28	0.00	0.00	0.94
time (sec)	N/A	0.213	0.185	2.905	0.397	0.300	0.000	0.000	14.867

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	13	31	0	0	47
N.S.	1	1.00	0.91	0.91	0.37	0.89	0.00	0.00	1.34
time (sec)	N/A	0.203	0.150	2.868	0.374	0.279	0.000	0.000	0.353

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	26	97	22	0	37
N.S.	1	1.00	1.00	0.89	0.96	3.59	0.81	0.00	1.37
time (sec)	N/A	0.152	0.036	2.253	0.325	0.411	10.108	0.000	0.296

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	43	65	116	0	0	0
N.S.	1	1.00	0.92	1.19	1.81	3.22	0.00	0.00	0.00
time (sec)	N/A	0.207	0.018	2.938	0.366	0.312	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	67	31	0	0	62
N.S.	1	1.00	0.91	0.91	1.91	0.89	0.00	0.00	1.77
time (sec)	N/A	0.215	0.019	2.985	0.394	0.298	0.000	0.000	14.559

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	60	52	87	670	207	0	0	0
N.S.	1	0.77	0.67	1.12	8.59	2.65	0.00	0.00	0.00
time (sec)	N/A	0.286	0.040	2.896	0.395	0.346	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	51	45	45	311	44	0	0	131
N.S.	1	0.67	0.59	0.59	4.09	0.58	0.00	0.00	1.72
time (sec)	N/A	0.234	0.049	3.241	0.413	0.266	0.000	0.000	16.408

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	86	66	106	1679	233	0	0	0
N.S.	1	0.74	0.57	0.91	14.47	2.01	0.00	0.00	0.00
time (sec)	N/A	0.385	0.055	2.937	0.418	0.332	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	77	58	65	49	182	0	0	78
N.S.	1	0.72	0.54	0.61	0.46	1.70	0.00	0.00	0.73
time (sec)	N/A	0.281	0.654	3.222	0.402	0.357	0.000	0.000	15.217

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	51	48	43	42	42	0	0	60
N.S.	1	0.67	0.63	0.57	0.55	0.55	0.00	0.00	0.79
time (sec)	N/A	0.230	0.082	2.762	0.378	0.313	0.000	0.000	0.678

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	51	48	45	25	157	0	0	65
N.S.	1	0.74	0.70	0.65	0.36	2.28	0.00	0.00	0.94
time (sec)	N/A	0.214	0.228	2.922	0.386	0.320	0.000	0.000	14.050

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	13	31	0	0	47
N.S.	1	1.00	1.00	0.91	0.37	0.89	0.00	0.00	1.34
time (sec)	N/A	0.210	0.161	2.867	0.388	0.273	0.000	0.000	13.901

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	26	97	0	0	37
N.S.	1	1.00	1.00	0.89	0.96	3.59	0.00	0.00	1.37
time (sec)	N/A	0.154	0.031	2.253	0.368	0.307	0.000	0.000	0.297

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	43	65	116	0	0	0
N.S.	1	1.00	0.92	1.19	1.81	3.22	0.00	0.00	0.00
time (sec)	N/A	0.211	0.029	2.756	0.384	0.333	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	32	67	31	0	0	87
N.S.	1	1.00	0.91	0.91	1.91	0.89	0.00	0.00	2.49
time (sec)	N/A	0.218	0.025	3.033	0.383	0.328	0.000	0.000	14.085

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	60	55	87	688	207	0	0	0
N.S.	1	0.77	0.71	1.12	8.82	2.65	0.00	0.00	0.00
time (sec)	N/A	0.290	0.057	3.194	0.452	0.302	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	51	45	45	343	44	0	0	131
N.S.	1	0.67	0.59	0.59	4.51	0.58	0.00	0.00	1.72
time (sec)	N/A	0.242	0.074	2.885	0.386	0.284	0.000	0.000	15.126

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	86	66	106	1729	233	0	0	0
N.S.	1	0.74	0.57	0.91	14.91	2.01	0.00	0.00	0.00
time (sec)	N/A	0.388	0.083	3.392	0.435	0.417	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.196	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.066	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.249	0.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.060	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.042	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.194	0.000	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.086	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.003	0.000	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	56	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.003	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.003	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	0.120	0.000	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.003	0.000	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	77	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.098	0.000	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	72	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	70	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	63	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	67	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.064	0.000	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.149	0.000	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	0.122	0.000	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	0.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.310	11.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	0	13	15
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.00	0.87	1.00
time (sec)	N/A	0.183	0.025	0.722	0.310	0.277	0.000	0.328	13.969

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	23	0	13	15
N.S.	1	1.00	1.00	0.82	0.76	1.35	0.00	0.76	0.88
time (sec)	N/A	0.185	0.030	0.602	0.217	0.280	0.000	0.337	14.252

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	53	88	0	67	0	0	0
N.S.	1	1.00	0.79	1.31	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.268	1.173	0.000	0.089	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	61	142	0	83	0	0	0
N.S.	1	1.00	0.91	2.12	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.328	0.265	1.211	0.000	0.097	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	22	0	14	16
N.S.	1	1.00	0.86	0.67	0.62	1.05	0.00	0.67	0.76
time (sec)	N/A	0.195	0.029	8.130	0.205	0.292	0.000	0.311	13.722

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	32	27	26	25	23	0	24	0
N.S.	1	0.97	0.82	0.79	0.76	0.70	0.00	0.73	0.00
time (sec)	N/A	0.213	0.068	0.737	0.269	0.316	0.000	0.305	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	29	26	25	33	0	26	0
N.S.	1	0.97	0.83	0.74	0.71	0.94	0.00	0.74	0.00
time (sec)	N/A	0.208	0.072	0.913	0.194	0.286	0.000	0.319	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	63	100	0	78	0	0	0
N.S.	1	1.05	0.68	1.09	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.432	0.306	1.222	0.000	0.093	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	63	152	0	95	0	0	0
N.S.	1	1.05	0.68	1.65	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.436	0.550	1.132	0.000	0.103	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	0	6	8
N.S.	1	1.00	1.00	0.70	0.60	0.60	0.00	0.60	0.80
time (sec)	N/A	0.164	0.011	0.715	0.209	0.244	0.000	0.301	13.425

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	47	24	41	95	0	42	0
N.S.	1	1.12	1.47	0.75	1.28	2.97	0.00	1.31	0.00
time (sec)	N/A	0.204	0.030	0.928	0.451	0.334	0.000	0.312	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	36	50	43	41	97	0	42	0
N.S.	1	1.16	1.61	1.39	1.32	3.13	0.00	1.35	0.00
time (sec)	N/A	0.207	0.037	1.015	0.410	0.320	0.000	0.302	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	49	123	0	82	0	0	0
N.S.	1	1.00	0.80	2.02	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.330	0.322	1.273	0.000	0.131	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	54	177	0	96	0	0	0
N.S.	1	1.00	0.87	2.85	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.316	0.282	1.365	0.000	0.100	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	69	74	73	65	131	0	66	0
N.S.	1	1.11	1.19	1.18	1.05	2.11	0.00	1.06	0.00
time (sec)	N/A	0.230	0.091	1.138	0.353	0.320	0.000	0.315	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	69	33	71	63	141	0	64	0
N.S.	1	1.11	0.53	1.15	1.02	2.27	0.00	1.03	0.00
time (sec)	N/A	0.231	0.035	1.056	0.335	0.311	0.000	0.312	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	91	64	168	0	98	0	0	0
N.S.	1	0.99	0.70	1.83	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.441	0.530	1.323	0.000	0.096	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	76	160	0	114	0	0	0
N.S.	1	1.00	0.83	1.74	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.461	0.354	1.767	0.000	0.096	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	105	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	32.631	0.000	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	10.466	0.000	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	10.540	0.000	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	68	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	10.641	0.000	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	10.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	312	0	0	0	0	0	0
N.S.	1	1.00	3.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	2.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	314	0	0	0	0	0	0
N.S.	1	1.00	3.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.335	1.001	0.000	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	314	0	0	0	0	0	0
N.S.	1	1.00	3.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.344	1.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	316	0	0	0	0	0	0
N.S.	1	1.00	3.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	1.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	76	94	0	0	0	0	0	0
N.S.	1	0.97	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	16.299	0.000	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	94	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	10.756	0.000	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	94	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	9.287	0.000	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	96	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	8.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	225	0	0	0	0	0	0
N.S.	1	1.00	2.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.328	28.334	0.000	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	116	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	11.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	125	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	11.379	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [51] had the largest ratio of [1.3000000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	3	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	5	5	1.11	8	0.625
5	A	4	3	0.93	8	0.375
6	A	7	7	1.15	8	0.875
7	A	4	3	0.85	8	0.375
8	A	9	9	1.17	8	1.125
9	A	6	6	1.08	10	0.600
10	A	4	4	1.00	10	0.400
11	A	4	4	1.00	10	0.400
12	A	2	2	1.00	10	0.200
13	A	2	2	1.00	10	0.200
14	A	4	4	1.00	10	0.400
15	A	4	4	1.00	10	0.400
16	A	6	6	1.02	10	0.600
17	A	8	8	1.06	12	0.667
18	A	6	6	1.00	12	0.500
19	A	6	6	1.00	12	0.500
20	A	4	4	1.00	12	0.333
21	A	4	4	1.00	12	0.333
22	A	6	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	6	6	1.00	12	0.500
24	A	8	8	1.04	12	0.667
25	A	2	2	1.00	10	0.200
26	A	2	2	1.00	10	0.200
27	A	2	2	1.00	10	0.200
28	A	2	2	1.00	10	0.200
29	A	2	2	1.00	10	0.200
30	A	2	2	1.00	10	0.200
31	A	2	2	1.00	12	0.167
32	A	2	2	1.00	12	0.167
33	A	2	2	1.00	12	0.167
34	A	2	2	1.00	12	0.167
35	A	2	2	1.00	12	0.167
36	A	2	2	1.00	12	0.167
37	A	2	2	1.00	8	0.250
38	A	2	2	1.00	10	0.200
39	A	8	8	1.06	10	0.800
40	A	6	6	1.00	10	0.600
41	A	4	4	1.00	10	0.400
42	A	4	4	1.00	10	0.400
43	A	6	6	1.00	10	0.600
44	A	8	8	1.13	10	0.800
45	A	12	12	0.81	10	1.200
46	A	8	8	0.88	10	0.800
47	A	6	6	0.93	10	0.600
48	A	6	6	0.88	10	0.600
49	A	8	8	0.86	10	0.800
50	A	12	12	0.78	10	1.200
51	A	13	13	0.70	10	1.300
52	A	9	9	0.77	10	0.900
53	A	5	5	0.81	10	0.500
54	A	6	5	1.00	10	0.500
55	A	6	5	0.60	10	0.500
56	A	6	5	0.48	10	0.500

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	4	1.00	12	0.333
58	A	4	4	1.00	14	0.286
59	A	4	4	1.00	14	0.286
60	A	4	4	1.19	14	0.286
61	A	4	4	1.00	14	0.286
62	A	4	4	1.00	14	0.286
63	A	4	4	1.00	14	0.286
64	A	4	4	1.00	14	0.286
65	A	4	4	1.00	14	0.286
66	A	11	11	1.15	21	0.524
67	A	9	9	1.11	21	0.429
68	A	9	9	1.14	21	0.429
69	A	7	7	1.07	21	0.333
70	A	7	7	1.10	19	0.368
71	A	4	4	1.00	12	0.333
72	A	5	5	1.00	19	0.263
73	A	7	7	1.14	21	0.333
74	A	7	7	1.09	21	0.333
75	A	9	9	1.14	21	0.429
76	A	9	9	1.14	21	0.429
77	A	11	11	1.17	21	0.524
78	A	11	11	1.13	21	0.524
79	A	9	9	1.14	21	0.429
80	A	9	9	1.10	21	0.429
81	A	7	7	1.10	19	0.368
82	A	6	6	1.00	12	0.500
83	A	5	5	1.00	19	0.263
84	A	5	5	1.00	21	0.238
85	A	7	7	1.09	21	0.333
86	A	7	7	1.06	21	0.333
87	A	9	9	1.10	21	0.429
88	A	9	9	1.12	21	0.429
89	A	11	11	1.14	21	0.524
90	A	11	11	1.14	21	0.524

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	9	9	1.10	21	0.429
92	A	9	9	1.11	19	0.474
93	A	6	6	1.00	12	0.500
94	A	7	7	1.00	19	0.368
95	A	5	5	1.00	21	0.238
96	A	5	5	1.00	21	0.238
97	A	7	7	1.06	21	0.333
98	A	7	7	1.06	21	0.333
99	A	9	9	1.08	21	0.429
100	A	9	9	1.12	21	0.429
101	A	11	11	1.12	21	0.524
102	A	8	8	1.06	12	0.667
103	A	11	11	1.14	21	0.524
104	A	9	9	1.08	21	0.429
105	A	9	9	1.11	21	0.429
106	A	7	7	1.03	21	0.333
107	A	7	7	1.07	21	0.333
108	A	5	5	1.00	19	0.263
109	A	4	4	1.00	12	0.333
110	A	7	7	1.08	19	0.368
111	A	7	7	1.13	21	0.333
112	A	9	9	1.11	21	0.429
113	A	9	9	1.18	21	0.429
114	A	11	11	1.15	21	0.524
115	A	11	11	1.11	21	0.524
116	A	9	9	1.08	21	0.429
117	A	9	9	1.08	21	0.429
118	A	7	7	1.03	21	0.333
119	A	7	7	1.03	21	0.333
120	A	5	5	1.00	21	0.238
121	A	5	5	1.00	19	0.263
122	A	6	6	1.00	12	0.500
123	A	7	7	1.07	19	0.368
124	A	9	9	1.10	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	9	9	1.15	21	0.429
126	A	11	11	1.14	21	0.524
127	A	11	11	1.11	21	0.524
128	A	9	9	1.08	21	0.429
129	A	9	9	1.08	21	0.429
130	A	7	7	1.03	21	0.333
131	A	7	7	1.03	21	0.333
132	A	5	5	1.00	21	0.238
133	A	5	5	1.00	21	0.238
134	A	7	7	1.06	19	0.368
135	A	6	6	1.00	12	0.500
136	A	9	9	1.09	19	0.474
137	A	9	9	1.14	21	0.429
138	A	11	11	1.15	21	0.524
139	A	8	8	1.04	12	0.667
140	A	6	6	0.76	23	0.261
141	A	5	4	0.69	23	0.174
142	A	4	4	0.76	23	0.174
143	A	3	3	1.00	23	0.130
144	A	2	2	1.00	23	0.087
145	A	3	3	1.00	23	0.130
146	A	5	4	1.00	23	0.174
147	A	5	5	0.79	23	0.217
148	A	5	4	0.69	23	0.174
149	A	7	7	0.78	23	0.304
150	A	6	6	0.74	23	0.261
151	A	5	4	0.68	23	0.174
152	A	4	4	0.75	23	0.174
153	A	3	3	1.00	23	0.130
154	A	2	2	1.00	23	0.087
155	A	3	3	1.00	23	0.130
156	A	5	4	1.00	23	0.174
157	A	5	5	0.78	23	0.217
158	A	5	4	0.68	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	7	7	0.76	23	0.304
160	A	5	4	0.54	23	0.174
161	A	6	6	0.72	23	0.261
162	A	5	4	0.67	23	0.174
163	A	4	4	0.74	23	0.174
164	A	3	3	1.00	23	0.130
165	A	2	2	1.00	23	0.087
166	A	3	3	1.00	23	0.130
167	A	5	4	1.00	23	0.174
168	A	5	5	0.77	23	0.217
169	A	5	4	0.67	23	0.174
170	A	7	7	0.74	23	0.304
171	A	5	4	0.56	23	0.174
172	A	6	6	0.76	23	0.261
173	A	5	4	0.69	23	0.174
174	A	4	4	0.76	23	0.174
175	A	3	3	1.00	23	0.130
176	A	2	2	1.00	23	0.087
177	A	3	3	1.00	23	0.130
178	A	5	4	1.00	23	0.174
179	A	5	5	0.79	23	0.217
180	A	5	4	0.69	23	0.174
181	A	7	7	0.78	23	0.304
182	A	6	6	0.72	23	0.261
183	A	5	4	0.67	23	0.174
184	A	4	4	0.74	23	0.174
185	A	3	3	1.00	23	0.130
186	A	2	2	1.00	23	0.087
187	A	3	3	1.00	23	0.130
188	A	5	4	1.00	23	0.174
189	A	5	5	0.77	23	0.217
190	A	5	4	0.67	23	0.174
191	A	7	7	0.74	23	0.304
192	A	6	6	0.72	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	5	4	0.67	23	0.174
194	A	4	4	0.74	23	0.174
195	A	3	3	1.00	23	0.130
196	A	2	2	1.00	23	0.087
197	A	3	3	1.00	23	0.130
198	A	5	4	1.00	23	0.174
199	A	5	5	0.77	23	0.217
200	A	5	4	0.67	23	0.174
201	A	7	7	0.74	23	0.304
202	A	3	3	1.00	21	0.143
203	A	3	3	1.00	21	0.143
204	A	3	3	1.00	19	0.158
205	A	2	2	1.00	12	0.167
206	A	3	3	1.00	19	0.158
207	A	3	3	1.00	21	0.143
208	A	3	3	1.00	21	0.143
209	A	3	3	1.00	21	0.143
210	A	3	3	1.00	21	0.143
211	A	3	3	1.00	19	0.158
212	A	2	2	1.00	12	0.167
213	A	3	3	1.00	19	0.158
214	A	3	3	1.00	21	0.143
215	A	3	3	1.00	21	0.143
216	A	3	3	1.00	21	0.143
217	A	3	3	1.00	21	0.143
218	A	3	3	1.00	19	0.158
219	A	2	2	1.00	12	0.167
220	A	3	3	1.00	19	0.158
221	A	3	3	1.00	21	0.143
222	A	3	3	1.00	21	0.143
223	A	3	3	1.00	21	0.143
224	A	3	3	1.00	21	0.143
225	A	3	3	1.00	19	0.158
226	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
227	A	3	3	1.00	19	0.158
228	A	3	3	1.00	21	0.143
229	A	3	3	1.00	21	0.143
230	A	3	3	1.00	21	0.143
231	A	3	3	1.00	21	0.143
232	A	3	3	1.00	19	0.158
233	A	2	2	1.00	12	0.167
234	A	3	3	1.00	19	0.158
235	A	3	3	1.00	21	0.143
236	A	3	3	1.00	21	0.143
237	A	3	3	1.00	21	0.143
238	A	3	3	1.00	21	0.143
239	A	3	3	1.00	19	0.158
240	A	2	2	1.00	12	0.167
241	A	3	3	1.00	19	0.158
242	A	3	3	1.00	21	0.143
243	A	3	3	1.00	21	0.143
244	A	3	3	1.00	21	0.143
245	A	3	3	1.00	19	0.158
246	A	3	3	1.00	17	0.176
247	A	2	2	1.00	10	0.200
248	A	3	3	1.00	17	0.176
249	A	3	3	1.00	19	0.158
250	A	3	3	1.00	19	0.158
251	A	3	3	1.00	19	0.158
252	A	3	3	1.00	21	0.143
253	A	3	3	1.00	21	0.143
254	A	3	3	1.00	21	0.143
255	A	3	3	1.00	21	0.143
256	A	3	3	1.00	21	0.143
257	A	3	3	1.00	21	0.143
258	A	3	3	1.00	21	0.143
259	A	3	3	1.00	21	0.143
260	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	4	3	1.00	17	0.176
262	A	4	3	1.00	17	0.176
263	A	6	6	1.00	19	0.316
264	A	6	6	1.00	19	0.316
265	A	6	5	1.00	11	0.455
266	A	6	5	0.97	19	0.263
267	A	6	5	0.97	19	0.263
268	A	8	8	1.05	19	0.421
269	A	8	8	1.05	19	0.421
270	A	4	3	1.00	9	0.333
271	A	8	7	1.12	17	0.412
272	A	8	7	1.16	17	0.412
273	A	6	6	1.00	19	0.316
274	A	6	6	1.00	19	0.316
275	A	8	7	1.11	19	0.368
276	A	8	7	1.11	19	0.368
277	A	8	8	0.99	19	0.421
278	A	8	8	1.00	19	0.421
279	A	4	4	1.00	21	0.190
280	A	4	4	1.00	21	0.190
281	A	4	4	1.00	21	0.190
282	A	4	4	1.00	21	0.190
283	A	4	4	1.00	21	0.190
284	A	4	4	1.00	17	0.235
285	A	4	4	1.00	19	0.211
286	A	4	4	1.00	19	0.211
287	A	4	4	1.00	21	0.190
288	A	4	4	0.97	23	0.174
289	A	4	4	1.00	23	0.174
290	A	4	4	1.00	23	0.174
291	A	4	4	1.00	23	0.174
292	A	4	4	1.00	23	0.174
293	A	4	4	1.00	23	0.174
294	A	4	4	1.00	23	0.174

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \cos(a+bx)\,dx$

3.1.1 Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cos(a+bx)\,dx = \frac{\sin(a+bx)}{b}$$

output
```
sin(b*x+a)/b
```

3.1.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(a+bx)\,dx = \frac{\cos(bx)\sin(a)}{b} + \frac{\cos(a)\sin(bx)}{b}$$

input
```
Integrate[Cos[a + b*x],x]
```

output
```
(Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

3.1.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x],x]
```

output
```
Sin[a + b*x]/b
```

3.1.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.1.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\sin(bx+a)}{b}$	11
default	$\frac{\sin(bx+a)}{b}$	11
risch	$\frac{\sin(bx+a)}{b}$	11
parallelrisch	$\frac{\sin(bx+a)}{b}$	11
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	30
meijerg	$\frac{\cos(a)\sin(bx)}{b}-\frac{\sin(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	35

input
```
int(cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
sin(b*x+a)/b
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx=\frac{\sin\left(bx+a\right)}{b}$$

input
```
integrate(cos(b*x+a),x, algorithm="fricas")
```

output
```
sin(b*x + a)/b
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \cos(a+bx)\,dx = \begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a),x)
```

output
```
Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx = \frac{\sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a),x, algorithm="maxima")
```

output
```
sin(b*x + a)/b
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx = \frac{\sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a),x, algorithm="giac")
```

output
```
sin(b*x + a)/b
```

3.1.9 Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\,dx = \frac{\sin(a+b\,x)}{b}$$

input
```
int(cos(a + b*x),x)
```

output
```
sin(a + b*x)/b
```

3.2 $\int \cos^2(a+bx)\,dx$

3.2.1 Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \cos^2(a+bx)\,dx=\frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \cos^2(a+bx)\,dx=\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Cos[a + b*x]^2,x]
```

output
```
(2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

3.2.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 3115$$

$$\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \; 24$$

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

input
```
Int[Cos[a + b*x]^2,x]
```

output
```
x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

3.2.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.2.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx+\sin(2bx+2a)}{4b}$	20
derivativedivides	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

input
```
int(cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4/b*sin(2*b*x+2*a)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\,dx = \frac{bx+\cos(bx+a)\sin(bx+a)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \cos^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b\neq 0 \\ x\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a
+ b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cos^2(a+bx)\,dx = \frac{2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(a+bx)\,dx = \frac{1}{2}\,x + \frac{\sin\left(2\,bx+2\,a\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

3.2.9 Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(a+bx)\,dx = \frac{x}{2} + \frac{\sin\left(2\,a+2\,b\,x\right)}{4\,b}$$

input
```
int(cos(a + b*x)^2,x)
```

output
```
x/2 + sin(2*a + 2*b*x)/(4*b)
```

3.3 $\int \cos^3(a+bx)\,dx$

3.3.1 Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \cos^3(a+bx)\,dx=\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

output
```
sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\,dx=\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]^3,x]
```

output
```
Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

3.3.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx$$
$$\Big\downarrow \text{ 3113}$$
$$-\frac{\int (1-\sin^2(a+bx))\,d(-\sin(a+bx))}{b}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3,x]
```

output
```
-((-Sin[a + b*x] + Sin[a + b*x]^3/3)/b)
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
default	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
parallelrisch	$\frac{9\sin(bx+a)+\sin(3bx+3a)}{12b}$	24
risch	$\frac{3\sin(bx+a)}{4b}+\frac{\sin(3bx+3a)}{12b}$	27

input
```
int(cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\,dx = \frac{\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \cos^3(a+bx)\,dx = \begin{cases} \frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos^2(a+bx)}{b} & \text{for } b\neq 0 \\ x\cos^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**3,x)
```

output
```
Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
 0)), (x*cos(a)**3, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\,dx = -\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\,dx = -\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.3.9 Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)\,dx = \frac{3\sin(a+b\,x) - \sin(a+b\,x)^3}{3b}$$

input
```
int(cos(a + b*x)^3,x)
```

output
```
(3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)
```

3.4 $\int \cos^4(a+bx)\,dx$

3.4.1 Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \cos^4(a+bx)\,dx = \frac{3x}{8}+\frac{3\cos(a+bx)\sin(a+bx)}{8b}+\frac{\cos^3(a+bx)\sin(a+bx)}{4b}$$

output
```
3/8*x+3/8*cos(b*x+a)*sin(b*x+a)/b+1/4*cos(b*x+a)^3*sin(b*x+a)/b
```

3.4.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \cos^4(a+bx)\,dx = \frac{12(a+bx)+8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

input
```
Integrate[Cos[a + b*x]^4,x]
```

output
```
(12*(a + b*x) + 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

3.4.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)$$

input
```
Int[Cos[a + b*x]^4,x]
```

output
```
(Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x]
)/(2*b)))/4
```

3.4.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.4.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sin(4bx+4a)+8\sin(2bx+2a)}{32b}$
risch	$\frac{3x}{8}+\frac{\sin(4bx+4a)}{32b}+\frac{\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
norman	$\frac{\frac{3x}{8}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{3\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{5\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{9x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{3x(\mathrm{ta}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

input
```
int(cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*b*x+sin(4*b*x+4*a)+8*sin(2*b*x+2*a))/b
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\,dx = \frac{3\,bx + \left(2\,\cos\left(bx+a\right)^3 + 3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

input
```
integrate(cos(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/8*(3*b*x + (2*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b*x + a))/b
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \cos^4(a+bx)\,dx$$
$$= \begin{cases} \frac{3x\sin^4{(a+bx)}}{8} + \frac{3x\sin^2{(a+bx)}\cos^2{(a+bx)}}{4} + \frac{3x\cos^4{(a+bx)}}{8} + \frac{3\sin^3{(a+bx)}\cos{(a+bx)}}{8b} + \frac{5\sin{(a+bx)}\cos^3{(a+bx)}}{8b} & \text{for } b \neq 0 \\ x\cos^4{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
 3*x*cos(a + b*x)**4/8 + 3*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*sin(a +
b*x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*cos(a)**4, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \cos^4(a+bx)\,dx = \frac{12\,bx + 12\,a + \sin\left(4\,bx + 4\,a\right) + 8\,\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

input
```
integrate(cos(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))/b
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \cos^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{\sin\left(4\,bx+4\,a\right)}{32\,b} + \frac{\sin\left(2\,bx+2\,a\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*b*x + 4*a)/b + 1/4*sin(2*b*x + 2*a)/b
```

3.4.9 Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cos^4(a+bx)\,dx = \frac{3\,x}{8} + \frac{\frac{\sin(2\,a+2\,b\,x)}{4} + \frac{\sin(4\,a+4\,b\,x)}{32}}{b}$$

input
```
int(cos(a + b*x)^4,x)
```

output
```
(3*x)/8 + (sin(2*a + 2*b*x)/4 + sin(4*a + 4*b*x)/32)/b
```

3.5 $\int \cos^5(a+bx)\,dx$

3.5.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \cos^5(a+bx)\,dx=\frac{\sin(a+bx)}{b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

output
```
sin(b*x+a)/b-2/3*sin(b*x+a)^3/b+1/5*sin(b*x+a)^5/b
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\,dx=\frac{\sin(a+bx)}{b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

input
```
Integrate[Cos[a + b*x]^5,x]
```

output
```
Sin[a + b*x]/b - (2*Sin[a + b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

3.5.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^5 dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(\sin^4(a+bx)-2\sin^2(a+bx)+1\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\sin^5(a+bx)+\frac{2}{3}\sin^3(a+bx)-\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^5,x]
```

output
```
-((-Sin[a + b*x] + (2*Sin[a + b*x]^3)/3 - Sin[a + b*x]^5/5)/b)
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.5.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{5b}$	32
default	$\frac{\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{5b}$	32
parallelrisch	$\frac{150\sin(bx+a)+3\sin(5bx+5a)+25\sin(3bx+3a)}{240b}$	37
risch	$\frac{5\sin(bx+a)}{8b}+\frac{\sin(5bx+5a)}{80b}+\frac{5\sin(3bx+3a)}{48b}$	41

input
```
int(cos(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5/b*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \cos^5(a+bx)\,dx = \frac{\left(3\,\cos\left(bx+a\right)^4+4\,\cos\left(bx+a\right)^2+8\right)\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/15*(3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/b
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \cos^5(a+bx)\,dx = \begin{cases} \frac{8\sin^5\left(a+bx\right)}{15b}+\frac{4\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)}{3b}+\frac{\sin\left(a+bx\right)\cos^4\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ x\cos^5\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**5,x)
```

output
```
Piecewise((8*sin(a + b*x)**5/(15*b) + 4*sin(a + b*x)**3*cos(a + b*x)**2/(3
*b) + sin(a + b*x)*cos(a + b*x)**4/b, Ne(b, 0)), (x*cos(a)**5, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \cos^5(a+bx)\,dx = \frac{3\,\sin\left(bx+a\right)^5-10\,\sin\left(bx+a\right)^3+15\,\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/15*(3*sin(b*x + a)^5 - 10*sin(b*x + a)^3 + 15*sin(b*x + a))/b
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \cos^5(a+bx)\,dx = \frac{3\,\sin\left(bx+a\right)^5-10\,\sin\left(bx+a\right)^3+15\,\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^5,x, algorithm="giac")
```

output
```
1/15*(3*sin(b*x + a)^5 - 10*sin(b*x + a)^3 + 15*sin(b*x + a))/b
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cos^5(a+bx)\,dx = \frac{\frac{\sin(a+b\,x)^5}{5}-\frac{2\sin(a+b\,x)^3}{3}+\sin\left(a+b\,x\right)}{b}$$

input
```
int(cos(a + b*x)^5,x)
```

output
```
(sin(a + b*x) - (2*sin(a + b*x)^3)/3 + sin(a + b*x)^5/5)/b
```

3.6 $\int \cos^6(a+bx)\,dx$

3.6.1 Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cos^6(a+bx)\,dx=\frac{5x}{16}+\frac{5\cos(a+bx)\sin(a+bx)}{16b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}$$

output
```
5/16*x+5/16*cos(b*x+a)*sin(b*x+a)/b+5/24*cos(b*x+a)^3*sin(b*x+a)/b+1/6*cos
(b*x+a)^5*sin(b*x+a)/b
```

3.6.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \cos^6(a+bx)\,dx=\frac{60a+60bx+45\sin(2(a+bx))+9\sin(4(a+bx))+\sin(6(a+bx))}{192b}$$

input
```
Integrate[Cos[a + b*x]^6,x]
```

output
```
(60*a + 60*b*x + 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] + Sin[6*(a + b*x
)])/(192*b)
```

3.6.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)$$

input
```
Int[Cos[a + b*x]^6,x]
```

output
```
(Cos[a + b*x]^5*Sin[a + b*x])/(6*b) + (5*((Cos[a + b*x]^3*Sin[a + b*x])/(4
*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/6
```

3.6.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.6.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{60bx+\sin(6bx+6a)+9\sin(4bx+4a)+45\sin(2bx+2a)}{192b}$
risch	$\frac{5x}{16}+\frac{\sin(6bx+6a)}{192b}+\frac{3\sin(4bx+4a)}{64b}+\frac{15\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
default	$\frac{\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
norman	$\frac{\frac{5x}{16}+\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{5\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{15\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{15\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}-\frac{11\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+15}{\left(1+\tan^2\left(\right.\right.}$

input
```
int(cos(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192*(60*b*x+sin(6*b*x+6*a)+9*sin(4*b*x+4*a)+45*sin(2*b*x+2*a))/b
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^6(a+bx)\,dx$$
$$= \frac{15\,bx + \left(8\,\cos\left(bx+a\right)^5 + 10\,\cos\left(bx+a\right)^3 + 15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

input
```
integrate(cos(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/48*(15*b*x + (8*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*si
n(b*x + a))/b
```

3.6.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \cos^6(a+bx)\,dx$$
$$= \begin{cases} \frac{5x\sin^6(a+bx)}{16} + \frac{15x\sin^4(a+bx)\cos^2(a+bx)}{16} + \frac{15x\sin^2(a+bx)\cos^4(a+bx)}{16} + \frac{5x\cos^6(a+bx)}{16} + \frac{5\sin^5(a+bx)\cos(a+bx)}{16b} + \frac{5\sin^3(a \\ x\cos^6(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**6,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/1
6 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 + 5*s
in(a + b*x)**5*cos(a + b*x)/(16*b) + 5*sin(a + b*x)**3*cos(a + b*x)**3/(6*
b) + 11*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*cos(a)**6, True
))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \cos^6(a+bx)\,dx$$
$$= -\frac{4\,\sin\left(2\,bx+2\,a\right)^3 - 60\,bx - 60\,a - 9\,\sin\left(4\,bx+4\,a\right) - 48\,\sin\left(2\,bx+2\,a\right)}{192\,b}$$

input
```
integrate(cos(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/192*(4*sin(2*b*x + 2*a)^3 - 60*b*x - 60*a - 9*sin(4*b*x + 4*a) - 48*sin
(2*b*x + 2*a))/b
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^6(a+bx)\,dx = \frac{5}{16}\,x + \frac{\sin\left(6\,bx+6\,a\right)}{192\,b} + \frac{3\,\sin\left(4\,bx+4\,a\right)}{64\,b} + \frac{15\,\sin\left(2\,bx+2\,a\right)}{64\,b}$$

input
```
integrate(cos(b*x+a)^6,x, algorithm="giac")
```

output
```
5/16*x + 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b + 15/64*sin(2*
b*x + 2*a)/b
```

3.6.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \cos^6(a+bx)\,dx = \frac{5\,x}{16} + \frac{\frac{15\,\sin(2\,a+2\,b\,x)}{64} + \frac{3\,\sin(4\,a+4\,b\,x)}{64} + \frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cos(a + b*x)^6,x)
```

output
```
(5*x)/16 + ((15*sin(2*a + 2*b*x))/64 + (3*sin(4*a + 4*b*x))/64 + sin(6*a +
 6*b*x)/192)/b
```

3.7 $\int \cos^7(a+bx)\,dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 54

$$\int \cos^7(a+bx)\,dx = \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{b} + \frac{3\sin^5(a+bx)}{5b} - \frac{\sin^7(a+bx)}{7b}$$

output
```
sin(b*x+a)/b-sin(b*x+a)^3/b+3/5*sin(b*x+a)^5/b-1/7*sin(b*x+a)^7/b
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \cos^7(a+bx)\,dx = \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{b} + \frac{3\sin^5(a+bx)}{5b} - \frac{\sin^7(a+bx)}{7b}$$

input
```
Integrate[Cos[a + b*x]^7,x]
```

output
```
Sin[a + b*x]/b - Sin[a + b*x]^3/b + (3*Sin[a + b*x]^5)/(5*b) - Sin[a + b*x
]^7/(7*b)
```

3.7.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^7(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^7 dx$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{\int \left(-\sin^6(a+bx)+3\sin^4(a+bx)-3\sin^2(a+bx)+1\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{7}\sin^7(a+bx)-\frac{3}{5}\sin^5(a+bx)+\sin^3(a+bx)-\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^7,x]
```

output
```
-((-Sin[a + b*x] + Sin[a + b*x]^3 - (3*Sin[a + b*x]^5)/5 + Sin[a + b*x]^7/
7)/b)
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.7.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx+a)\right)}{5}\right)\sin(bx+a)}{7b}$	42
default	$\frac{\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx+a)\right)}{5}\right)\sin(bx+a)}{7b}$	42
parallelrisch	$\frac{1225\sin(bx+a)+5\sin(7bx+7a)+49\sin(5bx+5a)+245\sin(3bx+3a)}{2240b}$	48
risch	$\frac{35\sin(bx+a)}{64b}+\frac{\sin(7bx+7a)}{448b}+\frac{7\sin(5bx+5a)}{320b}+\frac{7\sin(3bx+3a)}{64b}$	55

input
```
int(cos(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/7/b*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \cos^7(a+bx)\,dx = \frac{\left(5\,\cos\left(bx+a\right)^{6}+6\,\cos\left(bx+a\right)^{4}+8\,\cos\left(bx+a\right)^{2}+16\right)\sin\left(bx+a\right)}{35\,b}$$

input
```
integrate(cos(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/35*(5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x
 + a)/b
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \cos^7(a+bx)\,dx = \begin{cases} \frac{16\sin^{7}\left(a+bx\right)}{35b}+\frac{8\sin^{5}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{5b}+\frac{2\sin^{3}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{b}+\frac{\sin\left(a+bx\right)\cos^{6}\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ x\cos^{7}\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**7,x)
```

output
```
Piecewise((16*sin(a + b*x)**7/(35*b) + 8*sin(a + b*x)**5*cos(a + b*x)**2/(
5*b) + 2*sin(a + b*x)**3*cos(a + b*x)**4/b + sin(a + b*x)*cos(a + b*x)**6/
b, Ne(b, 0)), (x*cos(a)**7, True))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cos^7(a+bx)\,dx = -\frac{5\,\sin(bx+a)^7 - 21\,\sin(bx+a)^5 + 35\,\sin(bx+a)^3 - 35\,\sin(bx+a)}{35\,b}$$

input
```
integrate(cos(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/35*(5*sin(b*x + a)^7 - 21*sin(b*x + a)^5 + 35*sin(b*x + a)^3 - 35*sin(b
*x + a))/b
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cos^7(a+bx)\,dx = -\frac{5\,\sin(bx+a)^7 - 21\,\sin(bx+a)^5 + 35\,\sin(bx+a)^3 - 35\,\sin(bx+a)}{35\,b}$$

input
```
integrate(cos(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/35*(5*sin(b*x + a)^7 - 21*sin(b*x + a)^5 + 35*sin(b*x + a)^3 - 35*sin(b
*x + a))/b
```

3.7.9 Mupad [B] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \cos^7(a+bx)\,dx = -\frac{\sin(a+b\,x)\,\left(5\sin(a+b\,x)^6-21\sin(a+b\,x)^4+35\sin(a+b\,x)^2-35\right)}{35\,b}$$

input
```
int(cos(a + b*x)^7,x)
```

output
```
-(sin(a + b*x)*(35*sin(a + b*x)^2 - 21*sin(a + b*x)^4 + 5*sin(a + b*x)^6 -
 35))/(35*b)
```

3.8 $\int \cos^8(a+bx)\,dx$

3.8.1 Optimal result

Integrand size = 8, antiderivative size = 88

$$\int \cos^8(a+bx)\,dx = \frac{35x}{128}+\frac{35\cos(a+bx)\sin(a+bx)}{128b}+\frac{35\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{7\cos^5(a+bx)\sin(a+bx)}{48b}+\frac{\cos^7(a+bx)\sin(a+bx)}{8b}$$

output
```
35/128*x+35/128*cos(b*x+a)*sin(b*x+a)/b+35/192*cos(b*x+a)^3*sin(b*x+a)/b+7
/48*cos(b*x+a)^5*sin(b*x+a)/b+1/8*cos(b*x+a)^7*sin(b*x+a)/b
```

3.8.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \cos^8(a+bx)\,dx = \frac{840a+840bx+672\sin(2(a+bx))+168\sin(4(a+bx))+32\sin(6(a+bx))+3\sin(8(a+bx))}{3072b}$$

input
```
Integrate[Cos[a + b*x]^8,x]
```

output
```
(840*a + 840*b*x + 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] + 32*Sin[6*
(a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

3.8.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^8(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^8dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\int \cos^6(a+bx)dx+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)+\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \; 24$$

$$\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{7}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)$$

input
```
Int[Cos[a + b*x]^8,x]
```

output
```
(Cos[a + b*x]^7*Sin[a + b*x])/(8*b) + (7*((Cos[a + b*x]^5*Sin[a + b*x])/(6
*b) + (5*((Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Si
n[a + b*x])/(2*b)))/4))/6))/8
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.8.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{840bx+3\sin(8bx+8a)+32\sin(6bx+6a)+168\sin(4bx+4a)+672\sin(2bx+2a)}{3072b}$
derivativedivides	$\frac{\frac{\left(\cos^{7}(bx+a)+\frac{7\left(\cos^{5}(bx+a)\right)}{6}+\frac{35\left(\cos^{3}(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
default	$\frac{\frac{\left(\cos^{7}(bx+a)+\frac{7\left(\cos^{5}(bx+a)\right)}{6}+\frac{35\left(\cos^{3}(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
risch	$\frac{35x}{128}+\frac{\sin(8bx+8a)}{1024b}+\frac{\sin(6bx+6a)}{96b}+\frac{7\sin(4bx+4a)}{128b}+\frac{7\sin(2bx+2a)}{32b}$
norman	$\frac{35x}{128}+\frac{93\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{91\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{1799\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{1085\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{1085\left(\tan^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{1799\left(\tan^{11}\left(\frac{b}{2}\right.\right.}{192b}$

input
```
int(cos(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/3072*(840*b*x+3*sin(8*b*x+8*a)+32*sin(6*b*x+6*a)+168*sin(4*b*x+4*a)+672*
sin(2*b*x+2*a))/b
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \cos^8(a+bx)\,dx = \frac{105\,bx+\left(48\,\cos\left(bx+a\right)^{7}+56\,\cos\left(bx+a\right)^{5}+70\,\cos\left(bx+a\right)^{3}+105\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{384\,b}$$

input
```
integrate(cos(b*x+a)^8,x, algorithm="fricas")
```

output
```
1/384*(105*b*x + (48*cos(b*x + a)^7 + 56*cos(b*x + a)^5 + 70*cos(b*x + a)^
3 + 105*cos(b*x + a))*sin(b*x + a))/b
```

3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(82) = 164.

Time = 0.71 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.09

$$\int \cos^8(a+bx)\,dx$$
$$= \begin{cases} \frac{35x\sin^8(a+bx)}{128} + \frac{35x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{105x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{35x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{35x\cos^8(a+bx)}{128} + 3 \\ x\cos^8(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**8,x)
```

output
```
Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2
/32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(
a + b*x)**6/32 + 35*x*cos(a + b*x)**8/128 + 35*sin(a + b*x)**7*cos(a + b*x
)/(128*b) + 385*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 511*sin(a + b*x)
**3*cos(a + b*x)**5/(384*b) + 93*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(
b, 0)), (x*cos(a)**8, True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \cos^8(a+bx)\,dx = -\frac{128\sin(2bx+2a)^3 - 840bx - 840a - 3\sin(8bx+8a) - 168\sin(4bx+4a) - 768\sin(2bx+2a)}{3072b}$$

input
```
integrate(cos(b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/3072*(128*sin(2*b*x + 2*a)^3 - 840*b*x - 840*a - 3*sin(8*b*x + 8*a) - 1
68*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \cos^8(a+bx)\,dx = \frac{35}{128}x + \frac{\sin(8\,bx+8\,a)}{1024\,b} + \frac{\sin(6\,bx+6\,a)}{96\,b} + \frac{7\,\sin(4\,bx+4\,a)}{128\,b} + \frac{7\,\sin(2\,bx+2\,a)}{32\,b}$$

input
```
integrate(cos(b*x+a)^8,x, algorithm="giac")
```

output
```
35/128*x + 1/1024*sin(8*b*x + 8*a)/b + 1/96*sin(6*b*x + 6*a)/b + 7/128*sin
(4*b*x + 4*a)/b + 7/32*sin(2*b*x + 2*a)/b
```

3.8.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \cos^8(a+bx)\,dx = \frac{35\,x}{128} + \frac{\frac{7\,\sin(2\,a+2\,b\,x)}{32} + \frac{7\,\sin(4\,a+4\,b\,x)}{128} + \frac{\sin(6\,a+6\,b\,x)}{96} + \frac{\sin(8\,a+8\,b\,x)}{1024}}{b}$$

input
```
int(cos(a + b*x)^8,x)
```

output
```
(35*x)/128 + ((7*sin(2*a + 2*b*x))/32 + (7*sin(4*a + 4*b*x))/128 + sin(6*a
 + 6*b*x)/96 + sin(8*a + 8*b*x)/1024)/b
```

3.9 $\int \cos^{\frac{7}{2}}(a+bx)\,dx$

3.9.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx=\frac{10\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b}+\frac{10\sqrt{\cos(a+bx)}\sin(a+bx)}{21b}+\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}$$

output
```
10/21*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+
1/2*b*x),2^(1/2))/b+2/7*cos(b*x+a)^(5/2)*sin(b*x+a)/b+10/21*sin(b*x+a)*cos
(b*x+a)^(1/2)/b
```

3.9.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx=\frac{20\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}(23\sin(a+bx)+3\sin(3(a+bx)))}{42b}$$

input
```
Integrate[Cos[a + b*x]^(7/2),x]
```

output
```
(20*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(23*Sin[a + b*x] + 3*Si
n[3*(a + b*x)]))/(42*b)
```

3.9.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \; 3115$$

$$\frac{5}{7}\int \cos^{\frac{3}{2}}(a+bx)dx+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{7}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \; 3115$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \; 3120$$

$$\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)$$

input
```
Int[Cos[a + b*x]^(7/2),x]
```

output
```
(2*Cos[a + b*x]^(5/2)*Sin[a + b*x])/(7*b) + (5*((2*EllipticF[(a + b*x)/2,
2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b)))/7
```

3.9.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(81) = 162.

Time = 4.63 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(48\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-72\left(\cos^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right.}{21\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2(}}$

input
```
int(cos(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(48*cos(1/2
*b*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(1/2
*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(
1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-2*sin(
1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(-1+2*cos(
1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.9.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx = \frac{2\left(3\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)}\sin(bx+a) - 5i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(}{21\,b}$$

input
```
integrate(cos(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a))*sin(b*x + a) - 5*I*sqrt(
2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + 5*I*sqrt(2)
*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.9.7 Maxima [F]

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(7/2), x)
```

3.9.8 Giac [F]

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(7/2), x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \cos^{\frac{7}{2}}(a+bx)\,dx = -\frac{2\cos(a+b\,x)^{9/2}\,\sin(a+b\,x)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(a+b\,x)^2\right)}{9\,b\,\sqrt{\sin(a+b\,x)^2}}$$

input
```
int(cos(a + b*x)^(7/2),x)
```

output
```
-(2*cos(a + b*x)^(9/2)*sin(a + b*x)*hypergeom([1/2, 9/4], 13/4, cos(a + b*
x)^2))/(9*b*(sin(a + b*x)^2)^(1/2))
```

3.10 $\int \cos^{\frac{5}{2}}(a+bx)\,dx$

3.10.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx=\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{5b}$$

output
```
6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/
2*b*x),2^(1/2))/b+2/5*cos(b*x+a)^(3/2)*sin(b*x+a)/b
```

3.10.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx=\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\sin(2(a+bx))}{5b}$$

input
```
Integrate[Cos[a + b*x]^(5/2),x]
```

output
```
(6*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[2*(a + b*x)])/(5*b)
```

3.10.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}\int \sqrt{\cos(a+bx)}dx+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}$$

input
```
Int[Cos[a + b*x]^(5/2),x]
```

output
```
(6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Cos[a + b*x]^(3/2)*Sin[a + b*x])/
(5*b)
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(62) = 124$.

Time = 2.89 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.81

method	result
default	$-\frac{2\sqrt{\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(-8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+8\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\right.}}$

input
```
int(cos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*cos(1/2*
b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-
2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(
2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-2
*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(-1+2
*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.10.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx$$
$$=\frac{2\cos(bx+a)^{\frac{3}{2}}\sin(bx+a)+3i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin}{5b}$$

input
```
integrate(cos(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*cos(b*x + a)^(3/2)*sin(b*x + a) + 3*I*sqrt(2)*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 3*I*sqrt(2)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*
x + a))))/b
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(5/2),x)
```

output
```
Timed out
```

3.10.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(5/2), x)
```

3.10.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(5/2), x)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{5}{2}}(a+bx)\,dx = -\frac{2\cos\left(a+b\,x\right)^{7/2}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(a+b\,x\right)^2\right)}{7\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(cos(a + b*x)^(5/2),x)
```

output
```
-(2*cos(a + b*x)^(7/2)*sin(a + b*x)*hypergeom([1/2, 7/4], 11/4, cos(a + b*
x)^2))/(7*b*(sin(a + b*x)^2)^(1/2))
```

3.11 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

3.11.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}$$

output
```
2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/
2*b*x),2^(1/2))/b+2/3*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

3.11.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}\sin(a+bx)\right)}{3b}$$

input
```
Integrate[Cos[a + b*x]^(3/2),x]
```

output
```
(2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

3.11.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

input
```
Int[Cos[a + b*x]^(3/2),x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/
(3*b)
```

3.11.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(62) = 124$.

Time = 2.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.26

method	result
default	$-\frac{2\sqrt{\left(-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2}\right.}{3\sqrt{-2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$

input
```
int(cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b
*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(
sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos
(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(
1/2)/sin(1/2*b*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.11.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{2\sqrt{\cos(bx+a)}\sin(bx+a) - i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + i\sqrt{2}\text{weiers}}{3b}$$

input
```
integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
 0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0,
cos(b*x + a) - I*sin(b*x + a)))/b
```

3.11.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**(3/2), x)
```

3.11.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(3/2), x)
```

3.11.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(3/2), x)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{3\,b} + \frac{2\,\sqrt{\cos(a+b\,x)}\,\sin(a+b\,x)}{3\,b}$$

input
```
int(cos(a + b*x)^(3/2),x)
```

output
```
(2*ellipticF(a/2 + (b*x)/2, 2))/(3*b) + (2*cos(a + b*x)^(1/2)*sin(a + b*x)
)/(3*b)
```

3.12 $\int \sqrt{\cos(a+bx)}\,dx$

3.12.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

output
```
2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))/b
```

3.12.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Integrate[Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/b
```

3.12.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Int[Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/b
```

3.12.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.

Time = 1.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 8.31

method	result
default	$\frac{2\sqrt{\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$
risch	$-\frac{i\sqrt{2}\sqrt{\left(e^{2i(bx+a)}+1\right)e^{-i(bx+a)}}}{b}-\frac{i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{\left(e^{2i(bx+a)}+1\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(bx+a)}-i\right)}\sqrt{ie^{i(bx+a)}}\left(-2iE\left(\sqrt{-i(e\right.\right.}{\sqrt{e^{3i(bx+a)}+e^{i(bx+}}}}{b(e}$

input
```
int(cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*
a),2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b
*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.12.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \sqrt{\cos(a+bx)}\,dx$$
$$= \frac{i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\text{weierstrassZe}}{b}$$

input
```
integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
+ I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

3.12.6 Sympy [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cos(a + b*x)), x)
```

3.12.7 Maxima [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(b*x + a)), x)
```

3.12.8 Giac [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(b*x + a)), x)
```

3.12.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int(cos(a + b*x)^(1/2),x)
```

output
```
(2*ellipticE(a/2 + (b*x)/2, 2))/b
```

3.13 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

3.13.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

output
```
2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*
b*x),2^(1/2))/b
```

3.13.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

input
```
Integrate[1/Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/b
```

3.13.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

input
```
Int[1/Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/b
```

3.13.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.13.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2\,\mathrm{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2}\middle|\sqrt{2}\right)}{b}$	18

input
```
int(1/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

3.13.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$
$$= \frac{-i\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\sin{(bx+a)})+i\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cos{(bx -}}{b}$$

input
```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*
sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

3.13.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos{(a+bx)}}}\,dx$$

input
```
integrate(1/cos(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(cos(a + b*x)), x)
```

3.13.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cos(b*x + a)), x)
```

3.13.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos(bx+a)}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cos(b*x + a)), x)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|\,2\right)}{b}$$

input
```
int(1/cos(a + b*x)^(1/2),x)
```

output
```
(2*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.14 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

3.14.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

output
```
-2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2*sin(b*x+a)/b/cos(b*x+a)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(-3/2),x]
```

output
```
(-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

3.14.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 3116}$$
$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Int[Cos[a + b*x]^(-3/2),x]
```

output
```
(-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

3.14.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(62) = 124$.

Time = 1.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.79

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\right.\right.}}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$

input
```
int(1/cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(
1/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*
a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*Ellipti
cE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a
)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.14.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))+i\sqrt{}}{}$$

input
```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*cos(b*x + a)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sqrt
(cos(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

3.14.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cos(b*x+a)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**(-3/2), x)
```

3.14.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-3/2), x)
```

3.14.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-3/2), x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 14.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(1/cos(a + b*x)^(3/2),x)
```

output
```
(2*sin(a + b*x)*hypergeom([-1/4, 1/2], 3/4, cos(a + b*x)^2))/(b*cos(a + b*
x)^(1/2)*(sin(a + b*x)^2)^(1/2))
```

3.15 $\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

3.15.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

output
```
2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/
2*b*x),2^(1/2))/b+2/3*sin(b*x+a)/b/cos(b*x+a)^(3/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\frac{\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

input
```
Integrate[Cos[a + b*x]^(-5/2),x]
```

output
```
(2*(EllipticF[(a + b*x)/2, 2] + Sin[a + b*x]/Cos[a + b*x]^(3/2)))/(3*b)
```

3.15.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \ 3116$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3120$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Cos[a + b*x]^(-5/2),x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sin[a + b*x])/(3*b*Cos[a + b*x]^(
3/2))
```

3.15.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(62) = 124$.

Time = 1.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 5.07

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a}{2}}}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}$

input
```
int(1/cos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ell
ipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-2*sin(1/2*b*x+1/2*
a)^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))*((-1+2*cos(1/2*b*x+1/2*
a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2
*a)^2)^(1/2)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(3/2)/sin(1/2*b*x+1/2*a)/b
```

3.15.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.19

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\cos{(bx+a)}^2 \operatorname{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\,\sin{(bx+a)})+i\sqrt{2}\cos{(bx+a)}^2 \operatorname{weierstr}}{3\,b\cos{(bx+a)}^2}$$

input
```
integrate(1/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*x + a) + I
*sin(b*x + a)) + I*sqrt(2)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b
*x + a) - I*sin(b*x + a)) + 2*sqrt(cos(b*x + a))*sin(b*x + a))/(b*cos(b*x
+ a)^2)
```

3.15.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cos(b*x+a)**(5/2),x)
```

output
```
Integral(cos(a + b*x)**(-5/2), x)
```

3.15.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cos{(bx+a)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-5/2), x)
```

3.15.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-5/2), x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\,\sin(a+b\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(a+b\,x)^2\right)}{3\,b\cos(a+b\,x)^{3/2}\,\sqrt{\sin(a+b\,x)^2}}$$

input
```
int(1/cos(a + b*x)^(5/2),x)
```

output
```
(2*sin(a + b*x)*hypergeom([-3/4, 1/2], 1/4, cos(a + b*x)^2))/(3*b*cos(a +
b*x)^(3/2)*(sin(a + b*x)^2)^(1/2))
```

3.16 $\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

3.16.1 Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = -\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}$$

output
```
-6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1
/2*b*x),2^(1/2))/b+2/5*sin(b*x+a)/b/cos(b*x+a)^(5/2)+6/5*sin(b*x+a)/b/cos(
b*x+a)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{-6\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+3\sin(2(a+bx))+2\tan(a+bx)}{5b\cos^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^(-7/2),x]
```

output
```
(-6*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] + 3*Sin[2*(a + b*x)] + 2*
Tan[a + b*x])/(5*b*Cos[a + b*x]^(3/2))
```

3.16.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}dx\right)+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}\right)$$

input
```
Int[Cos[a + b*x]^(-7/2),x]
```

output
```
(2*Sin[a + b*x])/(5*b*Cos[a + b*x]^(5/2)) + (3*((-2*EllipticE[(a + b*x)/2,
 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])))/5
```

3.16.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.16.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(81) = 162$.

Time = 2.31 (sec) , antiderivative size = 358, normalized size of antiderivative = 5.51

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(24\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{}}{}$

input
```
int(1/cos(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*
b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x
+1/2*a)^3*(24*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6-12*(2*sin(1/2*b*x+1/
2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^
2)^(1/2)*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+1
2*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(
sin(1/2*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+8*sin(1/2*b*x+1/2*a)^2*co
s(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)
^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4+sin
(1/2*b*x+1/2*a)^2)^(1/2)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.16.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{-3i\sqrt{2}\cos(bx+a)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))+3}{}$$

input
```
integrate(1/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*cos(b*x + a)^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a
))) + 2*(3*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a))*sin(b*x + a))/(b*cos(b*x
 + a)^3)
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(b*x+a)**(7/2),x)
```

output
```
Timed out
```

3.16.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-7/2), x)
```

3.16.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-7/2), x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 14.85 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(a+b\,x\right)^2\right)}{5\,b\cos\left(a+b\,x\right)^{5/2}\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(1/cos(a + b*x)^(7/2),x)
```

output
```
(2*sin(a + b*x)*hypergeom([-5/4, 1/2], -1/4, cos(a + b*x)^2))/(5*b*cos(a +
 b*x)^(5/2)*(sin(a + b*x)^2)^(1/2))
```

3.17 $\int (c\cos(a+bx))^{7/2}\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 98

$$\int (c\cos(a+bx))^{7/2}\,dx = \frac{10c^4\sqrt{\cos(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b\sqrt{c\cos(a+bx)}} + \frac{10c^3\sqrt{c\cos(a+bx)}\sin(a+bx)}{21b} + \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b}$$

output
```
2/7*c*(c*cos(b*x+a))^(5/2)*sin(b*x+a)/b+10/21*c^4*(cos(1/2*a+1/2*b*x)^2)^(
1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(
1/2)/b/(c*cos(b*x+a))^(1/2)+10/21*c^3*sin(b*x+a)*(c*cos(b*x+a))^(1/2)/b
```

3.17.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int (c\cos(a+bx))^{7/2}\,dx = \frac{c^3\sqrt{c\cos(a+bx)}\left(20\,\mathrm{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}(23\sin(a+bx)+3\sin(3(a+bx$$
$$\overline{42b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[(c*Cos[a + b*x])^(7/2),x]
```

output
```
(c^3*Sqrt[c*Cos[a + b*x]]*(20*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x
]]*(23*Sin[a + b*x] + 3*Sin[3*(a + b*x)])))/(42*b*Sqrt[Cos[a + b*x]])
```

3.17.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}c^2\int (c\cos(a+bx))^{3/2}dx+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}c^2\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}c^2\left(\frac{1}{3}c^2\int \frac{1}{\sqrt{c\cos(a+bx)}}dx+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}\right)+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}c^2\left(\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}\right)+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{5}{7}c^2\left(\frac{c^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}\right)+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}c^2\left(\frac{c^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}\right)+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{7}c^2\left(\frac{2c^2\sqrt{\cos(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}\right)+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

input
```
Int[(c*Cos[a + b*x])^(7/2),x]
```

output
```
(2*c*(c*Cos[a + b*x])^(5/2)*Sin[a + b*x])/(7*b) + (5*c^2*((2*c^2*Sqrt[Cos[
a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[c*Cos[a + b*x]]) + (2*c*Sqr
t[c*Cos[a + b*x]]*Sin[a + b*x])/(3*b)))/7
```

3.17.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.17.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

method	result
default	$-\frac{2\sqrt{c\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, c^4\left(48\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-72\left(\cos^3\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{21\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\left(-1\right.}}$

input
```
int((c*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(c*(-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^4*(48*c
os(1/2*b*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*c
os(1/2*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^
2+1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-
c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/
(c*(-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)/b
```

3.17.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int (c\cos(a+bx))^{7/2}\,dx = \frac{-5i\sqrt{2}c^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+5i\sqrt{2}c^{\frac{7}{2}}\text{weierstrassPInve}}{}$$

input
```
integrate((c*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*c^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin
(b*x + a)) + 5*I*sqrt(2)*c^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) -
 I*sin(b*x + a)) + 2*(3*c^3*cos(b*x + a)^2 + 5*c^3)*sqrt(c*cos(b*x + a))*s
in(b*x + a))/b
```

3.17.6 Sympy [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((c*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

3.17.7 Maxima [F]

$$\int (c\cos(a+bx))^{7/2}\,dx = \int (c\cos(bx+a))^{\frac{7}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(7/2), x)
```

3.17.8 Giac [F]

$$\int (c\cos(a+bx))^{7/2}\,dx = \int (c\cos(bx+a))^{\frac{7}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(7/2), x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{7/2}\,dx = \int (c\,\cos(a+b\,x))^{7/2}\,dx$$

input
```
int((c*cos(a + b*x))^(7/2),x)
```

output
```
int((c*cos(a + b*x))^(7/2), x)
```

3.18 $\int (c\cos(a+bx))^{5/2}\,dx$

3.18.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (c\cos(a+bx))^{5/2}\,dx=\frac{6c^2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}}+\frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b}$$

output
```
2/5*c*(c*cos(b*x+a))^(3/2)*sin(b*x+a)/b+6/5*c^2*(cos(1/2*a+1/2*b*x)^2)^(1/
2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(c*cos(b*x+a))
^(1/2)/b/cos(b*x+a)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int (c\cos(a+bx))^{5/2}\,dx=\frac{(c\cos(a+bx))^{5/2}\left(6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\sin(2(a+bx))\right)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(c*Cos[a + b*x])^(5/2),x]
```

output
```
((c*Cos[a + b*x])^(5/2)*(6*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*
Sin[2*(a + b*x)]))/(5*b*Cos[a + b*x]^(5/2))
```

3.18.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}c^2\int \sqrt{c\cos(a+bx)}dx+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}c^2\int \sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3c^2\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3c^2\sqrt{c\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6c^2E\left(\frac{1}{2}(a+bx)|2\right)\sqrt{c\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

input
```
Int[(c*Cos[a + b*x])^(5/2),x]
```

output
```
(6*c^2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b
*x]]) + (2*c*(c*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b)
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.18.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(86) = 172.

Time = 2.78 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.04

method	result
default	$-\frac{2\sqrt{c\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, c^3\left(-8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+8\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{5\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\left(-1+2\right.}}$

input
```
int((c*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(c*(-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^3*(-8*co
s(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1
/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(
1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)
))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/
2*a)/(c*(-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)/b
```

3.18.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int (c\cos(a+bx))^{5/2}\,dx = \frac{2\sqrt{c\cos(bx+a)}c^2\cos(bx+a)\sin(bx+a) + 3i\sqrt{2}c^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInv}$$

input
```
integrate((c*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(c*cos(b*x + a))*c^2*cos(b*x + a)*sin(b*x + a) + 3*I*sqrt(2)*c^
(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*s
in(b*x + a))) - 3*I*sqrt(2)*c^(5/2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((c*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.18.7 Maxima [F]

$$\int (c\cos(a+bx))^{5/2}\,dx = \int (c\cos(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(5/2), x)
```

3.18.8 Giac [F]

$$\int (c\cos(a+bx))^{5/2}\,dx = \int (c\cos(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(5/2), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{5/2}\,dx = \int (c\,\cos(a+b\,x))^{5/2}\,dx$$

input
```
int((c*cos(a + b*x))^(5/2),x)
```

output
```
int((c*cos(a + b*x))^(5/2), x)
```

3.19 $\int (c\cos(a+bx))^{3/2}\,dx$

3.19.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (c\cos(a+bx))^{3/2}\,dx = \frac{2c^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{c\cos(a+bx)}} + \frac{2c\sqrt{c\cos(a+bx)}\sin(a+bx)}{3b}$$

output
```
2/3*c^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*
a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(c*cos(b*x+a))^(1/2)+2/3*c*sin(b*x+
a)*(c*cos(b*x+a))^(1/2)/b
```

3.19.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int (c\cos(a+bx))^{3/2}\,dx = \frac{2(c\cos(a+bx))^{3/2}\left(\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}\sin(a+bx)\right)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(c*Cos[a + b*x])^(3/2),x]
```

output
```
(2*(c*Cos[a + b*x])^(3/2)*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*
Sin[a + b*x]))/(3*b*Cos[a + b*x]^(3/2))
```

3.19.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}c^2\int \frac{1}{\sqrt{c\cos(a+bx)}}dx+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{c^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2c^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

input
```
Int[(c*Cos[a + b*x])^(3/2),x]
```

output
```
(2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[c*Cos[a + b
*x]]) + (2*c*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(3*b)
```

3.19.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.19.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.

Time = 2.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

method	result
default	$-\frac{2\sqrt{c\left(-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,c^{2}\left(4\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}}{3\sqrt{-c\left(2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\left(-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}}$

input
```
int((c*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(c*(-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^2*(4*sin
(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/
2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipti
cF(cos(1/2*b*x+1/2*a),2^(1/2)))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/
2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)/b
```

3.19.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int (c\cos(a+bx))^{3/2}\,dx = \frac{-i\sqrt{2}c^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}c^{\frac{3}{2}}\text{weierstrassPInvers}}{3b}$$

input
```
integrate((c*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*c^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*
x + a)) + I*sqrt(2)*c^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*si
n(b*x + a)) + 2*sqrt(c*cos(b*x + a))*c*sin(b*x + a))/b
```

3.19.6 Sympy [F]

$$\int (c\cos(a+bx))^{3/2}\,dx = \int (c\cos(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))**(3/2),x)
```

output
```
Integral((c*cos(a + b*x))**(3/2), x)
```

3.19.7 Maxima [F]

$$\int (c\cos(a+bx))^{3/2}\,dx = \int (c\cos(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(3/2), x)
```

3.19.8 Giac [F]

$$\int (c\cos(a+bx))^{3/2}\,dx = \int (c\cos(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(3/2), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{3/2}\,dx = \int (c\,\cos(a+b\,x))^{3/2}\,dx$$

input
```
int((c*cos(a + b*x))^(3/2),x)
```

output
```
int((c*cos(a + b*x))^(3/2), x)
```

3.20 $\int \sqrt{c\cos(a+bx)}\,dx$

3.20.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{c\cos(a+bx)}\,dx = \frac{2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

output
```
2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))*(c*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{c\cos(a+bx)}\,dx = \frac{2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Sqrt[c*Cos[a + b*x]],x]
```

output
```
(2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

3.20.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{c\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{c\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

input
```
Int[Sqrt[c*Cos[a + b*x]],x]
```

output
```
(2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

3.20.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(60) = 120$.

Time = 1.92 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.74

method	result
default	$\frac{2\sqrt{c\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, c\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\, E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\, b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{c\left(e^{2i(bx+a)}+1\right)e^{-i(bx+a)}}}{b}-\frac{i\left(-\frac{2\left(e^{2i(bx+a)}c+c\right)}{c\sqrt{e^{i(bx+a)}\left(e^{2i(bx+a)}c+c\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{ie^{i(bx+a)}}\,\left(-2iE\left(\sqrt{-\right.\right.}{\sqrt{e^{3i(bx+a)}c+e^{i(b}}}}{}$

input
```
int((c*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*(-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c*(sin(1/2*b*
x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+
1/2*a),2^(1/2))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/s
in(1/2*b*x+1/2*a)/(c*(-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)/b
```

3.20.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sqrt{c\cos(a+bx)}\,dx$$
$$= \frac{i\sqrt{2}\sqrt{c}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\sqrt{c}\text{weierst}}{b}$$

input
```
integrate((c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b
*x + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

3.20.6 Sympy [F]

$$\int \sqrt{c\cos(a+bx)}\,dx = \int \sqrt{c\cos(a+bx)}\,dx$$

input
```
integrate((c*cos(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*cos(a + b*x)), x)
```

3.20.7 Maxima [F]

$$\int \sqrt{c\cos(a+bx)}\,dx = \int \sqrt{c\cos(bx+a)}\,dx$$

input
```
integrate((c*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*cos(b*x + a)), x)
```

3.20.8 Giac [F]

$$\int \sqrt{c\cos(a+bx)}\,dx = \int \sqrt{c\cos\left(bx+a\right)}\,dx$$

input
```
integrate((c*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*cos(b*x + a)), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c\cos(a+bx)}\,dx = \int \sqrt{c\,\cos\left(a+b\,x\right)}\,dx$$

input
```
int((c*cos(a + b*x))^(1/2),x)
```

output
```
int((c*cos(a + b*x))^(1/2), x)
```

3.21 $\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx$

3.21.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx = \frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{c\cos(a+bx)}}$$

output
```
2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*
b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(c*cos(b*x+a))^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx = \frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{c\cos(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*Cos[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

3.21.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3121}$$
$$\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{\sqrt{c\cos(a+bx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{\sqrt{c\cos(a+bx)}}$$
$$\Big\downarrow \text{3120}$$
$$\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{c\cos(a+bx)}}$$

input
```
Int[1/Sqrt[c*Cos[a + b*x]],x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

3.21.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.21.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{2\sqrt{-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\operatorname{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2}\middle|\sqrt{2}\right)}{b\sqrt{c\left(-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}}$	54

input
```
int(1/(c*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/(c*(-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)*(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2
)*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

3.21.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{c}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{c}\text{weierstrassPInverse}(-4,0,\text{cc}}{bc}$$

input
```
integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x +
a)) + I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*
x + a)))/(b*c)
```

3.21.6 Sympy [F]

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos{(a+bx)}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(c*cos(a + b*x)), x)
```

3.21.7 Maxima [F]

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos{(bx+a)}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*cos(b*x + a)), x)
```

3.21.8 Giac [F]

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos{(bx+a)}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*cos(b*x + a)), x)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx = \frac{2\sqrt{\cos(a+b\,x)}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{c\,\cos(a+b\,x)}}$$

input
```
int(1/(c*cos(a + b*x))^(1/2),x)
```

output
```
(2*cos(a + b*x)^(1/2)*ellipticF(a/2 + (b*x)/2, 2))/(b*(c*cos(a + b*x))^(1/
2))
```

3.22 $\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 68

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = -\frac{2\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bc^2\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}$$

output
```
2*sin(b*x+a)/b/c/(c*cos(b*x+a))^(1/2)-2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1
/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(c*cos(b*x+a))^(1/2)/b
/c^2/cos(b*x+a)^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = \frac{2\left(-\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sin(a+bx)\right)}{bc\sqrt{c\cos(a+bx)}}$$

input
```
Integrate[(c*Cos[a + b*x])^(-3/2),x]
```

output
```
(2*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + Sin[a + b*x]))/(b*c*
Sqrt[c*Cos[a + b*x]])
```

3.22.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\int \sqrt{c\cos(a+bx)}dx}{c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\int \sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{c^2}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{c^2\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\sqrt{c\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{c^2\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{bc^2\sqrt{\cos(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(-3/2),x]
```

output
```
(-2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*c^2*Sqrt[Cos[a + b*
x]]) + (2*Sin[a + b*x])/(b*c*Sqrt[c*Cos[a + b*x]])
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.

Time = 1.45 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.91

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c}\,\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\right.\right.}}{c\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}}$

input
```
int(1/(c*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/c*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4*c+sin(1/2*b*x+1/2*a)^
2*c)^(1/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*
x+1/2*a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4*c+sin(1/2*b*x+1/2*a)^2*c)^(1/
2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(
1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(-1+2*cos(1/2*b*x+1/2*a)^2)
)^(1/2)/b
```

3.22.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{c}\cos(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+$$

input
```
integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(c)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*sqrt(c)*cos(b*x + a)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x
 + a))) + 2*sqrt(c*cos(b*x + a))*sin(b*x + a))/(b*c^2*cos(b*x + a))
```

3.22.6 Sympy [F]

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\cos(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))**(3/2),x)
```

output
```
Integral((c*cos(a + b*x))**(-3/2), x)
```

3.22.7 Maxima [F]

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(-3/2), x)
```

3.22.8 Giac [F]

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(-3/2), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\,\cos(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(c*cos(a + b*x))^(3/2),x)
```

output
```
int(1/(c*cos(a + b*x))^(3/2), x)
```

3.23 $\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx$

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx=\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

output
```
2/3*sin(b*x+a)/b/c/(c*cos(b*x+a))^(3/2)+2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/c
os(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b
/c^2/(c*cos(b*x+a))^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx=\frac{2\left(\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\tan(a+bx)\right)}{3bc^2\sqrt{c\cos(a+bx)}}$$

input
```
Integrate[(c*Cos[a + b*x])^(-5/2),x]
```

output
```
(2*(Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Tan[a + b*x]))/(3*b*c^2
*Sqrt[c*Cos[a + b*x]])
```

3.23.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{\int \frac{1}{\sqrt{c\cos(a+bx)}}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3c^2}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3c^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3c^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

input
```
Int[(c*Cos[a + b*x])^(-5/2),x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*c^2*Sqrt[c*Cos[a + b
*x]]) + (2*Sin[a + b*x])/(3*b*c*(c*Cos[a + b*x])^(3/2))
```

3.23.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.

Time = 1.65 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,F\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\cdots\right.}{3c^2\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\sin\cdots}$

input
```
int(1/(c*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ell
ipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-2*sin(1/2*b*x+1/2*
a)^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/c^2*(c*(-1+2*cos(1/2*b*
x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/
2*b*x+1/2*a)^2))^(1/2)/(-1+2*cos(1/2*b*x+1/2*a)^2)/sin(1/2*b*x+1/2*a)/(c*(
-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)/b
```

3.23.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{c}\cos{(bx+a)}^2\,\mathrm{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\,\sin{(bx+a)})+$$

input
```
integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(c)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*x
+ a) + I*sin(b*x + a)) + I*sqrt(2)*sqrt(c)*cos(b*x + a)^2*weierstrassPInve
rse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*sqrt(c*cos(b*x + a))*sin(b*x
 + a))/(b*c^3*cos(b*x + a)^2)
```

3.23.6 Sympy [F]

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\cos{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))**(5/2),x)
```

output
```
Integral((c*cos(a + b*x))**(-5/2), x)
```

3.23.7 Maxima [F]

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(-5/2), x)
```

3.23.8 Giac [F]

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(-5/2), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(c*cos(a + b*x))^(5/2),x)
```

output
```
int(1/(c*cos(a + b*x))^(5/2), x)
```

3.24 $\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx$

3.24.1 Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = -\frac{6\sqrt{c\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}$$

output
```
2/5*sin(b*x+a)/b/c/(c*cos(b*x+a))^(5/2)+6/5*sin(b*x+a)/b/c^3/(c*cos(b*x+a)
)^(1/2)-6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(
1/2*a+1/2*b*x),2^(1/2))*(c*cos(b*x+a))^(1/2)/b/c^4/cos(b*x+a)^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \frac{-6\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)+6\sin(a+bx)+2\sec(a+bx)\tan(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}$$

input
```
Integrate[(c*Cos[a + b*x])^(-7/2),x]
```

output
```
(-6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + 6*Sin[a + b*x] + 2*Sec[
a + b*x]*Tan[a + b*x])/(5*b*c^3*Sqrt[c*Cos[a + b*x]])
```

3.24.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{3\int \frac{1}{(c\cos(a+bx))^{3/2}}dx}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{3\left(\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\int \sqrt{c\cos(a+bx)}dx}{c^2}\right)}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\int \sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{c^2}\right)}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{3\left(\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{c^2\sqrt{\cos(a+bx)}}\right)}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{\sqrt{c\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{c^2\sqrt{\cos(a+bx)}}\right)}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

$$\Big\downarrow 3119$$

$$\frac{3\left(\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)|2\right)\sqrt{c\cos(a+bx)}}{bc^2\sqrt{\cos(a+bx)}}\right)}{5c^2}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

input
```
Int[(c*Cos[a + b*x])^(-7/2),x]
```

output
```
(2*Sin[a + b*x])/(5*b*c*(c*Cos[a + b*x])^(5/2)) + (3*((-2*Sqrt[c*Cos[a + b
*x]]*EllipticE[(a + b*x)/2, 2])/(b*c^2*Sqrt[Cos[a + b*x]]) + (2*Sin[a + b*
x])/(b*c*Sqrt[c*Cos[a + b*x]])))/(5*c^2)
```

3.24.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(112) = 224.

Time = 2.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.66

method	result
default	$-\frac{2\sqrt{c\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(24\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{}}{}$

input
```
int(1/(c*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(c*(-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)/c^4/sin(1/
2*b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b
*x+1/2*a)^2-1)*(24*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6-12*(2*sin(1/2*b
*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2
*a)+12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/
2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+8*sin(1/2*b*x+1/2*a)
^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^
4*c+sin(1/2*b*x+1/2*a)^2*c)^(1/2)/(c*(-1+2*cos(1/2*b*x+1/2*a)^2))^(1/2)/b
```

3.24.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{c}\cos(bx+a)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx}{}$$

input
```
integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(c)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*sqrt(c)*cos
(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
 - I*sin(b*x + a))) + 2*sqrt(c*cos(b*x + a))*(3*cos(b*x + a)^2 + 1)*sin(b*
x + a))/(b*c^4*cos(b*x + a)^3)
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

3.24.7 Maxima [F]

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(-7/2), x)
```

3.24.8 Giac [F]

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(-7/2), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\left(c\,\cos\left(a+b\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(c*cos(a + b*x))^(7/2),x)
```

output
```
int(1/(c*cos(a + b*x))^(7/2), x)
```

3.25 $\int \cos^{\frac{4}{3}}(a+bx)\,dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx=-\frac{3\cos^{\frac{7}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sin(a+bx)}{7b\sqrt{\sin^2(a+bx)}}$$

output
```
-3/7*cos(b*x+a)^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x+a)
/b/(sin(b*x+a)^2)^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx$$
$$=-\frac{3\cos^{\frac{7}{3}}(a+bx)\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{7b}$$

input
```
Integrate[Cos[a + b*x]^(4/3),x]
```

output
```
(-3*Cos[a + b*x]^(7/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[
a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(7*b)
```

3.25.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^{4/3}dx$$

$$\Big\downarrow \; 3122$$

$$-\frac{3\sin(a+bx)\cos^{\frac{7}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(4/3),x]
```

output
```
(-3*Cos[a + b*x]^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*S
in[a + b*x])/(7*b*Sqrt[Sin[a + b*x]^2])
```

3.25.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.25.4 Maple [F]

$$\int \left(\cos^{\frac{4}{3}}\left(bx+a\right)\right)dx$$

input
```
int(cos(b*x+a)^(4/3),x)
```

output
```
int(cos(b*x+a)^(4/3),x)
```

3.25.5 Fricas [F]

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx = \int \cos\left(bx+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^(4/3), x)
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(4/3),x)
```

output
```
Timed out
```

3.25.7 Maxima [F]

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(4/3), x)
```

3.25.8 Giac [F]

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{4}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(4/3), x)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 13.89 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx = -\frac{3\cos(a+b\,x)^{7/3}\,\sin(a+b\,x)\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ \cos(a+b\,x)^2\right)}{7\,b\,\sqrt{\sin(a+b\,x)^2}}$$

input
```
int(cos(a + b*x)^(4/3),x)
```

output
```
-(3*cos(a + b*x)^(7/3)*sin(a + b*x)*hypergeom([1/2, 7/6], 13/6, cos(a + b*
x)^2))/(7*b*(sin(a + b*x)^2)^(1/2))
```

3.26 $\int \cos^{\frac{2}{3}}(a+bx)\,dx$

3.26.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx=-\frac{3\cos^{\frac{5}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sin(a+bx)}{5b\sqrt{\sin^2(a+bx)}}$$

output
```
-3/5*cos(b*x+a)^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x+a)
/b/(sin(b*x+a)^2)^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx = -\frac{3\cos^{\frac{5}{3}}(a+bx)\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{5b}$$

input
```
Integrate[Cos[a + b*x]^(2/3),x]
```

output
```
(-3*Cos[a + b*x]^(5/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[
a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(5*b)
```

3.26.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^{2/3}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(a+bx)\cos^{\frac{5}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(2/3),x]
```

output
```
(-3*Cos[a + b*x]^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*S
in[a + b*x])/(5*b*Sqrt[Sin[a + b*x]^2])
```

3.26.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.26.4 Maple [F]

$$\int \left(\cos^{\frac{2}{3}}(bx+a)\right) dx$$

input
```
int(cos(b*x+a)^(2/3),x)
```

output
```
int(cos(b*x+a)^(2/3),x)
```

3.26.5 Fricas [F]

$$\int \cos^{\frac{2}{3}}(a+bx)\, dx = \int \cos(bx+a)^{\frac{2}{3}}\, dx$$

input
```
integrate(cos(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^(2/3), x)
```

3.26.6 Sympy [F]

$$\int \cos^{\frac{2}{3}}(a+bx)\, dx = \int \cos^{\frac{2}{3}}(a+bx)\, dx$$

input
```
integrate(cos(b*x+a)**(2/3),x)
```

output
```
Integral(cos(a + b*x)**(2/3), x)
```

3.26.7 Maxima [F]

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(2/3), x)
```

3.26.8 Giac [F]

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{2}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(2/3), x)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx = -\frac{3\cos(a+b\,x)^{5/3}\,\sin(a+b\,x)\;{}_2F_1\left(\frac{1}{2},\frac{5}{6};\;\frac{11}{6};\;\cos(a+b\,x)^2\right)}{5\,b\,\sqrt{\sin(a+b\,x)^2}}$$

input
```
int(cos(a + b*x)^(2/3),x)
```

output
```
-(3*cos(a + b*x)^(5/3)*sin(a + b*x)*hypergeom([1/2, 5/6], 11/6, cos(a + b*
x)^2))/(5*b*(sin(a + b*x)^2)^(1/2))
```

3.27 $\int \sqrt[3]{\cos(a+bx)}\,dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \sqrt[3]{\cos(a+bx)}\,dx = -\frac{3\cos^{\frac{4}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sin(a+bx)}{4b\sqrt{\sin^2(a+bx)}}$$

output
```
-3/4*cos(b*x+a)^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+a)/
b/(sin(b*x+a)^2)^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{\cos(a+bx)}\,dx = -\frac{3\cos^{\frac{4}{3}}(a+bx)\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{4b}$$

input
```
Integrate[Cos[a + b*x]^(1/3),x]
```

output
```
(-3*Cos[a + b*x]^(4/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(4*b)
```

3.27.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)\cos^{\frac{4}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(1/3),x]
```

output
```
(-3*Cos[a + b*x]^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Si
n[a + b*x])/(4*b*Sqrt[Sin[a + b*x]^2])
```

3.27.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.27.4 Maple [F]

$$\int \left(\cos^{\frac{1}{3}}\left(bx+a\right)\right)dx$$

input
```
int(cos(b*x+a)^(1/3),x)
```

output
```
int(cos(b*x+a)^(1/3),x)
```

3.27.5 Fricas [F]

$$\int \sqrt[3]{\cos(a+bx)}\,dx = \int \cos\left(bx+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^(1/3), x)
```

3.27.6 Sympy [F]

$$\int \sqrt[3]{\cos(a+bx)}\,dx = \int \sqrt[3]{\cos\left(a+bx\right)}\,dx$$

input
```
integrate(cos(b*x+a)**(1/3),x)
```

output
```
Integral(cos(a + b*x)**(1/3), x)
```

3.27.7 Maxima [F]

$$\int \sqrt[3]{\cos(a+bx)}\,dx = \int \cos\left(bx+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(1/3), x)
```

3.27.8 Giac [F]

$$\int \sqrt[3]{\cos(a+bx)}\,dx = \int \cos\left(bx+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(1/3), x)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \sqrt[3]{\cos(a+bx)}\,dx = -\frac{3\cos\left(a+b\,x\right)^{4/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{5}{3};\ \cos\left(a+b\,x\right)^2\right)}{4\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(cos(a + b*x)^(1/3),x)
```

output
```
-(3*cos(a + b*x)^(4/3)*sin(a + b*x)*hypergeom([1/2, 2/3], 5/3, cos(a + b*x
)^2))/(4*b*(sin(a + b*x)^2)^(1/2))
```

3.28 $\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx$

3.28.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos^{\frac{2}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sin(a+bx)}{2b\sqrt{\sin^2(a+bx)}}$$

output
```
-3/2*cos(b*x+a)^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+a)/
b/(sin(b*x+a)^2)^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos^{\frac{2}{3}}(a+bx)\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{2b}$$

input
```
Integrate[Cos[a + b*x]^(-1/3),x]
```

output
```
(-3*Cos[a + b*x]^(2/3)*Csc[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(2*b)
```

3.28.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt[3]{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{ 3122}$$
$$-\frac{3\sin(a+bx)\cos^{\frac{2}{3}}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(-1/3),x]
```

output
```
(-3*Cos[a + b*x]^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Si
n[a + b*x])/(2*b*Sqrt[Sin[a + b*x]^2])
```

3.28.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.28.4 Maple [F]

$$\int \frac{1}{\cos \left(b x+a\right)^{\frac{1}{3}}}d x$$

input
```
int(1/cos(b*x+a)^(1/3),x)
```

output
```
int(1/cos(b*x+a)^(1/3),x)
```

3.28.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^(-1/3), x)
```

3.28.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt[3]{\cos\left(a+bx\right)}}\,dx$$

input
```
integrate(1/cos(b*x+a)**(1/3),x)
```

output
```
Integral(cos(a + b*x)**(-1/3), x)
```

3.28.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-1/3), x)
```

3.28.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-1/3), x)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos\left(a+b\,x\right)^{2/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ \cos\left(a+b\,x\right)^2\right)}{2\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(1/cos(a + b*x)^(1/3),x)
```

output
```
-(3*cos(a + b*x)^(2/3)*sin(a + b*x)*hypergeom([1/3, 1/2], 4/3, cos(a + b*x
)^2))/(2*b*(sin(a + b*x)^2)^(1/2))
```

3.29 $\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

output
```
-3*cos(b*x+a)^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)/b/
(sin(b*x+a)^2)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos(a+bx)}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b}$$

input
```
Integrate[Cos[a + b*x]^(-2/3),x]
```

output
```
(-3*Cos[a + b*x]^(1/3)*Csc[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/b
```

3.29.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{2/3}}dx$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3\sin(a+bx)\sqrt[3]{\cos(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(-2/3),x]
```

output
```
(-3*Cos[a + b*x]^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

3.29.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.29.4 Maple [F]

$$\int \frac{1}{\cos (bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/cos(b*x+a)^(2/3),x)
```

output
```
int(1/cos(b*x+a)^(2/3),x)
```

3.29.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\cos (bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^(-2/3), x)
```

3.29.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(1/cos(b*x+a)**(2/3),x)
```

output
```
Integral(cos(a + b*x)**(-2/3), x)
```

3.29.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-2/3), x)
```

3.29.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-2/3), x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\cos\left(a+b\,x\right)^{1/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(1/cos(a + b*x)^(2/3),x)
```

output
```
-(3*cos(a + b*x)^(1/3)*sin(a + b*x)*hypergeom([1/6, 1/2], 7/6, cos(a + b*x
)^2))/(b*(sin(a + b*x)^2)^(1/2))
```

3.30 $\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx$

3.30.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
3*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*sin(b*x+a)/b/cos(b*x+a)^(1/3)/
(sin(b*x+a)^2)^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(-4/3),x]
```

output
```
(3*Csc[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sqrt[Sin
[a + b*x]^2])/(b*Cos[a + b*x]^(1/3))
```

3.30.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{4/3}}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3\sin(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\cos(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(-4/3),x]
```

output
```
(3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Cos[
a + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

3.30.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.30.4 Maple [F]

$$\int \frac{1}{\cos(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(1/cos(b*x+a)^(4/3),x)
```

output
```
int(1/cos(b*x+a)^(4/3),x)
```

3.30.5 Fricas [F]

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\cos(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^(-4/3), x)
```

3.30.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(1/cos(b*x+a)**(4/3),x)
```

output
```
Integral(cos(a + b*x)**(-4/3), x)
```

3.30.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-4/3), x)
```

3.30.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{1}{\cos\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-4/3), x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 15.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{2};\ \frac{5}{6};\ \cos\left(a+b\,x\right)^2\right)}{b\cos\left(a+b\,x\right)^{1/3}\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(1/cos(a + b*x)^(4/3),x)
```

output
```
(3*sin(a + b*x)*hypergeom([-1/6, 1/2], 5/6, cos(a + b*x)^2))/(b*cos(a + b*
x)^(1/3)*(sin(a + b*x)^2)^(1/2))
```

3.31 $\int (c\cos(a+bx))^{4/3}\,dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c\cos(a+bx))^{4/3}\,dx = -\frac{3(c\cos(a+bx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sin(a+bx)}{7bc\sqrt{\sin^2(a+bx)}}$$

output
```
-3/7*(c*cos(b*x+a))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*
x+a)/b/c/(sin(b*x+a)^2)^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c\cos(a+bx))^{4/3}\,dx = -\frac{3(c\cos(a+bx))^{4/3}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{7b}$$

input
```
Integrate[(c*Cos[a + b*x])^(4/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(4/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6,
Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(7*b)
```

3.31.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(a+bx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)}{7bc\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(4/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^
2]*Sin[a + b*x])/(7*b*c*Sqrt[Sin[a + b*x]^2])
```

3.31.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.31.4 Maple [F]

$$\int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

input
```
int((c*cos(b*x+a))^(4/3),x)
```

output
```
int((c*cos(b*x+a))^(4/3),x)
```

3.31.5 Fricas [F]

$$\int (c\cos(a+bx))^{4/3}\,dx = \int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(4/3),x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^(1/3)*c*cos(b*x + a), x)
```

3.31.6 Sympy [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((c*cos(b*x+a))**(4/3),x)
```

output
```
Timed out
```

3.31.7 Maxima [F]

$$\int (c\cos(a+bx))^{4/3}\,dx = \int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(4/3), x)
```

3.31.8 Giac [F]

$$\int (c\cos(a+bx))^{4/3}\,dx = \int (c\cos(bx+a))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(4/3),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(4/3), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{4/3}\,dx = \int (c\,\cos(a+b\,x))^{4/3}\,dx$$

input
```
int((c*cos(a + b*x))^(4/3),x)
```

output
```
int((c*cos(a + b*x))^(4/3), x)
```

3.32 $\int (c\cos(a+bx))^{2/3}\,dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c\cos(a+bx))^{2/3}\,dx = -\frac{3(c\cos(a+bx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sin(a+bx)}{5bc\sqrt{\sin^2(a+bx)}}$$

output
```
-3/5*(c*cos(b*x+a))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*
x+a)/b/c/(sin(b*x+a)^2)^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c\cos(a+bx))^{2/3}\,dx = -\frac{3(c\cos(a+bx))^{2/3}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{5b}$$

input
```
Integrate[(c*Cos[a + b*x])^(2/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(2/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6,
Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(5*b)
```

3.32.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(a+bx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)}{5bc\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(2/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^
2]*Sin[a + b*x])/(5*b*c*Sqrt[Sin[a + b*x]^2])
```

3.32.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.32.4 Maple [F]

$$\int \left(c \cos \left(b x +a \right)\right)^{\frac{2}{3}}d x$$

input
```
int((c*cos(b*x+a))^(2/3),x)
```

output
```
int((c*cos(b*x+a))^(2/3),x)
```

3.32.5 Fricas [F]

$$\int (c\cos(a+bx))^{2/3}\,dx = \int \left(c \cos \left(bx + a\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(2/3),x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^(2/3), x)
```

3.32.6 Sympy [F]

$$\int (c\cos(a+bx))^{2/3}\,dx = \int \left(c \cos \left(a + bx\right)\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))**(2/3),x)
```

output
```
Integral((c*cos(a + b*x))**(2/3), x)
```

3.32.7 Maxima [F]

$$\int (c\cos(a+bx))^{2/3}\,dx = \int (c\cos(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(2/3), x)
```

3.32.8 Giac [F]

$$\int (c\cos(a+bx))^{2/3}\,dx = \int (c\cos(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(2/3),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(2/3), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^{2/3}\,dx = \int (c\,\cos(a+b\,x))^{2/3}\,dx$$

input
```
int((c*cos(a + b*x))^(2/3),x)
```

output
```
int((c*cos(a + b*x))^(2/3), x)
```

3.33 $\int \sqrt[3]{c\cos(a+bx)}\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = -\frac{3(c\cos(a+bx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sin(a+bx)}{4bc\sqrt{\sin^2(a+bx)}}$$

output
```
-3/4*(c*cos(b*x+a))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x
+a)/b/c/(sin(b*x+a)^2)^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = -\frac{3\sqrt[3]{c\cos(a+bx)}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{4b}$$

input
```
Integrate[(c*Cos[a + b*x])^(1/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(1/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, C
os[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(4*b)
```

3.33.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{c\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)}{4bc\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(1/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2
]*Sin[a + b*x])/(4*b*c*Sqrt[Sin[a + b*x]^2])
```

3.33.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.33.4 Maple [F]

$$\int (c\cos(bx+a))^{\frac{1}{3}}\,dx$$

input
```
int((c*cos(b*x+a))^(1/3),x)
```

output
```
int((c*cos(b*x+a))^(1/3),x)
```

3.33.5 Fricas [F]

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = \int (c\cos(bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(1/3),x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^(1/3), x)
```

3.33.6 Sympy [F]

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = \int \sqrt[3]{c\cos(a+bx)}\,dx$$

input
```
integrate((c*cos(b*x+a))**(1/3),x)
```

output
```
Integral((c*cos(a + b*x))**(1/3), x)
```

3.33.7 Maxima [F]

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = \int (c\cos(bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(1/3), x)
```

3.33.8 Giac [F]

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = \int (c\cos(bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*cos(b*x+a))^(1/3),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(1/3), x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = \int (c\,\cos(a+b\,x))^{1/3}\,dx$$

input
```
int((c*cos(a + b*x))^(1/3),x)
```

output
```
int((c*cos(a + b*x))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx$

3.34.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = -\frac{3(c\cos(a+bx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sin(a+bx)}{2bc\sqrt{\sin^2(a+bx)}}$$

output
```
-3/2*(c*cos(b*x+a))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x
+a)/b/c/(sin(b*x+a)^2)^(1/2)
```

3.34.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = -\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{2b\sqrt[3]{c\cos(a+bx)}}$$

input
```
Integrate[(c*Cos[a + b*x])^(-1/3),x]
```

output
```
(-3*Cot[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sqrt[Sin
[a + b*x]^2])/(2*b*(c*Cos[a + b*x])^(1/3))
```

3.34.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{c\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)}{2bc\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(-1/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2
]*Sin[a + b*x])/(2*b*c*Sqrt[Sin[a + b*x]^2])
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.34.4 Maple [F]

$$\int \frac{1}{\left(c\cos\left(bx+a\right)\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(c*cos(b*x+a))^(1/3),x)
```

output
```
int(1/(c*cos(b*x+a))^(1/3),x)
```

3.34.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = \int \frac{1}{\left(c\cos\left(bx+a\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^(2/3)/(c*cos(b*x + a)), x)
```

3.34.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt[3]{c\cos\left(a+bx\right)}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))**(1/3),x)
```

output
```
Integral((c*cos(a + b*x))**(-1/3), x)
```

3.34.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(-1/3), x)
```

3.34.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(-1/3), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx = \int \frac{1}{(c\,\cos(a+b\,x))^{1/3}}\,dx$$

input
```
int(1/(c*cos(a + b*x))^(1/3),x)
```

output
```
int(1/(c*cos(a + b*x))^(1/3), x)
```

3.35 $\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx$

3.35.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = -\frac{3\sqrt[3]{c\cos(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt{\sin^2(a+bx)}}$$

output
```
-3*(c*cos(b*x+a))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a
)/b/c/(sin(b*x+a)^2)^(1/2)
```

3.35.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = -\frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(c\cos(a+bx))^{2/3}}$$

input
```
Integrate[(c*Cos[a + b*x])^(-2/3),x]
```

output
```
(-3*Cot[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sqrt[Sin
[a + b*x]^2])/(b*(c*Cos[a + b*x])^(2/3))
```

3.35.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(a+bx)\sqrt[3]{c\cos(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(-2/3),x]
```

output
```
(-3*(c*Cos[a + b*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(b*c*Sqrt[Sin[a + b*x]^2])
```

3.35.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.35.4 Maple [F]

$$\int \frac{1}{(c\cos(bx+a))^{\frac{2}{3}}}dx$$

input
```
int(1/(c*cos(b*x+a))^(2/3),x)
```

output
```
int(1/(c*cos(b*x+a))^(2/3),x)
```

3.35.5 Fricas [F]

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^(1/3)/(c*cos(b*x + a)), x)
```

3.35.6 Sympy [F]

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\cos(a+bx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))**(2/3),x)
```

output
```
Integral((c*cos(a + b*x))**(-2/3), x)
```

3.35.7 Maxima [F]

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(-2/3), x)
```

3.35.8 Giac [F]

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(-2/3), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\,\cos(a+b\,x))^{2/3}}\,dx$$

input
```
int(1/(c*cos(a + b*x))^(2/3),x)
```

output
```
int(1/(c*cos(a + b*x))^(2/3), x)
```

3.36 $\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx$

3.36.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt[3]{c\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
3*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*sin(b*x+a)/b/c/(c*cos(b*x+a))^
(1/3)/(sin(b*x+a)^2)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx = \frac{3\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(c\cos(a+bx))^{4/3}}$$

input
```
Integrate[(c*Cos[a + b*x])^(-4/3),x]
```

output
```
(3*Cot[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sqrt[Sin
[a + b*x]^2])/(b*(c*Cos[a + b*x])^(4/3))
```

3.36.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}\sqrt[3]{c\cos(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^(-4/3),x]
```

output
```
(3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(c
*Cos[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.36.4 Maple [F]

$$\int \frac{1}{(c \cos (bx + a))^{\frac{4}{3}}} dx$$

input
```
int(1/(c*cos(b*x+a))^(4/3),x)
```

output
```
int(1/(c*cos(b*x+a))^(4/3),x)
```

3.36.5 Fricas [F]

$$\int \frac{1}{(c \cos(a + bx))^{4/3}} dx = \int \frac{1}{(c \cos (bx + a))^{\frac{4}{3}}} dx$$

input
```
integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2), x)
```

3.36.6 Sympy [F]

$$\int \frac{1}{(c \cos(a + bx))^{4/3}} dx = \int \frac{1}{(c \cos (a + bx))^{\frac{4}{3}}} dx$$

input
```
integrate(1/(c*cos(b*x+a))**(4/3),x)
```

output
```
Integral((c*cos(a + b*x))**(-4/3), x)
```

3.36.7 Maxima [F]

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^(-4/3), x)
```

3.36.8 Giac [F]

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\cos(bx+a))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^(-4/3), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\,\cos(a+b\,x))^{4/3}}\,dx$$

input
```
int(1/(c*cos(a + b*x))^(4/3),x)
```

output
```
int(1/(c*cos(a + b*x))^(4/3), x)
```

3.37 $\int \cos^n(a+bx)\,dx$

3.37.1 Optimal result

Integrand size = 8, antiderivative size = 64

$$\int \cos^n(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{b(1+n)\sqrt{\sin^2(a+bx)}}$$

output
```
-cos(b*x+a)^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*sin
(b*x+a)/b/(1+n)/(sin(b*x+a)^2)^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \cos^n(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(1+n)}$$

input
```
Integrate[Cos[a + b*x]^n,x]
```

output
```
-((Cos[a + b*x]^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3
+ n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

3.37.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(a+bx)\cos^{n+1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{b(n+1)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^n,x]
```

output
```
-((Cos[a + b*x]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[a
 + b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

3.37.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.37.4 Maple [F]

$$\int (\cos^n (bx + a))\, dx$$

input
```
int(cos(b*x+a)^n,x)
```

output
```
int(cos(b*x+a)^n,x)
```

3.37.5 Fricas [F]

$$\int \cos^n(a + bx)\, dx = \int \cos{(bx + a)}^n\, dx$$

input
```
integrate(cos(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^n, x)
```

3.37.6 Sympy [F]

$$\int \cos^n(a + bx)\, dx = \int \cos^n{(a + bx)}\, dx$$

input
```
integrate(cos(b*x+a)**n,x)
```

output
```
Integral(cos(a + b*x)**n, x)
```

3.37.7 Maxima [F]

$$\int \cos^n(a+bx)\,dx = \int \cos(bx+a)^n\,dx$$

input
```
integrate(cos(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^n, x)
```

3.37.8 Giac [F]

$$\int \cos^n(a+bx)\,dx = \int \cos(bx+a)^n\,dx$$

input
```
integrate(cos(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^n, x)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 14.95 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \cos^n(a+bx)\,dx = -\frac{\cos(a+b\,x)^{n+1}\,\sin(a+b\,x)\;{}_2F_1\left(\frac{1}{2},\frac{n}{2}+\frac{1}{2};\;\frac{n}{2}+\frac{3}{2};\;\cos(a+b\,x)^2\right)}{b\,\sqrt{\sin(a+b\,x)^2}\;(n+1)}$$

input
```
int(cos(a + b*x)^n,x)
```

output
```
-(cos(a + b*x)^(n + 1)*sin(a + b*x)*hypergeom([1/2, n/2 + 1/2], n/2 + 3/2,
 cos(a + b*x)^2))/(b*(sin(a + b*x)^2)^(1/2)*(n + 1))
```

3.38 $\int (c\cos(a+bx))^n\,dx$

3.38.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int (c\cos(a+bx))^n\,dx = -\frac{(c\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bc(1+n)\sqrt{\sin^2(a+bx)}}$$

output
```
-(c*cos(b*x+a))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*sin(b*x+a)/b/c/(1+n)/(sin(b*x+a)^2)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int (c\cos(a+bx))^n\,dx = -\frac{(c\cos(a+bx))^n\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(1+n)}$$

input
```
Integrate[(c*Cos[a + b*x])^n,x]
```

output
```
-(((c*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

3.38.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(a+bx)(c\cos(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bc(n+1)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x])^n,x]
```

output
```
-(((c*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*c*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

3.38.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.38.4 Maple [F]

$$\int (c\cos(bx+a))^n\,dx$$

input
```
int((c*cos(b*x+a))^n,x)
```

output
```
int((c*cos(b*x+a))^n,x)
```

3.38.5 Fricas [F]

$$\int (c\cos(a+bx))^n\,dx = \int (c\cos(bx+a))^n\,dx$$

input
```
integrate((c*cos(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((c*cos(b*x + a))^n, x)
```

3.38.6 Sympy [F]

$$\int (c\cos(a+bx))^n\,dx = \int (c\cos(a+bx))^n\,dx$$

input
```
integrate((c*cos(b*x+a))**n,x)
```

output
```
Integral((c*cos(a + b*x))**n, x)
```

3.38.7 Maxima [F]

$$\int (c\cos(a+bx))^n\,dx = \int (c\cos(bx+a))^n\,dx$$

input
```
integrate((c*cos(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a))^n, x)
```

3.38.8 Giac [F]

$$\int (c\cos(a+bx))^n\,dx = \int (c\cos(bx+a))^n\,dx$$

input
```
integrate((c*cos(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a))^n, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos(a+bx))^n\,dx = \int (c\,\cos(a+b\,x))^n\,dx$$

input
```
int((c*cos(a + b*x))^n,x)
```

output
```
int((c*cos(a + b*x))^n, x)
```

3.39 $\int (a\cos^2(x))^{5/2}\,dx$

3.39.1 Optimal result

Integrand size = 10, antiderivative size = 53

$$\int (a\cos^2(x))^{5/2}\,dx = \frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x) + \frac{4}{15}a(a\cos^2(x))^{3/2}\tan(x) + \frac{1}{5}(a\cos^2(x))^{5/2}\tan(x)$$

output
```
4/15*a*(a*cos(x)^2)^(3/2)*tan(x)+1/5*(a*cos(x)^2)^(5/2)*tan(x)+8/15*a^2*(a
*cos(x)^2)^(1/2)*tan(x)
```

3.39.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int (a\cos^2(x))^{5/2}\,dx = \frac{1}{15}a^2\sqrt{a\cos^2(x)}\left(15-10\sin^2(x)+3\sin^4(x)\right)\tan(x)$$

input
```
Integrate[(a*Cos[x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cos[x]^2]*(15 - 10*Sin[x]^2 + 3*Sin[x]^4)*Tan[x])/15
```

3.39.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cos^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\cos^2(x)\right)^{3/2}dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}\right)+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}\right)$$

input
```
Int[(a*Cos[x]^2)^(5/2),x]
```

output
```
((a*Cos[x]^2)^(5/2)*Tan[x])/5 + (4*a*((2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((
a*Cos[x]^2)^(3/2)*Tan[x])/3))/5
```

3.39.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.39.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{a^{3}\cos(x)\sin(x)\left(3\left(\cos^{4}(x)\right)+4\left(\cos^{2}(x)\right)+8\right)}{15\sqrt{a\left(\cos^{2}(x)\right)}}$
risch	$-\frac{i a^{2} e^{6 i x}\sqrt{a\left(e^{2 i x}+1\right)^{2} e^{-2 i x}}}{160\left(e^{2 i x}+1\right)}-\frac{5 i a^{2} e^{2 i x}\sqrt{a\left(e^{2 i x}+1\right)^{2} e^{-2 i x}}}{16\left(e^{2 i x}+1\right)}+\frac{5 i a^{2}\sqrt{a\left(e^{2 i x}+1\right)^{2} e^{-2 i x}}}{16\left(e^{2 i x}+1\right)}+\frac{5 i a^{2} e^{-2 i x}\sqrt{a\left(e^{2 i x}+1\right)^{2} e^{-2 i x}}}{96\left(e^{2 i x}+1\right)}-$ 11

input
```
int((a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*cos(x)*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \left(a\cos^2(x)\right)^{5/2}\, dx = \frac{\left(3\,a^2\cos\left(x\right)^4 + 4\,a^2\cos\left(x\right)^2 + 8\,a^2\right)\sqrt{a\cos\left(x\right)^2}\sin\left(x\right)}{15\,\cos\left(x\right)}$$

input
```
integrate((a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos
(x)
```

3.39.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cos^2(x)\right)^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((a*cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \left(a\cos^2(x)\right)^{5/2}\, dx = \frac{1}{240}\left(3\,a^2\sin\left(5\,x\right) + 25\,a^2\sin\left(3\,x\right) + 150\,a^2\sin\left(x\right)\right)\sqrt{a}$$

input
```
integrate((a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \left(a\cos^2(x)\right)^{5/2} dx = \frac{1}{15}\left(3\,a^2\sin\left(x\right)^5 - 10\,a^2\sin\left(x\right)^3 + 15\,a^2\sin\left(x\right)\right)\sqrt{a}\mathrm{sgn}(\cos\left(x\right))$$

input
```
integrate((a*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/15*(3*a^2*sin(x)^5 - 10*a^2*sin(x)^3 + 15*a^2*sin(x))*sqrt(a)*sgn(cos(x)
)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cos^2(x)\right)^{5/2} dx = \int \left(a\cos\left(x\right)^2\right)^{5/2} dx$$

input
```
int((a*cos(x)^2)^(5/2),x)
```

output
```
int((a*cos(x)^2)^(5/2), x)
```

3.40 $\int (a\cos^2(x))^{3/2}\, dx$

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int (a\cos^2(x))^{3/2}\, dx = \frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x) + \frac{1}{3}(a\cos^2(x))^{3/2}\tan(x)$$

output
```
1/3*(a*cos(x)^2)^(3/2)*tan(x)+2/3*a*(a*cos(x)^2)^(1/2)*tan(x)
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int (a\cos^2(x))^{3/2}\, dx = -\frac{1}{3}a\sqrt{a\cos^2(x)}(-3+\sin^2(x))\tan(x)$$

input
```
Integrate[(a*Cos[x]^2)^(3/2),x]
```

output
```
-1/3*(a*Sqrt[a*Cos[x]^2]*(-3 + Sin[x]^2)*Tan[x])
```

3.40.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cos^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\cos^2(x)}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx+\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

input
```
Int[(a*Cos[x]^2)^(3/2),x]
```

output
```
(2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

3.40.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.40.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{a^{2} \cos \left(x \right) \sin \left(x \right) \left(\cos^{2}\left(x \right)+2\right)}{3 \sqrt{a \left(\cos^{2}\left(x \right)\right)}}$	24
risch	$-\frac{i a \,{\mathrm e}^{4 i x} \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \left({\mathrm e}^{2 i x}+1\right)}-\frac{3 i a \,{\mathrm e}^{2 i x} \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}+1\right)}+\frac{3 i a \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{8 \left({\mathrm e}^{2 i x}+1\right)}+\frac{i a \,{\mathrm e}^{-2 i x} \sqrt{a \left({\mathrm e}^{2 i x}+1\right)^{2} {\mathrm e}^{-2 i x}}}{24 \,{\mathrm e}^{2 i x}+24}$	141

input
```
int((a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*cos(x)*sin(x)*(cos(x)^2+2)/(a*cos(x)^2)^(1/2)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a\cos^2(x)\right)^{3/2}\,dx = \frac{\left(a\cos\left(x\right)^2+2\,a\right)\sqrt{a\cos\left(x\right)^2}\sin\left(x\right)}{3\,\cos\left(x\right)}$$

input
```
integrate((a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

3.40.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cos^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(x)**2)**(3/2),x)
```

output
```
Timed out
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \left(a\cos^2(x)\right)^{3/2}\,dx = \frac{1}{12}\left(a\sin\left(3\,x\right)+9\,a\sin\left(x\right)\right)\sqrt{a}$$

input
```
integrate((a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.50

$$\int \left(a\cos^2(x)\right)^{3/2}\,dx=-\frac{1}{3}\left(\sin\left(x\right)^3-3\,\sin\left(x\right)\right)a^{\frac{3}{2}}\mathrm{sgn}(\cos\left(x\right))$$

input
```
integrate((a*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))*a^(3/2)*sgn(cos(x))
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cos^2(x)\right)^{3/2}\,dx=\int \left(a\cos\left(x\right)^2\right)^{3/2}\,dx$$

input
```
int((a*cos(x)^2)^(3/2),x)
```

output
```
int((a*cos(x)^2)^(3/2), x)
```

3.41 $\int \sqrt{a \cos^2(x)}\,dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \sqrt{a \cos^2(x)}\,dx = \sqrt{a \cos^2(x)}\tan(x)$$

output
```
(a*cos(x)^2)^(1/2)*tan(x)
```

3.41.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a \cos^2(x)}\,dx = \sqrt{a \cos^2(x)}\tan(x)$$

input
```
Integrate[Sqrt[a*Cos[x]^2],x]
```

output
```
Sqrt[a*Cos[x]^2]*Tan[x]
```

3.41.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec(x)\sqrt{a\cos^2(x)}\int \cos(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(x)\sqrt{a\cos^2(x)}\int \sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\tan(x)\sqrt{a\cos^2(x)}$$

input
```
Int[Sqrt[a*Cos[x]^2],x]
```

output
```
Sqrt[a*Cos[x]^2]*Tan[x]
```

3.41.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.41.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a\cos(x)\sin(x)}{\sqrt{a(\cos^2(x))}}$	15
risch	$-\frac{i\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}\,\mathrm{e}^{2ix}}{2(\mathrm{e}^{2ix}+1)}+\frac{i\sqrt{a(\mathrm{e}^{2ix}+1)^2\mathrm{e}^{-2ix}}}{2\,\mathrm{e}^{2ix}+2}$	67

input
```
int((a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*cos(x)^2)^(1/2)*a*cos(x)*sin(x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int\sqrt{a\cos^2(x)}\,dx=\frac{\sqrt{a\cos(x)^2}\sin(x)}{\cos(x)}$$

input
```
integrate((a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a\cos^2(x)}\,dx = \frac{\sqrt{a\cos^2(x)}\sin(x)}{\cos(x)}$$

input
```
integrate((a*cos(x)**2)**(1/2),x)
```

output
```
sqrt(a*cos(x)**2)*sin(x)/cos(x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \sqrt{a\cos^2(x)}\,dx = \sqrt{a}\sin(x)$$

input
```
integrate((a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*sin(x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{a\cos^2(x)}\,dx = \sqrt{a}\,\mathrm{sgn}(\cos(x))\sin(x)$$

input
```
integrate((a*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a)*sgn(cos(x))*sin(x)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 14.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \sqrt{a\cos^2(x)}\,dx = \frac{\sqrt{2}\,\sqrt{a}\,\sqrt{\cos(2\,x)+1}\,(\cos(2\,x)-1+\sin(2\,x)\;1\mathrm{i})}{2\,\left(\cos(2\,x)\;1\mathrm{i}-\sin(2\,x)+1\mathrm{i}\right)}$$

input
```
int((a*cos(x)^2)^(1/2),x)
```

output
```
(2^(1/2)*a^(1/2)*(cos(2*x) + 1)^(1/2)*(cos(2*x) + sin(2*x)*1i - 1))/(2*(co
s(2*x)*1i - sin(2*x) + 1i))
```

3.42 $\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

output
```
arctanh(sin(x))*cos(x)/(a*cos(x)^2)^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Cos[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

3.42.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos^2(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \; 3686$$

$$\frac{\cos(x)\int \sec(x)dx}{\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{\cos(x)\text{arctanh}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

input
```
Int[1/Sqrt[a*Cos[x]^2],x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.42.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(14) = 28$.

Time = 0.90 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.00

method	result	size
default	$\frac{\cos(x)\sqrt{a\left(\sin^2(x)\right)}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)}{\sqrt{a}\,\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$	48
risch	$-\frac{2\ln\left(e^{ix}-i\right)\cos(x)}{\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}+\frac{2\ln\left(e^{ix}+i\right)\cos(x)}{\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}$	64

input
```
int(1/(a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(x)*(a*sin(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*sin(x)^2)^(1/2)+a)/cos(
x))/sin(x)/(a*cos(x)^2)^(1/2)
```

3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \left[-\frac{\sqrt{a\cos\left(x\right)^2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos\left(x\right)}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(x)^2}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

input
```
integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-
a)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

3.42.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos^2(x)}}\,dx$$

input
```
integrate(1/(a*cos(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cos(x)**2), x)
```

3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \frac{\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) - \log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)}{2\sqrt{a}}$$

input
```
integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2
*sin(x) + 1))/sqrt(a)
```

3.42.8 Giac [F]

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos{(x)}^2}}\,dx$$

input
```
integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cos(x)^2), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cos{(x)}^2}}\,dx$$

input
```
int(1/(a*cos(x)^2)^(1/2),x)
```

output
```
int(1/(a*cos(x)^2)^(1/2), x)
```

3.43 $\int \frac{1}{(a\cos^2(x))^{3/2}}\,dx$

3.43.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{(a\cos^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

output
```
1/2*arctanh(sin(x))*cos(x)/a/(a*cos(x)^2)^(1/2)+1/2*tan(x)/a/(a*cos(x)^2)^
(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a\cos^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(x))\cos(x) + \tan(x)}{2a\sqrt{a\cos^2(x)}}$$

input
```
Integrate[(a*Cos[x]^2)^(-3/2),x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x] + Tan[x])/(2*a*Sqrt[a*Cos[x]^2])
```

3.43.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a \cos^2(x)\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a \sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3683}$$

$$\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\cos(x)\int \sec(x)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cos(x)\int \csc\left(x+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\cos(x)\text{arctanh}(\sin(x))}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

input
```
Int[(a*Cos[x]^2)^(-3/2),x]
```

output
```
(ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x
]^2])
```

3.43.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.43.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 1.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\sqrt{a\left(\sin^2(x)\right)}\left(\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)a\left(\cos^2(x)\right)+\sqrt{a}\sqrt{a\left(\sin^2(x)\right)}\right)}{2a^{\frac{5}{2}}\cos(x)\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$	70
risch	$-\frac{i\left(\mathrm{e}^{2ix}-1\right)}{a\left(\mathrm{e}^{2ix}+1\right)\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}-\frac{\ln\left(\mathrm{e}^{ix}-i\right)\cos(x)}{a\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}+\frac{\ln\left(\mathrm{e}^{ix}+i\right)\cos(x)}{a\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}$	109

input
```
int(1/(a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^(5/2)/cos(x)*(a*sin(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*sin(x)^2)^(1/2)+a)
/cos(x))*a*cos(x)^2+a^(1/2)*(a*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx = -\frac{\sqrt{a\cos\left(x\right)^2}\left(\cos\left(x\right)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\,\sin\left(x\right)\right)}{4\,a^2\cos\left(x\right)^3}$$

input
```
integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x)
)/(a^2*cos(x)^3)
```

3.43.6 Sympy [F]

$$\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cos^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(x)**2)**(3/2),x)
```

output
```
Integral((a*cos(x)**2)**(-3/2), x)
```

3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(34) = 68.

Time = 0.40 (sec) , antiderivative size = 304, normalized size of antiderivative = 7.24

$$\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx = \frac{4\left(\sin\left(3\,x\right)-\sin\left(x\right)\right)\cos\left(4\,x\right)+\left(2\left(2\,\cos\left(2\,x\right)+1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^2+4\,\cos\left(2\,x\right.}{}$$

input
```
integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4
*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4
*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) +
1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x)
 + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x
)*sin(2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x
)^2 + 4*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)
^2 + 2*(2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a\cos^2(x))^{3/2}}\,dx = -\frac{\sin(x)}{2\left(\sin(x)^2-1\right)a^{\frac{3}{2}}\operatorname{sgn}(\cos(x))}$$

input
```
integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sin(x)/((sin(x)^2 - 1)*a^(3/2)*sgn(cos(x)))
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos^2(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos(x)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a*cos(x)^2)^(3/2),x)
```

output
```
int(1/(a*cos(x)^2)^(3/2), x)
```

3.44 $\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx$

3.44.1 Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx=\frac{3\text{arctanh}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

output
```
3/8*arctanh(sin(x))*cos(x)/a^2/(a*cos(x)^2)^(1/2)+1/4*tan(x)/a/(a*cos(x)^2
)^(3/2)+3/8*tan(x)/a^2/(a*cos(x)^2)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx=\frac{3\text{arctanh}(\sin(x))\cos(x)+\left(3+2\sec^2(x)\right)\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

input
```
Integrate[(a*Cos[x]^2)^(-5/2),x]
```

output
```
(3*ArcTanh[Sin[x]]*Cos[x] + (3 + 2*Sec[x]^2)*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^
2])
```

3.44.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos^2(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{(a\cos^2(x))^{3/2}}dx}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^2\right)^{3/2}}dx}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^2}}dx}{2a}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3686}$$

$$\frac{3\left(\frac{\cos(x)\int \sec(x)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a(a\cos^2(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\cos(x)\int\csc\left(x+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{3\left(\frac{\cos(x)\operatorname{arctanh}(\sin(x))}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\right)}{4a}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

input
```
Int[(a*Cos[x]^2)^(-5/2),x]
```

output
```
Tan[x]/(4*a*(a*Cos[x]^2)^(3/2)) + (3*((ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a
*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]^2])))/(4*a)
```

3.44.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.44.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\sqrt{a\left(\sin^{2}\left(x\right)\right)}\,\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(x\right)\right)}+2a}{\cos\left(x\right)}\right)a\left(\cos^{4}\left(x\right)\right)+3\sqrt{a\left(\sin^{2}\left(x\right)\right)}\,\left(\cos^{2}\left(x\right)\right)\sqrt{a}+2\sqrt{a}\,\sqrt{a\left(\sin^{2}\left(x\right)\right)}\right)}{8a^{\frac{7}{2}}\cos\left(x\right)^{3}\sin\left(x\right)\sqrt{a\left(\cos^{2}\left(x\right)\right)}}$	89
risch	$-\frac{i\left(3\,{\mathrm e}^{6 i x}+11\,{\mathrm e}^{4 i x}-11\,{\mathrm e}^{2 i x}-3\right)}{4a^{2}\left({\mathrm e}^{2 i x}+1\right)^{3}\sqrt{a\left({\mathrm e}^{2 i x}+1\right)^{2}{\mathrm e}^{-2 i x}}}+\frac{3\ln\left({\mathrm e}^{i x}+i\right)\cos\left(x\right)}{4a^{2}\sqrt{a\left({\mathrm e}^{2 i x}+1\right)^{2}{\mathrm e}^{-2 i x}}}-\frac{3\ln\left({\mathrm e}^{i x}-i\right)\cos\left(x\right)}{4a^{2}\sqrt{a\left({\mathrm e}^{2 i x}+1\right)^{2}{\mathrm e}^{-2 i x}}}$	126

input
```
int(1/(a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(7/2)/cos(x)^3*(a*sin(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*sin(x)^2)^(1/2
)+a)/cos(x))*a*cos(x)^4+3*(a*sin(x)^2)^(1/2)*cos(x)^2*a^(1/2)+2*a^(1/2)*(a
*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(a\cos^{2}(x)\right)^{5/2}}\,dx=-\frac{\left(3\,\cos\left(x\right)^{4}\log\left(-\frac{\sin\left(x\right)-1}{\sin\left(x\right)+1}\right)-2\,\left(3\,\cos\left(x\right)^{2}+2\right)\sin\left(x\right)\right)\sqrt{a\cos\left(x\right)^{2}}}{16\,a^{3}\cos\left(x\right)^{5}}$$

input
```
integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin
(x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

3.44.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cos^{2}(x)\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*cos(x)**2)**(5/2),x)
```

output
```
Timed out
```

3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(49) = 98$.

Time = 0.62 (sec) , antiderivative size = 933, normalized size of antiderivative = 15.30

$$\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*
(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin
(3*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(1
1*sin(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*
x) + 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) +
 16*cos(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x
)^2 + 4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*
(3*sin(4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*si
n(4*x)*sin(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2
+ 2*sin(x) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x)
 + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 +
 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6
*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*
sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x)
+ 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
- 4*(3*cos(7*x) + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*c
os(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos
(3*x) - 3*cos(x))*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 2
4*(11*cos(3*x) + 3*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*c
os(3*x)*sin(2*x) + 48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))...
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a\cos^2(x))^{5/2}}\,dx = -\frac{3\sin(x)^3 - 5\sin(x)}{8\left(\sin(x)^2 - 1\right)^2 a^{\frac{5}{2}}\operatorname{sgn}(\cos(x))}$$

input
```
integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/8*(3*sin(x)^3 - 5*sin(x))/((sin(x)^2 - 1)^2*a^(5/2)*sgn(cos(x)))
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos^2(x))^{5/2}}\,dx = \int \frac{1}{\left(a\cos(x)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a*cos(x)^2)^(5/2),x)
```

output
```
int(1/(a*cos(x)^2)^(5/2), x)
```

3.45 $\int \left(a\cos^3(x)\right)^{5/2}\,dx$

3.45.1 Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \left(a\cos^3(x)\right)^{5/2}\,dx = \frac{26a^2\sqrt{a\cos^3(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)}{77\cos^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\sin(x) + \frac{26}{77}a^2\sqrt{a\cos^3(x)}\tan(x)$$

output
```
26/77*a^2*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*(a
*cos(x)^3)^(1/2)/cos(x)^(3/2)+78/385*a^2*cos(x)*sin(x)*(a*cos(x)^3)^(1/2)+
26/165*a^2*cos(x)^3*sin(x)*(a*cos(x)^3)^(1/2)+2/15*a^2*cos(x)^5*sin(x)*(a*
cos(x)^3)^(1/2)+26/77*a^2*(a*cos(x)^3)^(1/2)*tan(x)
```

3.45.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.52

$$\int \left(a\cos^3(x)\right)^{5/2}\,dx = \frac{a\left(a\cos^3(x)\right)^{3/2}\left(12480\,\text{EllipticF}\left(\frac{x}{2},2\right) + \sqrt{\cos(x)}(15465\sin(x) + 3657\sin(3x) + 749\right.}{36960\cos^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Cos[x]^3)^(5/2),x]
```

output
```
(a*(a*Cos[x]^3)^(3/2)*(12480*EllipticF[x/2, 2] + Sqrt[Cos[x]]*(15465*Sin[x
] + 3657*Sin[3*x] + 749*Sin[5*x] + 77*Sin[7*x])))/(36960*Cos[x]^(9/2))
```

3.45.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cos^3(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^3\right)^{5/2}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a^2\sqrt{a\cos^3(x)}\int \cos^{\frac{15}{2}}(x)dx}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cos^3(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{15/2}dx}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cos^3(x)}\left(\frac{13}{15}\int \cos^{\frac{11}{2}}(x)dx+\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cos^3(x)}\left(\frac{13}{15}\int \sin\left(x+\frac{\pi}{2}\right)^{11/2}dx+\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cos^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int \cos^{\frac{7}{2}}(x)dx+\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cos^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int \sin\left(x+\frac{\pi}{2}\right)^{7/2}dx+\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3115$$

$$\frac{a^2\sqrt{a\cos^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(x)dx+\frac{2}{7}\sin(x)\cos^{\frac{5}{2}}(x)\right)+\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\sqrt{a\cos^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int \sin\left(x+\frac{\pi}{2}\right)^{3/2}dx+\frac{2}{7}\sin(x)\cos^{\frac{5}{2}}(x)\right)+\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3115$$

$$\frac{a^2\sqrt{a\cos^3(x)}\Big(\frac{13}{15}\Big(\frac{9}{11}\Big(\frac{5}{7}\Big(\frac{1}{3}\int \frac{1}{\sqrt{\cos(x)}}dx+\frac{2}{3}\sin(x)\sqrt{\cos(x)}\Big)+\frac{2}{7}\sin(x)\cos^{\frac{5}{2}}(x)\Big)+\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)\Big)+\frac{2}{15}\sin(x)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\sqrt{a\cos^3(x)}\Bigg(\frac{13}{15}\Bigg(\frac{9}{11}\Bigg(\frac{5}{7}\Bigg(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\sin(x)\sqrt{\cos(x)}\Bigg)+\frac{2}{7}\sin(x)\cos^{\frac{5}{2}}(x)\Bigg)+\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)\Bigg)+\frac{2}{15}}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3120$$

$$\frac{a^2\sqrt{a\cos^3(x)}\Big(\frac{2}{15}\sin(x)\cos^{\frac{13}{2}}(x)+\frac{13}{15}\Big(\frac{2}{11}\sin(x)\cos^{\frac{9}{2}}(x)+\frac{9}{11}\Big(\frac{2}{7}\sin(x)\cos^{\frac{5}{2}}(x)+\frac{5}{7}\Big(\frac{2\,\text{EllipticF}\left(\frac{x}{2},2\right)}{3}+\frac{2}{3}\sin(x)\sqrt{\text{co}}}{\cos^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Cos[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cos[x]^3]*((2*Cos[x]^(13/2)*Sin[x])/15 + (13*((2*Cos[x]^(9/2)*
Sin[x])/11 + (9*((2*Cos[x]^(5/2)*Sin[x])/7 + (5*((2*EllipticF[x/2, 2])/3 +
 (2*Sqrt[Cos[x]]*Sin[x])/3))/7))/11))/15))/Cos[x]^(3/2)
```

3.45. $\int (a\cos^3(x))^{5/2}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.45.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.33 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

method	result
default	$-\frac{2\sqrt{a\left(\cos^{3}\left(x\right)\right)}\, a^{2}\left(-77\left(\cos^{5}\left(x\right)\right)\sin\left(x\right)-91\left(\cos^{3}\left(x\right)\right)\sin\left(x\right)+195 i \sec\left(x\right) F\left(i\left(\csc\left(x\right)-\cot\left(x\right)\right), i\right)\sqrt{\frac{1}{\cos\left(x\right)+1}}\, \sqrt{\frac{\cos\left(x\right)}{\cos\left(x\right)+1}}+195 i\left(\sec^{2}(x\right.\right.}{1155}$

input
```
int((a*cos(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1155*(a*cos(x)^3)^(1/2)*a^2*(-77*cos(x)^5*sin(x)-91*cos(x)^3*sin(x)+195
*I*sec(x)*EllipticF(I*(csc(x)-cot(x)),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos
(x)+1))^(1/2)+195*I*sec(x)^2*EllipticF(I*(csc(x)-cot(x)),I)*(1/(cos(x)+1))
^(1/2)*(cos(x)/(cos(x)+1))^(1/2)-117*cos(x)*sin(x)-195*tan(x))
```

3.45.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \left(a\cos^3(x)\right)^{5/2}\,dx = \frac{195i\sqrt{2}a^{\frac{5}{2}}\cos(x)\,\text{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)) - 195i\sqrt{2}a^{\frac{5}{2}}\cos(x)}{}$$

input
```
integrate((a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/1155*(195*I*sqrt(2)*a^(5/2)*cos(x)*weierstrassPInverse(-4, 0, cos(x) + I
*sin(x)) - 195*I*sqrt(2)*a^(5/2)*cos(x)*weierstrassPInverse(-4, 0, cos(x)
- I*sin(x)) + 2*(77*a^2*cos(x)^6 + 91*a^2*cos(x)^4 + 117*a^2*cos(x)^2 + 19
5*a^2)*sqrt(a*cos(x)^3)*sin(x))/cos(x)
```

3.45.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cos^3(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(x)**3)**(5/2),x)
```

output
```
Timed out
```

3.45.7 Maxima [F]

$$\int \left(a\cos^3(x)\right)^{5/2}\,dx = \int \left(a\cos(x)^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cos(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^3)^(5/2), x)
```

3.45.8 Giac [F]

$$\int \left(a\cos^3(x)\right)^{5/2} dx = \int \left(a\cos(x)^3\right)^{\frac{5}{2}} dx$$

input
```
integrate((a*cos(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^3)^(5/2), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cos^3(x)\right)^{5/2} dx = \int \left(a\cos(x)^3\right)^{5/2} dx$$

input
```
int((a*cos(x)^3)^(5/2),x)
```

output
```
int((a*cos(x)^3)^(5/2), x)
```

3.46 $\int (a\cos^3(x))^{3/2}\,dx$

3.46.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int (a\cos^3(x))^{3/2}\,dx = \frac{14a\sqrt{a\cos^3(x)}E\left(\frac{x}{2}\middle|2\right)}{15\cos^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cos^3(x)}\sin(x) + \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\sin(x)$$

output
```
14/15*a*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2))*(a*c
os(x)^3)^(1/2)/cos(x)^(3/2)+14/45*a*sin(x)*(a*cos(x)^3)^(1/2)+2/9*a*cos(x)
^2*sin(x)*(a*cos(x)^3)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

$$\int (a\cos^3(x))^{3/2}\,dx = \frac{(a\cos^3(x))^{3/2}\left(168E\left(\frac{x}{2}\middle|2\right)+\sqrt{\cos(x)}(38\sin(2x)+5\sin(4x))\right)}{180\cos^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Cos[x]^3)^(3/2),x]
```

output
```
((a*Cos[x]^3)^(3/2)*(168*EllipticE[x/2, 2] + Sqrt[Cos[x]]*(38*Sin[2*x] + 5
*Sin[4*x])))/(180*Cos[x]^(9/2))
```

3.46.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cos^3(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)^3\right)^{3/2}dx$$

$$\Big\downarrow \; 3686$$

$$\frac{a\sqrt{a\cos^3(x)}\int \cos^{\frac{9}{2}}(x)dx}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sqrt{a\cos^3(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{9/2}dx}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3115$$

$$\frac{a\sqrt{a\cos^3(x)}\left(\frac{7}{9}\int \cos^{\frac{5}{2}}(x)dx+\frac{2}{9}\sin(x)\cos^{\frac{7}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sqrt{a\cos^3(x)}\left(\frac{7}{9}\int \sin\left(x+\frac{\pi}{2}\right)^{5/2}dx+\frac{2}{9}\sin(x)\cos^{\frac{7}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3115$$

$$\frac{a\sqrt{a\cos^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\cos(x)}dx+\frac{2}{5}\sin(x)\cos^{\frac{3}{2}}(x)\right)+\frac{2}{9}\sin(x)\cos^{\frac{7}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\sqrt{a\cos^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2}{5}\sin(x)\cos^{\frac{3}{2}}(x)\right)+\frac{2}{9}\sin(x)\cos^{\frac{7}{2}}(x)\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \; 3119$$

$$\frac{a\sqrt{a\cos^3(x)}\left(\frac{2}{9}\sin(x)\cos^{\frac{7}{2}}(x)+\frac{7}{9}\left(\frac{6E\left(\frac{x}{2}|2\right)}{5}+\frac{2}{5}\sin(x)\cos^{\frac{3}{2}}(x)\right)\right)}{\cos^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Cos[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Cos[x]^3]*((2*Cos[x]^(7/2)*Sin[x])/9 + (7*((6*EllipticE[x/2, 2])
/5 + (2*Cos[x]^(3/2)*Sin[x])/5))/9))/Cos[x]^(3/2)
```

3.46.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.46.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.04 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.04

method	result
default	$\frac{2\sqrt{a(\cos^3(x))}\,a\left(5(\cos^3(x))\sin(x)-21iF(i(\csc(x)-\cot(x)),i)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}+21i\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,E(i(\csc(x)-\cot(x))\right.}{}$

input
```
int((a*cos(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(a*cos(x)^3)^(1/2)*a/(cos(x)+1)*(5*cos(x)^3*sin(x)-21*I*EllipticF(I*(
csc(x)-cot(x)),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)+21*I*(1/(
cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)+
5*cos(x)^2*sin(x)-42*I*sec(x)*EllipticF(I*(csc(x)-cot(x)),I)*(1/(cos(x)+1)
)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)+42*I*sec(x)*(1/(cos(x)+1))^(1/2)*(cos(x)
/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)+7*cos(x)*sin(x)-21*I*sec
(x)^2*EllipticF(I*(csc(x)-cot(x)),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+
1))^(1/2)+21*I*sec(x)^2*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*Ell
ipticE(I*(csc(x)-cot(x)),I)+7*sin(x)+21*tan(x))
```

3.46.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \left(a\cos^3(x)\right)^{3/2}\,dx =\\ &-\frac{7}{15}i\,\sqrt{2}a^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(x)}+i\,\sin{(x)}))\\ &+\frac{7}{15}i\,\sqrt{2}a^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(x)}-i\,\sin{(x)}))\\ &+\frac{2}{45}\sqrt{a\cos{(x)}^3}\left(5\,a\cos{(x)}^2+7\,a\right)\sin{(x)}\end{aligned}$$

input
```
integrate((a*cos(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
-7/15*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(x) + I*sin(x))) + 7/15*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(x) - I*sin(x))) + 2/45*sqrt(a*cos(x)^3)*(5*a*cos(
x)^2 + 7*a)*sin(x)
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cos^3(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a*cos(x)**3)**(3/2),x)
```

output
```
Timed out
```

3.46.7 Maxima [F]

$$\int \left(a\cos^3(x)\right)^{3/2} dx = \int \left(a\cos\left(x\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*cos(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^3)^(3/2), x)
```

3.46.8 Giac [F]

$$\int \left(a\cos^3(x)\right)^{3/2} dx = \int \left(a\cos\left(x\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*cos(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^3)^(3/2), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cos^3(x)\right)^{3/2} dx = \int \left(a\cos(x)^3\right)^{3/2} dx$$

input
```
int((a*cos(x)^3)^(3/2),x)
```

output
```
int((a*cos(x)^3)^(3/2), x)
```

3.47 $\int \sqrt{a\cos^3(x)}\,dx$

3.47.1 Optimal result

Integrand size = 10, antiderivative size = 44

$$\int \sqrt{a\cos^3(x)}\,dx = \frac{2\sqrt{a\cos^3(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)}{3\cos^{\frac{3}{2}}(x)} + \frac{2}{3}\sqrt{a\cos^3(x)}\tan(x)$$

output
```
2/3*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*(a*cos(x
)^3)^(1/2)/cos(x)^(3/2)+2/3*(a*cos(x)^3)^(1/2)*tan(x)
```

3.47.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \sqrt{a\cos^3(x)}\,dx = \frac{2\sqrt{a\cos^3(x)}\left(\text{EllipticF}\left(\frac{x}{2},2\right)+\sqrt{\cos(x)}\sin(x)\right)}{3\cos^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sqrt[a*Cos[x]^3],x]
```

output
```
(2*Sqrt[a*Cos[x]^3]*(EllipticF[x/2, 2] + Sqrt[Cos[x]]*Sin[x]))/(3*Cos[x]^(
3/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos^3(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^3}\,dx$$

$$\Big\downarrow \ 3686$$

$$\frac{\sqrt{a\cos^3(x)}\int \cos^{\frac{3}{2}}(x)dx}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a\cos^3(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{3/2}dx}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3115$$

$$\frac{\sqrt{a\cos^3(x)}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(x)}}dx+\frac{2}{3}\sin(x)\sqrt{\cos(x)}\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a\cos^3(x)}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\sin(x)\sqrt{\cos(x)}\right)}{\cos^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \ 3120$$

$$\frac{\sqrt{a\cos^3(x)}\left(\frac{2\,\text{EllipticF}\left(\frac{x}{2},2\right)}{3}+\frac{2}{3}\sin(x)\sqrt{\cos(x)}\right)}{\cos^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Cos[x]^3],x]
```

output
```
(Sqrt[a*Cos[x]^3]*((2*EllipticF[x/2, 2])/3 + (2*Sqrt[Cos[x]]*Sin[x])/3))/C
os[x]^(3/2)
```

3.47.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.47.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.09

method	result
default	$-\frac{2\sqrt{a\left(\cos^{3}\left(x\right)\right)}\,\left(i\sec\left(x\right)F\left(i\left(\csc\left(x\right)-\cot\left(x\right)\right),i\right)\sqrt{\frac{1}{\cos\left(x\right)+1}}\,\sqrt{\frac{\cos\left(x\right)}{\cos\left(x\right)+1}}+i\left(\sec^{2}\left(x\right)\right)F\left(i\left(\csc\left(x\right)-\cot\left(x\right)\right),i\right)\sqrt{\frac{1}{\cos\left(x\right)+1}}\,\sqrt{\frac{\cos\left(x\right)}{\cos\left(x\right)+1}}-\tan$

input
```
int((a*cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a*cos(x)^3)^(1/2)*(I*sec(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))
^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)+I*sec(x)^2*(1/(cos(x)+1))^(1/2)*(cos
(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)-tan(x))
```

3.47.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \sqrt{a\cos^3(x)}\,dx$$
$$= \frac{i\sqrt{2}\sqrt{a}\cos(x)\,\text{weierstrassPInverse}(-4,0,\cos(x)+i\,\sin(x)) - i\sqrt{2}\sqrt{a}\cos(x)\,\text{weierstrassPInverse}(-4,0}{3\,\cos(x)}$$

input
```
integrate((a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(I*sqrt(2)*sqrt(a)*cos(x)*weierstrassPInverse(-4, 0, cos(x) + I*sin(x)
) - I*sqrt(2)*sqrt(a)*cos(x)*weierstrassPInverse(-4, 0, cos(x) - I*sin(x))
 + 2*sqrt(a*cos(x)^3)*sin(x))/cos(x)
```

3.47.6 Sympy [F]

$$\int \sqrt{a\cos^3(x)}\,dx = \int \sqrt{a\cos^3(x)}\,dx$$

input
```
integrate((a*cos(x)**3)**(1/2),x)
```

output
```
Integral(sqrt(a*cos(x)**3), x)
```

3.47.7 Maxima [F]

$$\int \sqrt{a\cos^3(x)}\,dx = \int \sqrt{a\cos(x)^3}\,dx$$

input
```
integrate((a*cos(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cos(x)^3), x)
```

3.47.8 Giac [F]

$$\int \sqrt{a\cos^3(x)}\,dx = \int \sqrt{a\cos(x)^3}\,dx$$

input
```
integrate((a*cos(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(x)^3), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cos^3(x)}\,dx = \int \sqrt{a\cos(x)^3}\,dx$$

input
```
int((a*cos(x)^3)^(1/2),x)
```

output
```
int((a*cos(x)^3)^(1/2), x)
```

3.48 $\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = -\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}}+\frac{2\cos(x)\sin(x)}{\sqrt{a\cos^3(x)}}$$

output
```
-2*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/
2))/(a*cos(x)^3)^(1/2)+2*cos(x)*sin(x)/(a*cos(x)^3)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = \frac{-2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)+\sin(2x)}{\sqrt{a\cos^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Cos[x]^3],x]
```

output
```
(-2*Cos[x]^(3/2)*EllipticE[x/2, 2] + Sin[2*x])/Sqrt[a*Cos[x]^3]
```

3.48.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos^3(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^3}}dx$$
$$\Big\downarrow \text{3686}$$
$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{3}{2}}(x)}dx}{\sqrt{a\cos^3(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{3/2}}dx}{\sqrt{a\cos^3(x)}}$$
$$\Big\downarrow \text{3116}$$
$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{\sqrt{\cos(x)}}-\int \sqrt{\cos(x)}dx\right)}{\sqrt{a\cos^3(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{\sqrt{\cos(x)}}-\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}dx\right)}{\sqrt{a\cos^3(x)}}$$
$$\Big\downarrow \text{3119}$$
$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{\sqrt{\cos(x)}}-2E\left(\frac{x}{2}\middle|2\right)\right)}{\sqrt{a\cos^3(x)}}$$

input
```
Int[1/Sqrt[a*Cos[x]^3],x]
```

output
```
(Cos[x]^(3/2)*(-2*EllipticE[x/2, 2] + (2*Sin[x])/Sqrt[Cos[x]]))/Sqrt[a*Cos
[x]^3]
```

3.48.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.48.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.81

method	result
default	$\frac{2\left(i\sqrt{\frac{\cos(x)}{\cos(x)+1}}\sqrt{\frac{1}{\cos(x)+1}}F(i(\csc(x)-\cot(x)),i)(\cos^2(x))-i\sqrt{\frac{\cos(x)}{\cos(x)+1}}\sqrt{\frac{1}{\cos(x)+1}}E(i(\csc(x)-\cot(x)),i)(\cos^2(x))+2i\sqrt{\frac{\cos(x)}{\cos(x)+1}}\right.}{}$

input
```
int(1/(a*cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(I*(cos(x)/(cos(x)+1))^(1/2)*(1/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-co
t(x)),I)*cos(x)^2-I*(cos(x)/(cos(x)+1))^(1/2)*(1/(cos(x)+1))^(1/2)*Ellipti
cE(I*(csc(x)-cot(x)),I)*cos(x)^2+2*I*(cos(x)/(cos(x)+1))^(1/2)*(1/(cos(x)+
1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)*cos(x)-2*I*(cos(x)/(cos(x)+1))^(1
/2)*(1/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)*cos(x)+I*(cos(x)/(
cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)*(1/(cos(x)+1))^(1/2)-I*(co
s(x)/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)*(1/(cos(x)+1))^(1/2)
+sin(x))*cos(x)/(cos(x)+1)/(a*cos(x)^3)^(1/2)
```

3.48.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$$
$$= \frac{i\sqrt{2}\sqrt{a}\cos(x)^2 \operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x))) - i\sqrt{2}\sqrt{a}\cos(x)^2}{a\cos(x)^2}$$

input
```
integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(a)*cos(x)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(x) + I*sin(x))) - I*sqrt(2)*sqrt(a)*cos(x)^2*weierstrassZeta(-4, 0
, weierstrassPInverse(-4, 0, cos(x) - I*sin(x))) + 2*sqrt(a*cos(x)^3)*sin(
x))/(a*cos(x)^2)
```

3.48.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$$

input
```
integrate(1/(a*cos(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cos(x)**3), x)
```

3.48.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cos(x)^3}}\,dx$$

input
```
integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cos(x)^3), x)
```

3.48.8 Giac [F]

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cos(x)^3}}\,dx$$

input
```
integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cos(x)^3), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cos(x)^3}}\,dx$$

input
```
int(1/(a*cos(x)^3)^(1/2),x)
```

output
```
int(1/(a*cos(x)^3)^(1/2), x)
```

3.49 $\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx$

3.49.1 Optimal result

Integrand size = 10, antiderivative size = 71

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx=\frac{10\cos^{\frac{3}{2}}(x)\,\text{EllipticF}\left(\frac{x}{2},2\right)}{21a\sqrt{a\cos^3(x)}}+\frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}}$$

output
```
10/21*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^
(1/2))/a/(a*cos(x)^3)^(1/2)+10/21*sin(x)/a/(a*cos(x)^3)^(1/2)+2/7*sec(x)*t
an(x)/a/(a*cos(x)^3)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx=\frac{2\cos^2(x)\left(5\cos^{\frac{5}{2}}(x)\,\text{EllipticF}\left(\frac{x}{2},2\right)+5\cos(x)\sin(x)+3\tan(x)\right)}{21\,(a\cos^3(x))^{3/2}}$$

input
```
Integrate[(a*Cos[x]^3)^(-3/2),x]
```

output
```
(2*Cos[x]^2*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*Sin[x] + 3*Tan[x]
))/(21*(a*Cos[x]^3)^(3/2))
```

3.49.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{9}{2}}(x)}dx}{a\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{9/2}}dx}{a\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{5}{7}\int \frac{1}{\cos^{\frac{5}{2}}(x)}dx+\frac{2\sin(x)}{7\cos^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{5}{7}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{5/2}}dx+\frac{2\sin(x)}{7\cos^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(x)}}dx+\frac{2\sin(x)}{3\cos^{\frac{3}{2}}(x)}\right)+\frac{2\sin(x)}{7\cos^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2\sin(x)}{3\cos^{\frac{3}{2}}(x)}\right)+\frac{2\sin(x)}{7\cos^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{7\cos^{\frac{7}{2}}(x)}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{x}{2},2\right)}{3}+\frac{2\sin(x)}{3\cos^{\frac{3}{2}}(x)}\right)\right)}{a\sqrt{a\cos^3(x)}}$$

input
```
Int[(a*Cos[x]^3)^(-3/2),x]
```

output
```
(Cos[x]^(3/2)*((2*Sin[x])/(7*Cos[x]^(7/2)) + (5*((2*EllipticF[x/2, 2])/3 +
 (2*Sin[x])/(3*Cos[x]^(3/2))))/7))/(a*Sqrt[a*Cos[x]^3])
```

3.49.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.49.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

method	result
default	$-\frac{2\left(5i\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{\frac{1}{\cos(x)+1}}\,F(i(\csc(x)-\cot(x)),i)(\cos^2(x))+5i\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{\frac{1}{\cos(x)+1}}\,F(i(\csc(x)-\cot(x)),i)\cos(x)-5\sin(x)-3\,\mathrm{se}}{21a\sqrt{a(\cos^3(x))}}$

input
```
int(1/(a*cos(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/a/(a*cos(x)^3)^(1/2)*(5*I*cos(x)^2*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos
(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)+5*I*cos(x)*(1/(cos(x)+1))^(1/
2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)-5*sin(x)-3*sec
(x)*tan(x))
```

3.49.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a\cos^3(x)\right)^{3/2}}\,dx = \frac{5i\,\sqrt{2}\sqrt{a}\cos\left(x\right)^5 \operatorname{weierstrassPInverse}(-4,0,\cos\left(x\right)+i\,\sin\left(x\right))-5i\,\sqrt{2}\sqrt{a}\cos\left(x\right)^5\,\mathrm{w}}{21\,a^2\,\mathrm{co}}$$

input
```
integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(5*I*sqrt(2)*sqrt(a)*cos(x)^5*weierstrassPInverse(-4, 0, cos(x) + I*s
in(x)) - 5*I*sqrt(2)*sqrt(a)*cos(x)^5*weierstrassPInverse(-4, 0, cos(x) -
I*sin(x)) + 2*sqrt(a*cos(x)^3)*(5*cos(x)^2 + 3)*sin(x))/(a^2*cos(x)^5)
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(x)**3)**(3/2),x)
```

output
```
Timed out
```

3.49.7 Maxima [F]

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^3)^(-3/2), x)
```

3.49.8 Giac [F]

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^3)^(-3/2), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cos{(x)}^3\right)^{3/2}}\,dx$$

input
```
int(1/(a*cos(x)^3)^(3/2),x)
```

output
```
int(1/(a*cos(x)^3)^(3/2), x)
```

3.50 $\int \frac{1}{(a\cos^3(x))^{5/2}}\,dx$

3.50.1 Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \frac{1}{(a\cos^3(x))^{5/2}}\,dx = -\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}}+\frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}}+\frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}}$$

output
```
-154/195*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x)
,2^(1/2))/a^2/(a*cos(x)^3)^(1/2)+154/195*cos(x)*sin(x)/a^2/(a*cos(x)^3)^(1
/2)+154/585*tan(x)/a^2/(a*cos(x)^3)^(1/2)+22/117*sec(x)^2*tan(x)/a^2/(a*co
s(x)^3)^(1/2)+2/13*sec(x)^4*tan(x)/a^2/(a*cos(x)^3)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a\cos^3(x))^{5/2}}\,dx = \frac{-462\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)+462\cos(x)\sin(x)+2(77+55\sec^2(x)+45\sec^4(x))\tan(x)}{585a^2\sqrt{a\cos^3(x)}}$$

input
```
Integrate[(a*Cos[x]^3)^(-5/2),x]
```

output
```
(-462*Cos[x]^(3/2)*EllipticE[x/2, 2] + 462*Cos[x]*Sin[x] + 2*(77 + 55*Sec[
x]^2 + 45*Sec[x]^4)*Tan[x])/(585*a^2*Sqrt[a*Cos[x]^3])
```

3.50.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{15/2}}dx}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\cos^{\frac{11}{2}}(x)}dx+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{11/2}}dx+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int \frac{1}{\cos^{\frac{7}{2}}(x)}dx+\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{7/2}}dx+\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \; 3116$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\cos^{\frac{3}{2}}(x)}dx+\frac{2\sin(x)}{5\cos^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(x)}{5\cos^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \; 3116$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(\frac{2\sin(x)}{\sqrt{\cos(x)}}-\int\sqrt{\cos(x)}dx\right)+\frac{2\sin(x)}{5\cos^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(\frac{2\sin(x)}{\sqrt{\cos(x)}}-\int\sqrt{\sin\left(x+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(x)}{5\cos^{\frac{5}{2}}(x)}\right)+\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}\right)+\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cos^3(x)}}$$

$$\Big\downarrow \; 3119$$

$$\frac{\cos^{\frac{3}{2}}(x)\left(\frac{2\sin(x)}{13\cos^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sin(x)}{9\cos^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{3}{5}\left(\frac{2\sin(x)}{\sqrt{\cos(x)}}-2E\left(\frac{x}{2}\middle|2\right)\right)\right)\right)\right)}{a^2\sqrt{a\cos^3(x)}}$$

input
```
Int[(a*Cos[x]^3)^(-5/2),x]
```

output
```
(Cos[x]^(3/2)*((2*Sin[x])/(13*Cos[x]^(13/2)) + (11*((2*Sin[x])/(9*Cos[x]^(
9/2)) + (7*((2*Sin[x])/(5*Cos[x]^(5/2)) + (3*(-2*EllipticE[x/2, 2] + (2*Si
n[x])/Sqrt[Cos[x]]))/5))/9))/13))/(a^2*Sqrt[a*Cos[x]^3])
```

3.50. $\int \frac{1}{(a\cos^3(x))^{5/2}}\,dx$

3.50.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.50.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.31 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.54

method	result
default	$-\frac{2i\left(231\left(\cos^3(x)\right)\sqrt{\frac{1}{\cos(x)+1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}E(i(\csc(x)-\cot(x)),i)-231\left(\cos^3(x)\right)\sqrt{\frac{1}{\cos(x)+1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}F(i(\csc(x)-\cot(x)),i)+462(c\right.}{}$

input
```
int(1/(a*cos(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*I/(cos(x)+1)/a^2/(a*cos(x)^3)^(1/2)*(231*cos(x)^3*(1/(cos(x)+1))^(1
/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)-231*cos(x)^3*
(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x))
,I)+462*cos(x)^2*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(
I*(csc(x)-cot(x)),I)-462*cos(x)^2*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))
^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)+231*cos(x)*(1/(cos(x)+1))^(1/2)*(cos
(x)/(cos(x)+1))^(1/2)*EllipticE(I*(csc(x)-cot(x)),I)-231*cos(x)*(1/(cos(x)
+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(csc(x)-cot(x)),I)+231*I*
cos(x)*sin(x)+77*I*sin(x)+77*I*tan(x)+55*I*sec(x)*tan(x)+55*I*tan(x)*sec(x
)^2+45*I*tan(x)*sec(x)^3+45*I*tan(x)*sec(x)^4)
```

3.50.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \frac{231i\,\sqrt{2}\sqrt{a}\cos(x)^8\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(x)+i\,\sin(}{}$$

input
```
integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/585*(231*I*sqrt(2)*sqrt(a)*cos(x)^8*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(x) + I*sin(x))) - 231*I*sqrt(2)*sqrt(a)*cos(x)^8*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) - I*sin(x))) + 2*(231*co
s(x)^6 + 77*cos(x)^4 + 55*cos(x)^2 + 45)*sqrt(a*cos(x)^3)*sin(x))/(a^3*cos
(x)^8)
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(x)**3)**(5/2),x)
```

output
```
Timed out
```

3.50.7 Maxima [F]

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(x)^3)^(-5/2), x)
```

3.50.8 Giac [F]

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(x)^3)^(-5/2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cos\left(x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(a*cos(x)^3)^(5/2),x)
```

output
```
int(1/(a*cos(x)^3)^(5/2), x)
```

3.51 $\int \left(a\cos^4(x)\right)^{5/2} \, dx$

3.51.1 Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \left(a\cos^4(x)\right)^{5/2} \, dx = \frac{63}{256}a^2x\sqrt{a\cos^4(x)}\sec^2(x) + \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{10}a^2\cos^7(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{63}{256}a^2\sqrt{a\cos^4(x)}\tan(x)$$

output
```
63/256*a^2*x*sec(x)^2*(a*cos(x)^4)^(1/2)+21/128*a^2*cos(x)*sin(x)*(a*cos(x
)^4)^(1/2)+21/160*a^2*cos(x)^3*sin(x)*(a*cos(x)^4)^(1/2)+9/80*a^2*cos(x)^5
*sin(x)*(a*cos(x)^4)^(1/2)+1/10*a^2*cos(x)^7*sin(x)*(a*cos(x)^4)^(1/2)+63/
256*a^2*(a*cos(x)^4)^(1/2)*tan(x)
```

3.51.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\cos^4(x)\right)^{5/2} \, dx = \frac{a\left(a\cos^4(x)\right)^{3/2}\sec^6(x)(2520x + 2100\sin(2x) + 600\sin(4x) + 150\sin(6x) + 25\sin(8x}{10240}$$

input
```
Integrate[(a*Cos[x]^4)^(5/2),x]
```

output
```
(a*(a*Cos[x]^4)^(3/2)*Sec[x]^6*(2520*x + 2100*Sin[2*x] + 600*Sin[4*x] + 15
0*Sin[6*x] + 25*Sin[8*x] + 2*Sin[10*x]))/10240
```

3.51.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \cos^4(x)\right)^{5/2} dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a \sin\left(x+\frac{\pi}{2}\right)^4\right)^{5/2} dx$$

$$\Big\downarrow \; 3686$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \int \cos^{10}(x)dx$$

$$\Big\downarrow \; 3042$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \int \sin\left(x+\frac{\pi}{2}\right)^{10} dx$$

$$\Big\downarrow \; 3115$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \left(\frac{9}{10}\int \cos^8(x)dx+\frac{1}{10}\sin(x)\cos^9(x)\right)$$

$$\Big\downarrow \; 3042$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \left(\frac{9}{10}\int \sin\left(x+\frac{\pi}{2}\right)^8 dx+\frac{1}{10}\sin(x)\cos^9(x)\right)$$

$$\Big\downarrow \; 3115$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \left(\frac{9}{10}\left(\frac{7}{8}\int \cos^6(x)dx+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)$$

$$\Big\downarrow \; 3042$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \left(\frac{9}{10}\left(\frac{7}{8}\int \sin\left(x+\frac{\pi}{2}\right)^6 dx+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)$$

$$\Big\downarrow \; 3115$$

$$a^2 \sec^2(x)\sqrt{a \cos^4(x)} \left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)+\frac{1}{10}\sin(x)\cos^9(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x)\right)+$$

$$\Big\downarrow \text{3042}$$

$$a^2\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin(x)\cos^7(x$$

$$\Big\downarrow \text{3115}$$

$$a^2\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)+\frac{1}{8}\sin($$

$$\Big\downarrow \text{24}$$

$$a^2\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{1}{10}\sin(x)\cos^9(x)+\frac{9}{10}\left(\frac{1}{8}\sin(x)\cos^7(x)+\frac{7}{8}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\right(\right.\right.\right.\right.$$

input
```
Int[(a*Cos[x]^4)^(5/2),x]
```

output
```
a^2*Sqrt[a*Cos[x]^4]*Sec[x]^2*((Cos[x]^9*Sin[x])/10 + (9*((Cos[x]^7*Sin[x]
)/8 + (7*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4 + (3*(x/2 + (Cos[x
]*Sin[x])/2))/4))/6))/8))/10)
```

3.51.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.51.4 Maple [A] (verified)

Time = 13.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.42

method	result
default	$\frac{a^2\sqrt{a(\cos^4(x))}\,(128(\cos^7(x))\sin(x)+144(\cos^5(x))\sin(x)+168(\cos^3(x))\sin(x)+210\cos(x)\sin(x)+315\tan(x)+315(\sec^2(x))x)}{1280}$
risch	$\frac{63a^2 e^{2ix}\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}\,x}{256(e^{2ix}+1)^2}-\frac{ia^2 e^{12ix}\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}}{10240(e^{2ix}+1)^2}-\frac{5ia^2 e^{10ix}\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}}{4096(e^{2ix}+1)^2}-\frac{105ia^2 e^{4ix}\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}}{1024(e^{2ix}+1)^2}$

input
```
int((a*cos(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*a^2*(a*cos(x)^4)^(1/2)*(128*cos(x)^7*sin(x)+144*cos(x)^5*sin(x)+168
*cos(x)^3*sin(x)+210*cos(x)*sin(x)+315*tan(x)+315*sec(x)^2*x)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.52

$$\int \left(a\cos^4(x)\right)^{5/2}\,dx = \frac{\sqrt{a\cos(x)^4}\left(315\,a^2x + \left(128\,a^2\cos(x)^9 + 144\,a^2\cos(x)^7 + 168\,a^2\cos(x)^5 + 210\,a^2\cos\right.\right.}{1280\,\cos(x)^2}$$

input
```
integrate((a*cos(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/1280*sqrt(a*cos(x)^4)*(315*a^2*x + (128*a^2*cos(x)^9 + 144*a^2*cos(x)^7
+ 168*a^2*cos(x)^5 + 210*a^2*cos(x)^3 + 315*a^2*cos(x))*sin(x))/cos(x)^2
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cos^4(x)\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a*cos(x)**4)**(5/2),x)
```

output
```
Timed out
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int \left(a\cos^4(x)\right)^{5/2}\,dx=\frac{63}{256}\,a^{\frac{5}{2}}x+\frac{315\,a^{\frac{5}{2}}\tan\left(x\right)^{9}+1470\,a^{\frac{5}{2}}\tan\left(x\right)^{7}+2688\,a^{\frac{5}{2}}\tan\left(x\right)^{5}+2370\,a^{\frac{5}{2}}\tan\left(x\right)^{3}+965\,a^{\frac{5}{2}}\tan\left(x\right)}{1280\,\left(\tan\left(x\right)^{10}+5\,\tan\left(x\right)^{8}+10\,\tan\left(x\right)^{6}+10\,\tan\left(x\right)^{4}+5\,\tan\left(x\right)^{2}+1\right)}$$

input
```
integrate((a*cos(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
63/256*a^(5/2)*x + 1/1280*(315*a^(5/2)*tan(x)^9 + 1470*a^(5/2)*tan(x)^7 +
2688*a^(5/2)*tan(x)^5 + 2370*a^(5/2)*tan(x)^3 + 965*a^(5/2)*tan(x))/(tan(x
)^10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

$$\int \left(a\cos^4(x)\right)^{5/2}\,dx = \frac{1}{10240}\left(2520\,a^2x + 2\,a^2\sin\left(10\,x\right) + 25\,a^2\sin\left(8\,x\right) + 150\,a^2\sin\left(6\,x\right) + 600\,a^2\sin\left(4\,x\right)\right.$$

input
```
integrate((a*cos(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/10240*(2520*a^2*x + 2*a^2*sin(10*x) + 25*a^2*sin(8*x) + 150*a^2*sin(6*x)
 + 600*a^2*sin(4*x) + 2100*a^2*sin(2*x))*sqrt(a)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cos^4(x)\right)^{5/2}\,dx = \int \left(a\cos\left(x\right)^4\right)^{5/2}\,dx$$

input
```
int((a*cos(x)^4)^(5/2),x)
```

output
```
int((a*cos(x)^4)^(5/2), x)
```

3.52 $\int (a\cos^4(x))^{3/2}\,dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int (a\cos^4(x))^{3/2}\,dx = \frac{5}{16}ax\sqrt{a\cos^4(x)}\sec^2(x)+\frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\sin(x)+\frac{5}{16}a\sqrt{a\cos^4(x)}\tan(x)$$

output
```
5/16*a*x*sec(x)^2*(a*cos(x)^4)^(1/2)+5/24*a*cos(x)*sin(x)*(a*cos(x)^4)^(1/
2)+1/6*a*cos(x)^3*sin(x)*(a*cos(x)^4)^(1/2)+5/16*a*(a*cos(x)^4)^(1/2)*tan(
x)
```

3.52.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int (a\cos^4(x))^{3/2}\,dx = \frac{1}{192}(a\cos^4(x))^{3/2}\sec^6(x)(60x+45\sin(2x)+9\sin(4x)+\sin(6x))$$

input
```
Integrate[(a*Cos[x]^4)^(3/2),x]
```

output
```
((a*Cos[x]^4)^(3/2)*Sec[x]^6*(60*x + 45*Sin[2*x] + 9*Sin[4*x] + Sin[6*x]))
/192
```

3.52.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \cos^4(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin\left(x+\frac{\pi}{2}\right)^4\right)^{3/2} dx$$

$$\Big\downarrow \text{3686}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)} \int \cos^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)} \int \sin\left(x+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{3115}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)}\left(\frac{5}{6}\int \cos^4(x)dx+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)}\left(\frac{5}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4 dx+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2 dx+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$a \sec^2(x)\sqrt{a \cos^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)+\frac{1}{6}\sin(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)\right)$$

input
```
Int[(a*Cos[x]^4)^(3/2),x]
```

output
```
a*Sqrt[a*Cos[x]^4]*Sec[x]^2*((Cos[x]^5*Sin[x])/6 + (5*((Cos[x]^3*Sin[x])/4
 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4))/6)
```

3.52.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.52.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

method	result
default	$\frac{a\sqrt{a(\cos^4(x))}\,(8(\cos^3(x))\sin(x)+10\cos(x)\sin(x)+15\tan(x)+15(\sec^2(x))x)}{48}$
risch	$\frac{5a\,e^{2ix}\sqrt{a(e^{2ix}+1)^4e^{-4ix}}\,x}{16(e^{2ix}+1)^2}-\frac{ia\,e^{8ix}\sqrt{a(e^{2ix}+1)^4e^{-4ix}}}{384(e^{2ix}+1)^2}-\frac{3ia\,e^{6ix}\sqrt{a(e^{2ix}+1)^4e^{-4ix}}}{128(e^{2ix}+1)^2}-\frac{15ia\,e^{4ix}\sqrt{a(e^{2ix}+1)^4e^{-4ix}}}{128(e^{2ix}+1)^2}+\frac{15i}{}$

input
```
int((a*cos(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*a*(a*cos(x)^4)^(1/2)*(8*cos(x)^3*sin(x)+10*cos(x)*sin(x)+15*tan(x)+15
*sec(x)^2*x)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \frac{\sqrt{a\cos(x)^4}\left(15\,ax + \left(8\,a\cos(x)^5 + 10\,a\cos(x)^3 + 15\,a\cos(x)\right)\sin(x)\right)}{48\,\cos(x)^2}$$

input
```
integrate((a*cos(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/48*sqrt(a*cos(x)^4)*(15*a*x + (8*a*cos(x)^5 + 10*a*cos(x)^3 + 15*a*cos(x
))*sin(x))/cos(x)^2
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a*cos(x)**4)**(3/2),x)
```

output
```
Timed out
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \frac{5}{16}\,a^{\frac{3}{2}}x + \frac{15\,a^{\frac{3}{2}}\tan(x)^5 + 40\,a^{\frac{3}{2}}\tan(x)^3 + 33\,a^{\frac{3}{2}}\tan(x)}{48\left(\tan(x)^6 + 3\,\tan(x)^4 + 3\,\tan(x)^2 + 1\right)}$$

input
```
integrate((a*cos(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
5/16*a^(3/2)*x + 1/48*(15*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 33*a^(3
/2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.32

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \frac{1}{192} a^{\frac{3}{2}}(60\,x + \sin(6\,x) + 9\,\sin(4\,x) + 45\,\sin(2\,x))$$

input
```
integrate((a*cos(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/192*a^(3/2)*(60*x + sin(6*x) + 9*sin(4*x) + 45*sin(2*x))
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \int \left(a\cos(x)^4\right)^{3/2} dx$$

input
```
int((a*cos(x)^4)^(3/2),x)
```

output
```
int((a*cos(x)^4)^(3/2), x)
```

3.53 $\int \sqrt{a\cos^4(x)}\,dx$

3.53.1 Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a\cos^4(x)}\,dx = \frac{1}{2}x\sqrt{a\cos^4(x)}\sec^2(x) + \frac{1}{2}\sqrt{a\cos^4(x)}\tan(x)$$

output
```
1/2*x*sec(x)^2*(a*cos(x)^4)^(1/2)+1/2*(a*cos(x)^4)^(1/2)*tan(x)
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sqrt{a\cos^4(x)}\,dx = \frac{1}{2}\sqrt{a\cos^4(x)}\sec^2(x)(x+\cos(x)\sin(x))$$

input
```
Integrate[Sqrt[a*Cos[x]^4],x]
```

output
```
(Sqrt[a*Cos[x]^4]*Sec[x]^2*(x + Cos[x]*Sin[x]))/2
```

3.53.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3686}$$

$$\sec^2(x)\sqrt{a\cos^4(x)}\int \cos^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec^2(x)\sqrt{a\cos^4(x)}\int \sin\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{24}$$

$$\sec^2(x)\sqrt{a\cos^4(x)}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[Sqrt[a*Cos[x]^4],x]
```

output
```
Sqrt[a*Cos[x]^4]*Sec[x]^2*(x/2 + (Cos[x]*Sin[x])/2)
```

3.53.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.53.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\sqrt{a(\cos^4(x))}\,(\tan(x)+(\sec^2(x))x)}{2}$	20
risch	$\frac{\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}\,e^{2ix}x}{2(e^{2ix}+1)^2}-\frac{i\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}\,e^{4ix}}{8(e^{2ix}+1)^2}+\frac{i\sqrt{a(e^{2ix}+1)^4 e^{-4ix}}}{8(e^{2ix}+1)^2}$	102

input
```
int((a*cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*cos(x)^4)^(1/2)*(tan(x)+sec(x)^2*x)
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

$$\int \sqrt{a\cos^4(x)}\,dx = \frac{\sqrt{a\cos(x)^4}(\cos(x)\sin(x)+x)}{2\cos(x)^2}$$

input
```
integrate((a*cos(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(a*cos(x)^4)*(cos(x)*sin(x) + x)/cos(x)^2
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{a\cos^4(x)}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(x)**4)**(1/2),x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \sqrt{a\cos^4(x)}\,dx = \frac{1}{2}\sqrt{a}x + \frac{\sqrt{a}\tan(x)}{2(\tan(x)^2+1)}$$

input
```
integrate((a*cos(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*x + 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

$$\int \sqrt{a\cos^4(x)}\,dx = \frac{1}{4}\sqrt{a}(2x + \sin(2x))$$

input
```
integrate((a*cos(x)^4)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a)*(2*x + sin(2*x))
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cos^4(x)}\,dx = \int \sqrt{a\cos(x)^4}\,dx$$

input
```
int((a*cos(x)^4)^(1/2),x)
```

output
```
int((a*cos(x)^4)^(1/2), x)
```

3.54 $\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx$

3.54.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\cos(x)\sin(x)}{\sqrt{a\cos^4(x)}}$$

output
```
cos(x)*sin(x)/(a*cos(x)^4)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\cos(x)\sin(x)}{\sqrt{a\cos^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Cos[x]^4],x]
```

output
```
(Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

3.54.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos^4(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)^4}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos^2(x)\int \sec^2(x)dx}{\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^2(x)\int \csc\left(x+\frac{\pi}{2}\right)^2dx}{\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos^2(x)\int 1d(-\tan(x))}{\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(x)\cos(x)}{\sqrt{a\cos^4(x)}}$$

input
```
Int[1/Sqrt[a*Cos[x]^4],x]
```

output
```
(Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

3.54.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.54.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\cos(x)\sin(x)}{\sqrt{a(\cos^4(x))}}$	14
risch	$\frac{2i(1+\mathrm{e}^{-2ix})}{\sqrt{a(\mathrm{e}^{2ix}+1)^4\mathrm{e}^{-4ix}}}$	29

input
```
int(1/(a*cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(x)*sin(x)/(a*cos(x)^4)^(1/2)
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\sqrt{a\cos{(x)}^4}\sin{(x)}}{a\cos{(x)}^3}$$

input
```
integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*cos(x)^4)*sin(x)/(a*cos(x)^3)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(x)**4)**(1/2),x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\tan{(x)}}{\sqrt{a}}$$

input
```
integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
tan(x)/sqrt(a)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\tan{(x)}}{\sqrt{a}}$$

input
```
integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="giac")
```

output
```
tan(x)/sqrt(a)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\tan(x)}{\sqrt{a}}$$

input
```
int(1/(a*cos(x)^4)^(1/2),x)
```

output
```
tan(x)/a^(1/2)
```

3.55 $\int \frac{1}{(a\cos^4(x))^{3/2}}\,dx$

3.55.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{(a\cos^4(x))^{3/2}}\,dx = \frac{\cos(x)\sin(x)}{a\sqrt{a\cos^4(x)}} + \frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}} + \frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}$$

output
```
cos(x)*sin(x)/a/(a*cos(x)^4)^(1/2)+2/3*sin(x)^2*tan(x)/a/(a*cos(x)^4)^(1/2
)+1/5*sin(x)^2*tan(x)^3/a/(a*cos(x)^4)^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a\cos^4(x))^{3/2}}\,dx = \frac{\cos(x)(8+6\cos(2x)+\cos(4x))\sin(x)}{15\,(a\cos^4(x))^{3/2}}$$

input
```
Integrate[(a*Cos[x]^4)^(-3/2),x]
```

output
```
(Cos[x]*(8 + 6*Cos[2*x] + Cos[4*x])*Sin[x])/(15*(a*Cos[x]^4)^(3/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos^2(x)\int \sec^6(x)dx}{a\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^2(x)\int \csc\left(x+\frac{\pi}{2}\right)^6dx}{a\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos^2(x)\int \left(\tan^4(x)+2\tan^2(x)+1\right)d(-\tan(x))}{a\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos^2(x)\left(-\frac{1}{5}\tan^5(x)-\frac{2\tan^3(x)}{3}-\tan(x)\right)}{a\sqrt{a\cos^4(x)}}$$

input
```
Int[(a*Cos[x]^4)^(-3/2),x]
```

output
```
-((Cos[x]^2*(-Tan[x] - (2*Tan[x]^3)/3 - Tan[x]^5/5))/(a*Sqrt[a*Cos[x]^4]))
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.55.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{8\cos(x)\sin(x)+4\tan(x)+3\tan(x)\left(\sec^{2}(x)\right)}{15a\sqrt{a\left(\cos^{4}(x)\right)}}$	33
risch	$\frac{16i(5+11\cos(2x)+9i\sin(2x))}{15a\left(e^{2ix}+1\right)^{3}\sqrt{a\left(e^{2ix}+1\right)^{4}e^{-4ix}}}$	49

input
```
int(1/(a*cos(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/a/(a*cos(x)^4)^(1/2)*(8*cos(x)*sin(x)+4*tan(x)+3*tan(x)*sec(x)^2)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx = \frac{\sqrt{a\cos(x)^4}\left(8\cos(x)^4+4\cos(x)^2+3\right)\sin(x)}{15\,a^2\cos(x)^7}$$

input
```
integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/15*sqrt(a*cos(x)^4)*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^7)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(x)**4)**(3/2),x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx = \frac{3\tan(x)^5+10\tan(x)^3+15\tan(x)}{15\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^(3/2)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx = \frac{3\,\tan(x)^5 + 10\,\tan(x)^3 + 15\,\tan(x)}{15\,a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="giac")
```

output
```
1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^(3/2)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 14.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.54

$$\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx = \frac{4\,\sin(x)}{5\,a^{3/2}\cos(x)^3} + \frac{\sin(x)}{5\,a^{3/2}\cos(x)^5} - \frac{8\sin(x)^3}{15\,a^{3/2}\cos(x)^3}$$

input
```
int(1/(a*cos(x)^4)^(3/2),x)
```

output
```
(4*sin(x))/(5*a^(3/2)*cos(x)^3) + sin(x)/(5*a^(3/2)*cos(x)^5) - (8*sin(x)^
3)/(15*a^(3/2)*cos(x)^3)
```

3.56 $\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx$

3.56.1 Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx=\frac{\cos(x)\sin(x)}{a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}}+\frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^7(x)}{9a^2\sqrt{a\cos^4(x)}}$$

output
```
cos(x)*sin(x)/a^2/(a*cos(x)^4)^(1/2)+4/3*sin(x)^2*tan(x)/a^2/(a*cos(x)^4)^
(1/2)+6/5*sin(x)^2*tan(x)^3/a^2/(a*cos(x)^4)^(1/2)+4/7*sin(x)^2*tan(x)^5/a
^2/(a*cos(x)^4)^(1/2)+1/9*sin(x)^2*tan(x)^7/a^2/(a*cos(x)^4)^(1/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx=\frac{(128+130\cos(2x)+46\cos(4x)+10\cos(6x)+\cos(8x))\sec^6(x)\tan(x)}{315a^2\sqrt{a\cos^4(x)}}$$

input
```
Integrate[(a*Cos[x]^4)^(-5/2),x]
```

output
```
((128 + 130*Cos[2*x] + 46*Cos[4*x] + 10*Cos[6*x] + Cos[8*x])*Sec[x]^6*Tan[
x])/(315*a^2*Sqrt[a*Cos[x]^4])
```

3.56.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos^4(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(x+\frac{\pi}{2}\right)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cos^2(x)\int \sec^{10}(x)dx}{a^2\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^2(x)\int \csc\left(x+\frac{\pi}{2}\right)^{10}dx}{a^2\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\cos^2(x)\int \left(\tan^8(x)+4\tan^6(x)+6\tan^4(x)+4\tan^2(x)+1\right)d(-\tan(x))}{a^2\sqrt{a\cos^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos^2(x)\left(-\frac{1}{9}\tan^9(x)-\frac{4\tan^7(x)}{7}-\frac{6\tan^5(x)}{5}-\frac{4\tan^3(x)}{3}-\tan(x)\right)}{a^2\sqrt{a\cos^4(x)}}$$

input
```
Int[(a*Cos[x]^4)^(-5/2),x]
```

output
```
-((Cos[x]^2*(-Tan[x] - (4*Tan[x]^3)/3 - (6*Tan[x]^5)/5 - (4*Tan[x]^7)/7 -
Tan[x]^9/9))/(a^2*Sqrt[a*Cos[x]^4]))
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.39

method	result	size
default	$\frac{\tan(x)(\sec^6(x))(128(\cos^8(x))+64(\cos^6(x))+48(\cos^4(x))+40(\cos^2(x))+35)}{315a^2\sqrt{a(\cos^4(x))}}$	46
risch	$\frac{256i(126\,\mathrm{e}^{6ix}+84\,\mathrm{e}^{4ix}+9+37\cos(2x)+35i\sin(2x))}{315a^2(\mathrm{e}^{2ix}+1)^7\sqrt{a(\mathrm{e}^{2ix}+1)^4\mathrm{e}^{-4ix}}}$	63

input
```
int(1/(a*cos(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*tan(x)*sec(x)^6*(128*cos(x)^8+64*cos(x)^6+48*cos(x)^4+40*cos(x)^2+35
)/a^2/(a*cos(x)^4)^(1/2)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx = \frac{\left(128\cos(x)^8+64\cos(x)^6+48\cos(x)^4+40\cos(x)^2+35\right)\sqrt{a\cos(x)^4}\sin(x)}{315\,a^3\cos(x)^{11}}$$

input
```
integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sqrt(a
*cos(x)^4)*sin(x)/(a^3*cos(x)^11)
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(x)**4)**(5/2),x)
```

output
```
Timed out
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx = \frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))/a^(5/2)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a\cos^4(x))^{5/2}}\,dx = \frac{35\tan(x)^9 + 180\tan(x)^7 + 378\tan(x)^5 + 420\tan(x)^3 + 315\tan(x)}{315\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="giac")
```

output
```
1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(
x))/a^(5/2)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a\cos^4(x))^{5/2}}\,dx = \frac{e^{x\,4i}\sqrt{a\left(\frac{e^{-x\,1i}}{2}+\frac{e^{x\,1i}}{2}\right)^4}\,2048i}{5\,a^3\left(e^{x\,2i}+1\right)^5\left(e^{x\,2i}+2\,e^{x\,4i}+e^{x\,6i}\right)} - \frac{e^{x\,4i}\sqrt{a\left(\frac{e^{-x\,1i}}{2}+\frac{e^{x\,1i}}{2}\right)^4}\,4096i}{3\,a^3\left(e^{x\,2i}+1\right)^6\left(e^{x\,2i}+2\,e^{x\,4i}+e^{x\,6i}\right)} + \frac{e^{x\,4i}\sqrt{a\left(\frac{e^{-x\,1i}}{2}+\frac{e^{x\,1i}}{2}\right)^4}\,12288i}{7\,a^3\left(e^{x\,2i}+1\right)^7\left(e^{x\,2i}+2\,e^{x\,4i}+e^{x\,6i}\right)} - \frac{e^{x\,4i}\sqrt{a\left(\frac{e^{-x\,1i}}{2}+\frac{e^{x\,1i}}{2}\right)^4}\,1024i}{a^3\left(e^{x\,2i}+1\right)^8\left(e^{x\,2i}+2\,e^{x\,4i}+e^{x\,6i}\right)} + \frac{e^{x\,4i}\sqrt{a\left(\frac{e^{-x\,1i}}{2}+\frac{e^{x\,1i}}{2}\right)^4}\,2048i}{9\,a^3\left(e^{x\,2i}+1\right)^9\left(e^{x\,2i}+2\,e^{x\,4i}+e^{x\,6i}\right)}$$

input
```
int(1/(a*cos(x)^4)^(5/2),x)
```

output
```
(exp(x*4i)*(a*(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*2048i)/(5*a^3*(exp(x*2
i) + 1)^5*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) - (exp(x*4i)*(a*(exp(-x*1
i)/2 + exp(x*1i)/2)^4)^(1/2)*4096i)/(3*a^3*(exp(x*2i) + 1)^6*(exp(x*2i) +
2*exp(x*4i) + exp(x*6i))) + (exp(x*4i)*(a*(exp(-x*1i)/2 + exp(x*1i)/2)^4)^
(1/2)*12288i)/(7*a^3*(exp(x*2i) + 1)^7*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i
))) - (exp(x*4i)*(a*(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*1024i)/(a^3*(exp
(x*2i) + 1)^8*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) + (exp(x*4i)*(a*(exp(
-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*2048i)/(9*a^3*(exp(x*2i) + 1)^9*(exp(x*2i
) + 2*exp(x*4i) + exp(x*6i)))
```

3.57 $\int (b\cos^m(c+dx))^n\, dx$

3.57.1 Optimal result

Integrand size = 12, antiderivative size = 78

$$\int (b\cos^m(c+dx))^n\, dx = -\frac{\cos(c+dx)\,(b\cos^m(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+mn)\sqrt{\sin^2(c+dx)}}$$

output
```
-cos(d*x+c)*(b*cos(d*x+c)^m)^n*hypergeom([1/2, 1/2*m*n+1/2],[1/2*m*n+3/2],
cos(d*x+c)^2)*sin(d*x+c)/d/(m*n+1)/(sin(d*x+c)^2)^(1/2)
```

3.57.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int (b\cos^m(c+dx))^n\, dx = -\frac{(b\cos^m(c+dx))^n \cot(c+dx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(1+mn)}$$

input
```
Integrate[(b*Cos[c + d*x]^m)^n,x]
```

output
```
-(((b*Cos[c + d*x]^m)^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + m*n)/2, (
3 + m*n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + m*n)))
```

3.57.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos^m(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)^m\right)^n dx$$

$$\Big\downarrow \text{3687}$$

$$\cos^{-mn}(c+dx)\,(b\cos^m(c+dx))^n\int \cos^{mn}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-mn}(c+dx)\,(b\cos^m(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{mn}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)\cos(c+dx)\,(b\cos^m(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(mn+1),\frac{1}{2}(mn+3),\cos^2(c+dx)\right)}{d(mn+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x]^m)^n,x]
```

output
```
-((Cos[c + d*x]*(b*Cos[c + d*x]^m)^n*Hypergeometric2F1[1/2, (1 + m*n)/2, (
3 + m*n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m*n)*Sqrt[Sin[c + d*x]^2
]))
```

3.57.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.57.4 Maple [F]

$$\int (b(\cos^m(dx+c)))^n dx$$

input
```
int((b*cos(d*x+c)^m)^n,x)
```

output
```
int((b*cos(d*x+c)^m)^n,x)
```

3.57.5 Fricas [F]

$$\int (b\cos^m(c+dx))^n \, dx = \int (b\cos{(dx+c)}^m)^n \, dx$$

input
```
integrate((b*cos(d*x+c)^m)^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)^m)^n, x)
```

3.57.6 Sympy [F]

$$\int (b\cos^m(c+dx))^n\,dx = \int (b\cos^m{(c+dx)})^n\,dx$$

input
```
integrate((b*cos(d*x+c)**m)**n,x)
```

output
```
Integral((b*cos(c + d*x)**m)**n, x)
```

3.57.7 Maxima [F]

$$\int (b\cos^m(c+dx))^n\,dx = \int (b\cos{(dx+c)}^m)^n\,dx$$

input
```
integrate((b*cos(d*x+c)^m)^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c)^m)^n, x)
```

3.57.8 Giac [F]

$$\int (b\cos^m(c+dx))^n\,dx = \int (b\cos{(dx+c)}^m)^n\,dx$$

input
```
integrate((b*cos(d*x+c)^m)^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c)^m)^n, x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos^m(c+dx))^n\,dx = \int (b\cos(c+d\,x)^m)^n\,dx$$

input
```
int((b*cos(c + d*x)^m)^n,x)
```

output
```
int((b*cos(c + d*x)^m)^n, x)
```

3.58 $\int (c\cos^m(a+bx))^{5/2}\,dx$

3.58.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (c\cos^m(a+bx))^{5/2}\,dx = -\frac{2c^2\cos^{1+2m}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\cos^2(a+bx)\right)\sin(a+$$
$$b(2+5m)\sqrt{\sin^2(a+bx)}$$

output
```
-2*c^2*cos(b*x+a)^(1+2*m)*hypergeom([1/2, 1/2+5/4*m],[3/2+5/4*m],cos(b*x+a
)^2)*sin(b*x+a)*(c*cos(b*x+a)^m)^(1/2)/b/(2+5*m)/(sin(b*x+a)^2)^(1/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int (c\cos^m(a+bx))^{5/2}\,dx = -\frac{2(c\cos^m(a+bx))^{5/2}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\cos^2(a+bx)\right)\sqrt{\sin^2(a+}$$
$$b(2+5m)$$

input
```
Integrate[(c*Cos[a + b*x]^m)^(5/2),x]
```

output
```
(-2*(c*Cos[a + b*x]^m)^(5/2)*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)
/4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + 5*m))
```

3.58.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos^m(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)^m\right)^{5/2}dx$$

$$\Big\downarrow \text{3687}$$

$$c^2\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\int \cos^{\frac{5m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$c^2\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{5m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2c^2\sin(a+bx)\cos^{2m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5m+2),\frac{1}{4}(5m+6),\cos^2(a+bx)\right)}{b(5m+2)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x]^m)^(5/2),x]
```

output
```
(-2*c^2*Cos[a + b*x]^(1 + 2*m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/
2, (2 + 5*m)/4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 5*m)*Sq
rt[Sin[a + b*x]^2])
```

3.58.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.58.4 Maple [F]

$$\int (c(\cos^m (bx+a)))^{\frac{5}{2}}dx$$

input
```
int((c*cos(b*x+a)^m)^(5/2),x)
```

output
```
int((c*cos(b*x+a)^m)^(5/2),x)
```

3.58.5 Fricas [F(-2)]

Exception generated.

$$\int (c\cos^m(a+bx))^{5/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int (c \cos^m(a+bx))^{5/2} \, dx = \text{Timed out}$$

input
```
integrate((c*cos(b*x+a)**m)**(5/2),x)
```

output
```
Timed out
```

3.58.7 Maxima [F]

$$\int (c \cos^m(a+bx))^{5/2} \, dx = \int (c \cos{(bx+a)}^m)^{\frac{5}{2}} \, dx$$

input
```
integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a)^m)^(5/2), x)
```

3.58.8 Giac [F]

$$\int (c \cos^m(a+bx))^{5/2} \, dx = \int (c \cos{(bx+a)}^m)^{\frac{5}{2}} \, dx$$

input
```
integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a)^m)^(5/2), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int (c\cos^m(a+bx))^{5/2}\,dx = \int (c\cos(a+b\,x)^m)^{5/2}\,dx$$

input
```
int((c*cos(a + b*x)^m)^(5/2),x)
```

output
```
int((c*cos(a + b*x)^m)^(5/2), x)
```

3.59 $\int (c\cos^m(a+bx))^{3/2}\,dx$

3.59.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int (c\cos^m(a+bx))^{3/2}\,dx = -\frac{2c\cos^{1+m}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\cos^2(a+bx)\right)\sin(a+bx)}{b(2+3m)\sqrt{\sin^2(a+bx)}}$$

output
```
-2*c*cos(b*x+a)^(1+m)*hypergeom([1/2, 1/2+3/4*m],[3/2+3/4*m],cos(b*x+a)^2)
*sin(b*x+a)*(c*cos(b*x+a)^m)^(1/2)/b/(2+3*m)/(sin(b*x+a)^2)^(1/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int (c\cos^m(a+bx))^{3/2}\,dx = -\frac{2(c\cos^m(a+bx))^{3/2}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(2+3m)}$$

input
```
Integrate[(c*Cos[a + b*x]^m)^(3/2),x]
```

output
```
(-2*(c*Cos[a + b*x]^m)^(3/2)*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)
/4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + 3*m))
```

3.59.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos^m(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)^m\right)^{3/2}dx$$

$$\Big\downarrow \text{3687}$$

$$c\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\int \cos^{\frac{3m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$c\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2c\sin(a+bx)\cos^{m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3m+2),\frac{3(m+2)}{4},\cos^2(a+bx)\right)}{b(3m+2)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x]^m)^(3/2),x]
```

output
```
(-2*c*Cos[a + b*x]^(1 + m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (
2 + 3*m)/4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 3*m)*Sqrt
[Sin[a + b*x]^2])
```

3.59.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.59.4 Maple [F]

$$\int \left(c(\cos^{m}\left(bx+a\right))\right)^{\frac{3}{2}}dx$$

input
```
int((c*cos(b*x+a)^m)^(3/2),x)
```

output
```
int((c*cos(b*x+a)^m)^(3/2),x)
```

3.59.5 Fricas [F(-2)]

Exception generated.

$$\int (c\cos^m(a+bx))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.59.6 Sympy [F]

$$\int (c\cos^m(a+bx))^{3/2}\,dx = \int (c\cos^m{(a+bx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a)**m)**(3/2),x)
```

output
```
Integral((c*cos(a + b*x)**m)**(3/2), x)
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int (c\cos^m(a+bx))^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: BINDING-STACK overflow at size
 10240. Stack can probably be resized.Proceed with caution.
```

3.59.8 Giac [F]

$$\int (c\cos^m(a+bx))^{3/2}\,dx = \int (c\cos{(bx+a)}^m)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a)^m)^(3/2), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \left(c\cos^m(a+bx)\right)^{3/2}\,dx=\int {\left(c\,{\cos\left(a+b\,x\right)}^m\right)}^{3/2}\,dx$$

input
```
int((c*cos(a + b*x)^m)^(3/2),x)
```

output
```
int((c*cos(a + b*x)^m)^(3/2), x)
```

3.60 $\int \sqrt{c\cos^m(a+bx)}\,dx$

3.60.1 Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \sqrt{c\cos^m(a+bx)}\,dx = -\frac{2\cos(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\cos^2(a+bx)\right)\sin(a+bx)}{b(2+m)\sqrt{\sin^2(a+bx)}}$$

output
```
-2*cos(b*x+a)*hypergeom([1/2, 1/2+1/4*m],[3/2+1/4*m],cos(b*x+a)^2)*sin(b*x
+a)*(c*cos(b*x+a)^m)^(1/2)/b/(2+m)/(sin(b*x+a)^2)^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \sqrt{c\cos^m(a+bx)}\,dx = -\frac{2\sqrt{c\cos^m(a+bx)}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(2+m)}$$

input
```
Integrate[Sqrt[c*Cos[a + b*x]^m],x]
```

output
```
(-2*Sqrt[c*Cos[a + b*x]^m]*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4,
(6 + m)/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + m))
```

3.60.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\cos^m(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)^m}dx$$

$$\Big\downarrow \text{3687}$$

$$\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\int \cos^{\frac{m}{2}}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{m/2}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(a+bx)\cos^{\frac{m+2}{2}-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{4},\frac{m+6}{4},\cos^2(a+bx)\right)}{b(m+2)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[Sqrt[c*Cos[a + b*x]^m],x]
```

output
```
(-2*Cos[a + b*x]^(-1/2*m + (2 + m)/2)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometri
c2F1[1/2, (2 + m)/4, (6 + m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + m)*S
qrt[Sin[a + b*x]^2])
```

3.60.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.60.4 Maple [F]

$$\int \sqrt{c\left(\cos^{m}\left(bx+a\right)\right)}dx$$

input
```
int((c*cos(b*x+a)^m)^(1/2),x)
```

output
```
int((c*cos(b*x+a)^m)^(1/2),x)
```

3.60.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c\cos^{m}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.60.6 Sympy [F]

$$\int \sqrt{c\cos^m(a+bx)}\,dx = \int \sqrt{c\cos^m\left(a+bx\right)}\,dx$$

input
```
integrate((c*cos(b*x+a)**m)**(1/2),x)
```

output
```
Integral(sqrt(c*cos(a + b*x)**m), x)
```

3.60.7 Maxima [F]

$$\int \sqrt{c\cos^m(a+bx)}\,dx = \int \sqrt{c\cos\left(bx+a\right)^m}\,dx$$

input
```
integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*cos(b*x + a)^m), x)
```

3.60.8 Giac [F]

$$\int \sqrt{c\cos^m(a+bx)}\,dx = \int \sqrt{c\cos\left(bx+a\right)^m}\,dx$$

input
```
integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*cos(b*x + a)^m), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c\cos^m(a+bx)}\,dx = \int \sqrt{c\cos\left(a+b\,x\right)^m}\,dx$$

input
```
int((c*cos(a + b*x)^m)^(1/2),x)
```

output
```
int((c*cos(a + b*x)^m)^(1/2), x)
```

3.61 $\int \frac{1}{\sqrt{c \cos^m(a+bx)}} dx$

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{1}{\sqrt{c \cos^m(a+bx)}} dx = -\frac{2\cos(a+bx) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\cos^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
-2*cos(b*x+a)*hypergeom([1/2, 1/2-1/4*m],[3/2-1/4*m],cos(b*x+a)^2)*sin(b*x
+a)/b/(2-m)/(c*cos(b*x+a)^m)^(1/2)/(sin(b*x+a)^2)^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c \cos^m(a+bx)}} dx = \frac{2\cot(a+bx) \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(-2+m)\sqrt{c\cos^m(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*Cos[a + b*x]^m],x]
```

output
```
(2*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]^
2]*Sqrt[Sin[a + b*x]^2])/(b*(-2 + m)*Sqrt[c*Cos[a + b*x]^m])
```

3.61.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin\left(a+bx+\frac{\pi}{2}\right)^m}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{m}{2}}(a+bx)dx}{\sqrt{c\cos^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{m}{2}}(a+bx)\int \sin\left(a+bx+\frac{\pi}{2}\right)^{-m/2}dx}{\sqrt{c\cos^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(a+bx)\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\cos^2(a+bx)\right)}{b(2-m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

input
```
Int[1/Sqrt[c*Cos[a + b*x]^m],x]
```

output
```
(-2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]
^2]*Sin[a + b*x])/(b*(2 - m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[Sin[a + b*x]^2])
```

3.61.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.61.4 Maple [F]

$$\int \frac{1}{\sqrt{c\left(\cos^{m}\left(bx+a\right)\right)}}dx$$

input
```
int(1/(c*cos(b*x+a)^m)^(1/2),x)
```

output
```
int(1/(c*cos(b*x+a)^m)^(1/2),x)
```

3.61.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c\cos^{m}(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.61.6 Sympy [F]

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)**m)**(1/2),x)
```

output
```
Integral(1/sqrt(c*cos(a + b*x)**m), x)
```

3.61.7 Maxima [F]

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos(bx+a)^m}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*cos(b*x + a)^m), x)
```

3.61.8 Giac [F]

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos(bx+a)^m}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*cos(b*x + a)^m), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\cos\left(a+b\,x\right)^m}}\,dx$$

input
```
int(1/(c*cos(a + b*x)^m)^(1/2),x)
```

output
```
int(1/(c*cos(a + b*x)^m)^(1/2), x)
```

3.62 $\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx$

3.62.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = -\frac{2\cos^{1-m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\cos^2(a+bx)\right)\sin(a+bx)}{bc(2-3m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
-2*cos(b*x+a)^(1-m)*hypergeom([1/2, 1/2-3/4*m],[3/2-3/4*m],cos(b*x+a)^2)*s
in(b*x+a)/b/c/(2-3*m)/(c*cos(b*x+a)^m)^(1/2)/(sin(b*x+a)^2)^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = -\frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),-\frac{3}{4}(-2+m),\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{\left(b-\frac{3bm}{2}\right)(c\cos^m(a+bx))^{3/2}}$$

input
```
Integrate[(c*Cos[a + b*x]^m)^(-3/2),x]
```

output
```
-((Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/((b - (3*b*m)/2)*(c*Cos[a + b*x]^m)^(3/2)
))
```

3.62.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)^m\right)^{3/2}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{3m}{2}}(a+bx)dx}{c\sqrt{c\cos^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{m}{2}}(a+bx)\int \sin\left(a+bx+\frac{\pi}{2}\right)^{-3m/2}dx}{c\sqrt{c\cos^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(a+bx)\cos^{1-m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\cos^2(a+bx)\right)}{bc(2-3m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x]^m)^(-3/2),x]
```

output
```
(-2*Cos[a + b*x]^(1 - m)*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4
, Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(2 - 3*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt
[Sin[a + b*x]^2])
```

3.62.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.62.4 Maple [F]

$$\int \frac{1}{\left(c \left(\cos^{m}\left(b x +a \right)\right)\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(c*cos(b*x+a)^m)^(3/2),x)
```

output
```
int(1/(c*cos(b*x+a)^m)^(3/2),x)
```

3.62.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.62.6 Sympy [F]

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\cos^m\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)**m)**(3/2),x)
```

output
```
Integral((c*cos(a + b*x)**m)**(-3/2), x)
```

3.62.7 Maxima [F]

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\cos\left(bx+a\right)^m\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a)^m)^(-3/2), x)
```

3.62.8 Giac [F]

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\cos\left(bx+a\right)^m\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a)^m)^(-3/2), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = \int \frac{1}{\left(c\cos\left(a+b\,x\right)^m\right)^{3/2}}\,dx$$

input
```
int(1/(c*cos(a + b*x)^m)^(3/2),x)
```

output
```
int(1/(c*cos(a + b*x)^m)^(3/2), x)
```

3.63 $\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx$

3.63.1 Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = -\frac{2\cos^{1-2m}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\cos^2(a+bx)\right)\sin(a+bx)}{bc^2(2-5m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

output
```
-2*cos(b*x+a)^(1-2*m)*hypergeom([1/2, 1/2-5/4*m],[3/2-5/4*m],cos(b*x+a)^2)
*sin(b*x+a)/b/c^2/(2-5*m)/(c*cos(b*x+a)^m)^(1/2)/(sin(b*x+a)^2)^(1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = -\frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{\left(b-\frac{5bm}{2}\right)(c\cos^m(a+bx))^{5/2}}$$

input
```
Integrate[(c*Cos[a + b*x]^m)^(-5/2),x]
```

output
```
-((Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Cos[a + b
*x]^2]*Sqrt[Sin[a + b*x]^2])/((b - (5*b*m)/2)*(c*Cos[a + b*x]^m)^(5/2)))
```

3.63.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\sin\left(a+bx+\frac{\pi}{2}\right)^m\right)^{5/2}}dx$$

$$\Big\downarrow \text{3687}$$

$$\frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{5m}{2}}(a+bx)dx}{c^2\sqrt{c\cos^m(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{m}{2}}(a+bx)\int \sin\left(a+bx+\frac{\pi}{2}\right)^{-5m/2}dx}{c^2\sqrt{c\cos^m(a+bx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(a+bx)\cos^{1-2m}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\cos^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

input
```
Int[(c*Cos[a + b*x]^m)^(-5/2),x]
```

output
```
(-2*Cos[a + b*x]^(1 - 2*m)*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4
, Cos[a + b*x]^2]*Sin[a + b*x])/(b*c^2*(2 - 5*m)*Sqrt[c*Cos[a + b*x]^m]*Sq
rt[Sin[a + b*x]^2])
```

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.63.4 Maple [F]

$$\int \frac{1}{\left(c\left(\cos^{m}\left(bx+a\right)\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(c*cos(b*x+a)^m)^(5/2),x)
```

output
```
int(1/(c*cos(b*x+a)^m)^(5/2),x)
```

3.63.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c\cos^{m}(a+bx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.63.6 Sympy [F]

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\cos^m{(a+bx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)**m)**(5/2),x)
```

output
```
Integral((c*cos(a + b*x)**m)**(-5/2), x)
```

3.63.7 Maxima [F]

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\cos{(bx+a)}^m)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a)^m)^(-5/2), x)
```

3.63.8 Giac [F]

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\cos{(bx+a)}^m)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*cos(b*x + a)^m)^(-5/2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(c\cos\left(a+b\,x\right)^m\right)^{5/2}}\,dx$$

input
```
int(1/(c*cos(a + b*x)^m)^(5/2),x)
```

output
```
int(1/(c*cos(a + b*x)^m)^(5/2), x)
```

3.64 $\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx$

3.64.1 Optimal result

Integrand size = 14, antiderivative size = 24

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx = \frac{(c\cos^m(a+bx))^{\frac{1}{m}}\tan(a+bx)}{b}$$

output
```
(c*cos(b*x+a)^m)^(1/m)*tan(b*x+a)/b
```

3.64.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx = \frac{(c\cos^m(a+bx))^{\frac{1}{m}}\tan(a+bx)}{b}$$

input
```
Integrate[(c*Cos[a + b*x]^m)^m^(-1),x]
```

output
```
((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

3.64.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3687, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(c\sin\left(a+bx+\frac{\pi}{2}\right)^m\right)^{\frac{1}{m}}dx$$

$$\Big\downarrow \text{3687}$$

$$\sec(a+bx)\,(c\cos^m(a+bx))^{\frac{1}{m}}\int \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(a+bx)\,(c\cos^m(a+bx))^{\frac{1}{m}}\int \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\tan(a+bx)\,(c\cos^m(a+bx))^{\frac{1}{m}}}{b}$$

input
```
Int[(c*Cos[a + b*x]^m)^m^(-1),x]
```

output
```
((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

3.64.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.64.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

method	result	size
parallelrisch	$-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(c(\cos^m(bx+a))\right)^{\frac{1}{m}}}{b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}$	44

input
```
int((c*cos(b*x+a)^m)^(1/m),x,method=_RETURNVERBOSE)
```

output
```
-2/b*tan(1/2*b*x+1/2*a)*(c*cos(b*x+a)^m)^(1/m)/(tan(1/2*b*x+1/2*a)^2-1)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx=\frac{c^{\left(\frac{1}{m}\right)}\sin{(bx+a)}}{b}$$

input
```
integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="fricas")
```

output
```
c^(1/m)*sin(b*x + a)/b
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx = \begin{cases} x(c\cos^m(a))^{\frac{1}{m}} & \text{for } b=0 \\ x(0^m c)^{\frac{1}{m}} & \text{for } a=-bx+\frac{\pi}{2} \vee a=-bx+\frac{3\pi}{2} \\ \frac{(c\cos^m(a+bx))^{\frac{1}{m}}\sin(a+bx)}{b\cos(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate((c*cos(b*x+a)**m)**(1/m),x)
```

output
```
Piecewise((x*(c*cos(a)**m)**(1/m), Eq(b, 0)), (x*(0**m*c)**(1/m), Eq(a, -b
*x + pi/2) | Eq(a, -b*x + 3*pi/2)), ((c*cos(a + b*x)**m)**(1/m)*sin(a + b*
x)/(b*cos(a + b*x)), True))
```

3.64.7 Maxima [F]

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx = \int (c\cos(bx+a)^m)^{\left(\frac{1}{m}\right)}\,dx$$

input
```
integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="maxima")
```

output
```
integrate((c*cos(b*x + a)^m)^(1/m), x)
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(24) = 48$.

Time = 0.56 (sec) , antiderivative size = 300, normalized size of antiderivative = 12.50

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx$$
$$= \frac{2\left(|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^3-|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\right.}{b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+2b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{}{4}\right.}$$

input
```
integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="giac")
```

output
```
2*(abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/
2*b*x + 1/2*a)^3 - abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/
4*pi/m)^2*tan(1/2*b*x + 1/2*a) + 4*abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*
pi*sgn(c)/m - 1/4*pi/m)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/m)*tan(1/2*b*x
+ 1/2*a)^3 + abs(c)^(1/m)*tan(1/2*b*x + 1/2*a))/(b*tan(1/2*b*x + 1/2*a + 1
/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/2*b*x + 1/2*a)^4 + 2*b*tan(1/2*b*x + 1/
2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/2*b*x + 1/2*a)^2 + b*tan(1/2*b*x
 + 1/2*a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2 + 2*b*
tan(1/2*b*x + 1/2*a)^2 + b)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 14.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx = \frac{\sin(2\,a+2\,b\,x)\,\left(c\cos(a+b\,x)^m\right)^{1/m}}{b\,\left(\cos(2\,a+2\,b\,x)+1\right)}$$

input
```
int((c*cos(a + b*x)^m)^(1/m),x)
```

output
```
(sin(2*a + 2*b*x)*(c*cos(a + b*x)^m)^(1/m))/(b*(cos(2*a + 2*b*x) + 1))
```

3.65 $\int (a(b\cos(c+dx))^p)^n \, dx$

3.65.1 Optimal result

Integrand size = 14, antiderivative size = 80

$$\int (a(b\cos(c+dx))^p)^n \, dx = -\frac{\cos(c+dx)\left(a(b\cos(c+dx))^p\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\cos^2(c+dx)\right)\sin(c+dx}{d(1+np)\sqrt{\sin^2(c+dx)}}$$

output
```
-cos(d*x+c)*(a*(b*cos(d*x+c))^p)^n*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3
/2],cos(d*x+c)^2)*sin(d*x+c)/d/(n*p+1)/(sin(d*x+c)^2)^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int (a(b\cos(c+dx))^p)^n \, dx = -\frac{(a(b\cos(c+dx))^p)^n\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\cos^2(c+dx)\right)\sqrt{\sin^2(c+}}{d(1+np)}$$

input
```
Integrate[(a*(b*Cos[c + d*x])^p)^n,x]
```

output
```
-(((a*(b*Cos[c + d*x])^p)^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/
2, (3 + n*p)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + n*p)))
```

3.65.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3687, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a(b\cos(c+dx))^p)^n \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^p\right)^n dx$$

$$\Big\downarrow \text{3687}$$

$$(b\cos(c+dx))^{-np}(a(b\cos(c+dx))^p)^n \int (b\cos(c+dx))^{np}dx$$

$$\Big\downarrow \text{3042}$$

$$(b\cos(c+dx))^{-np}(a(b\cos(c+dx))^p)^n \int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{np} dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)\cos(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(np+1),\frac{1}{2}(np+3),\cos^2(c+dx)\right)(a(b\cos(c+dx))^p)^n}{d(np+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(a*(b*Cos[c + d*x])^p)^n,x]
```

output
```
-((Cos[c + d*x]*(a*(b*Cos[c + d*x])^p)^n*Hypergeometric2F1[1/2, (1 + n*p)/
2, (3 + n*p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + n*p)*Sqrt[Sin[c + d*
x]^2]))
```

3.65.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3687
```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> S
imp[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p]))   Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{
b, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Ma
tchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
 cos, tan, cot, sec, csc}, trig]])
```

3.65.4 Maple [F]

$$\int (a(\cos(dx+c)\, b)^p)^n \, dx$$

input
```
int((a*(cos(d*x+c)*b)^p)^n,x)
```

output
```
int((a*(cos(d*x+c)*b)^p)^n,x)
```

3.65.5 Fricas [F]

$$\int (a(b\cos(c+dx))^p)^n \, dx = \int ((b\cos(dx+c))^p\, a)^n \, dx$$

input
```
integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="fricas")
```

output
```
integral(((b*cos(d*x + c))^p*a)^n, x)
```

3.65.6 Sympy [F]

$$\int (a(b\cos(c+dx))^p)^n \, dx = \int (a(b\cos(c+dx))^p)^n \, dx$$

input
```
integrate((a*(b*cos(d*x+c))**p)**n,x)
```

output
```
Integral((a*(b*cos(c + d*x))**p)**n, x)
```

3.65.7 Maxima [F]

$$\int (a(b\cos(c+dx))^p)^n \, dx = \int \left((b\cos(dx+c))^p a\right)^n \, dx$$

input
```
integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="maxima")
```

output
```
integrate(((b*cos(d*x + c))^p*a)^n, x)
```

3.65.8 Giac [F]

$$\int (a(b\cos(c+dx))^p)^n \, dx = \int \left((b\cos(dx+c))^p a\right)^n \, dx$$

input
```
integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="giac")
```

output
```
integrate(((b*cos(d*x + c))^p*a)^n, x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int (a(b\cos(c+dx))^p)^n \, dx = \int (a\,(b\,\cos\,(c+d\,x))^p)^n \, dx$$

input
```
int((a*(b*cos(c + d*x))^p)^n,x)
```

output
```
int((a*(b*cos(c + d*x))^p)^n, x)
```

3.66 $\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{30b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^4d}$$

output
```
18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/11*(b*cos(d*x+c))^(9/2)*sin(
d*x+c)/b^4/d+30/77*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+3
0/77*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.66.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(240\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(\right.}{616d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^5*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Sqr
t[Cos[c + d*x]])
```

3.66.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{11/2}dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\int (b\cos(c+dx))^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^5}$$

$\downarrow$ 3115

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^5}$$

$\downarrow$ 3042

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/}}{11d}}{b^5}$$

$\downarrow$ 3121

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))}{11d}}{b^5}$$

$\downarrow$ 3042

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c}{11d}}{b^5}$$

$\downarrow$ 3120

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(}{11d}}{b^5}$$

input
```
Int[Cos[c + d*x]^5*Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*d) + (9*b^2*((2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^5
```

3.66. $\int\cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.66.4 Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.90

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2040\left(\cos^{7}\left(\cdots\right.\right.\right.}{77\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\cdots\right.\right.\right.}}$

input
```
int(cos(d*x+c)^5*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(448*cos
(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3
+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/
2*c)^2-1)*b)^(1/2)/d
```

3.66.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2\left(7\cos(dx+c)^4+9\cos(dx+c)^2+15\right)\sqrt{b\cos(dx+c)}\sin(dx+c)-15i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-}{77\,d}$$

input
```
integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*si
n(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)))/d
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.66.7 Maxima [F]

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

3.66.8 Giac [F]

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^5\,dx$$

input
```
integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \cos(c+d\,x)^5\,\sqrt{b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2), x)
```

3.67 $\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx=\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

output
```
14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*x
+c)/b^3/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}(38\sin(2(c+dx))+5\sin(4(c+dx)))\right)}{180d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Sqrt[Cos[c + d*x]])
```

3.67.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{9/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\int (b\cos(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{7}{9}b^2\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^4}$$

input
```
Int[Cos[c + d*x]^4*Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*d) + (7*b^2*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^4
```

3.67.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(109) = 218$.

Time = 4.48 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.28

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}\right.}$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(160*cos
(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-24*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.67.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$=\frac{2\left(5\cos\left(dx+c\right)^3+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weiers}}{}$$

input
```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
 c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.67.7 Maxima [F]

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

3.67.8 Giac [F]

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^4\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2), x)
```

3.68 $\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

output
```
2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.68.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(20\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(23\sin(c+dx)+3\sin(3(c+dx)))\right)}{42d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(
23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

3.68.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{7/2}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^3}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^3}$$

input
```
Int[Cos[c + d*x]^3*Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^3
```

3.68.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.68.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.19

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(c\right.\right.}}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*cos(
1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2
*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.68.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$=\frac{2\sqrt{b\cos(dx+c)}\left(3\cos(dx+c)^2+5\right)\sin(dx+c)-5i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i}{21\,d}$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqr
t(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)))/d
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.68.7 Maxima [F]

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^3, x)
```

3.68.8 Giac [F]

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^3, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

input
```
int(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2), x)
```

3.69 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

output
```
2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/d/cos(d*x+c)^(1/2)
```

3.69.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Si
n[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

3.69.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{5/2}dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^2}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^2}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^2}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]],x]
```

output
```
((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d))/b^2
```

3.69.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(85) = 170$.

Time = 2.84 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\, (-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{(2(\cos^2}}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-8*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.69.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c) + 3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\text{co}$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.69.6 Sympy [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(c+dx)}\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*cos(c + d*x)**2, x)
```

3.69.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

3.69.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \cos(c+d\,x)^2\,\sqrt{b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2), x)
```

3.70 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.70.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{2b\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d
```

3.70.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{2(b\cos(c+dx))^{3/2}\left(\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
Sin[c + d*x]))/(3*b*d*Cos[c + d*x]^(3/2))
```

3.70.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b}$$

input
```
Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/b
```

3.70.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.70.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(83) = 166$.

Time = 2.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.81

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.70.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\text{co}}{3d}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

3.70.6 Sympy [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(c+dx)}\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*cos(c + d*x), x)
```

3.70.7 Maxima [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

3.70.8 Giac [F]

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \int \cos(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/2), x)
```

3.71 $\int \sqrt{b\cos(c+dx)}\,dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{b\cos(c+dx)}\,dx=\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.71.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{b\cos(c+dx)}\,dx=\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

3.71.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

3.71.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(60) = 120$.

Time = 2.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.74

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
risch	$-\frac{i\sqrt{2}\,\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)b\,\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(b\,\mathrm{e}^{2i(dx+c)}+b\right)}{b\sqrt{\mathrm{e}^{i(dx+c)}\left(b\,\mathrm{e}^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\left(\sqrt{-\cdots}\right.\right.}{\sqrt{b\,\mathrm{e}^{3i(dx+c)}+b\,\mathrm{e}^{i\cdots}}}\right.}{\cdots}$

input
```
int((cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.71.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \sqrt{b\cos(c+dx)}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\text{weierst}}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.71.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x)), x)
```

3.71.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c)), x)
```

3.71.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\cos(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c)), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\,dx = \int \sqrt{b\;\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2),x)
```

output
```
int((b*cos(c + d*x))^(1/2), x)
```

3.72 $\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx$

3.72.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = \frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]
)
```

3.72.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 2030, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{b\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]
)
```

3.72.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(61) = 122.

Time = 1.44 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.64

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$	142

input
```
int(sec(d*x+c)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/s
in(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.72.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\text{co}}{d}$$

input
```
integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/d
```

3.72.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{b\cos{(c+dx)}}\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*sec(c + d*x), x)
```

3.72.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{b\cos{(dx+c)}}\sec{(dx+c)}\,dx$$

input
```
integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

3.72.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt{b\cos(dx+c)}\sec(dx+c)\ dx$$

input
```
integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = \int \frac{\sqrt{b\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.73 $\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx = -\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d
/cos(d*x+c)^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{2b\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
(2*b*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d*
Sqrt[b*Cos[c + d*x]])
```

3.73.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$b^2\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b^2\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^2\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$b^2\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

output
```
b^2*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

3.73.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(83) = 166$.

Time = 1.78 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.11

method	result
default	$-\frac{2b\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\right.}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}}$

input
```
int(sec(d*x+c)^2*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)
^(1/2)/d
```

3.73.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.60

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+}{}$$

input
```
integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

3.73.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**2, x)
```

3.73.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.73.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^2\,dx$$

input
```
integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.74 $\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{2b\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{2b\left(\sqrt{\cos(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
(2*b*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*S
qrt[b*Cos[c + d*x]])
```

3.74.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt{b\cos(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$b^3\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b^3\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$b^3\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

output
```
b^3*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

3.74.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(86) = 172$.

Time = 1.84 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.41

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

input
```
int(sec(d*x+c)^3*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b*((2*cos(1/2*d*x+1/2*c
)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(
1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.74.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\cos{(dx+c)}^2\ \text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\ \sin{(dx+c)})+i\sqrt{2}\sqrt{b}\cos{(dx+c)}^2\ \text{we}}{3\,d\cos{(dx+c)}^2}$$

input
```
integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(d*cos(d*x + c)^2)
```

3.74.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{b\cos\left(c+dx\right)}\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**3, x)
```

3.74.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

3.74.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3\,dx$$

input
```
integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.75 $\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx$

3.75.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*b*sin(d*x+c)/d/(b*cos(d*x+c)
)^(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.75.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = \frac{2\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(-3\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{3}{2}\sin(2(c+dx))+\tan(c+dx)\right)}{5d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-3*Cos[c + d*x]^(3/2)*EllipticE[(c
 + d*x)/2, 2] + (3*Sin[2*(c + d*x)])/2 + Tan[c + d*x]))/(5*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^4\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

output
```
b^4*((2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.75.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(107) = 214$.

Time = 2.55 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.83

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\mathrm{co}\right.}{}$

input
```
int(sec(d*x+c)^4*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b
+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.75.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.24

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx$$
$$= \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))}{}$$

input
```
integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*
x + c))/(d*cos(d*x + c)^3)
```

3.75.6 Sympy [F]

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = \int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**4, x)
```

3.75.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = \int \sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.75.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = \int \sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

input
```
integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx = \int \frac{\sqrt{b\cos(c+d\,x)}}{\cos(c+d\,x)^4}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

3.76 $\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx = \frac{10b\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/7*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b^2*sin(d*x+c)/d/(b*cos(d*
x+c))^(3/2)+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx = \frac{\sqrt{b\cos(c+dx)}\sec^3(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
 d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

3.76.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{5\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{5\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^5\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5,x]
```

output
```
b^5*((2*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*Sin[
c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

3.76.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(110) = 220$.

Time = 2.14 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.04

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos}{}}\right.}{}$

input
```
int(sec(d*x+c)^5*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^4+40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))*b*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)^3/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/
d
```

3.76.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx$$
$$= \frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\sqrt{b}\cos(dx+c)^4}{21\,d\,\text{co}}$$

input
```
integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5
*cos(d*x + c)^2 + 3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.76.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.76.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx = \int \sqrt{b\cos(dx+c)}\sec(dx+c)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

3.76.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx = \int \sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^5\,dx$$

input
```
integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^5(c+dx)\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^5,x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^5, x)
```

3.77 $\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx = -\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

output
```
2/9*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^3*sin(d*x+c)/d/(b*cos(d*
x+c))^(5/2)+14/15*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx$$

$$= \frac{\sqrt{b\cos(c+dx)}\sec^5(c+dx)\left(-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150\sin(c+dx)+91\sin(3(c+dx))+\right.}{360d}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^6,x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5*(-336*Cos[c + d*x]^(9/2)*EllipticE[(c
 + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x
)]))/(360*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)\sqrt{b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$b^6\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{7\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{7\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^6\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

3.77. $\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx$

$$\downarrow 3119$$

$$b^6\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

input
```
Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^6,x]
```

output
```
b^6*((2*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + (7*((2*Sin[c + d*x]
)/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b
*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

3.77.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.77.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(131) = 262$.

Time = 3.06 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.37

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{144b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^5}-\frac{7\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\right.\right.}}{180b\left(\cos^2\left(\right.\right.}\right.}{}$

input
```
int(sec(d*x+c)^6*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1/144*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/
15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b
*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

3.77.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx$$
$$= \frac{-21i\,\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^5 \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))}{}$$

input
```
integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*
cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt
(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.77.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.77.7 Maxima [F]

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx = \int \sqrt{b\cos(dx+c)}\sec(dx+c)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

3.77.8 Giac [F]

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx = \int \sqrt{b\cos(dx+c)}\sec(dx+c)^6\,dx$$

input
```
integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx=\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^6}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^6,x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^6, x)
```

3.78 $\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.78.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{30b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30b\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^3d}$$

output
```
18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d*
x+c)/b^3/d+30/77*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+3
0/77*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.78.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{(b\cos(c+dx))^{3/2}\left(240\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(290\sin(c+dx)+57\sin(3(c+\right.}{616d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^4*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*C
os[c + d*x]^(3/2))
```

3.78.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{11/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\int (b\cos(c+dx))^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^4}$$

input

```
Int[Cos[c + d*x]^4*(b*Cos[c + d*x])^(3/2),x]
```

output

```
((2*b*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*d) + (9*b^2*((2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^4
```

3.78. $\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.78.4 Maple [A] (verified)

Time = 5.70 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.87

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2040\left(\cos^7\left(\right.\right.\right.}{77\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(448*c
os(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-
2040*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)
^3+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+
1/2*c)^2-1)*b)^(1/2)/d
```

3.78.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{-15i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPI}}{}$$

input
```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/77*(-15*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 15*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 2*(7*b*cos(d*x + c)^4 + 9*b*cos(d*x + c)^2 + 15*b)*sq
rt(b*cos(d*x + c))*sin(d*x + c))/d
```

3.78.6 Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4*(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.78.7 Maxima [F]

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

3.78.8 Giac [F]

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^4\,dx$$

input
```
integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^4\,(b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2), x)
```

3.79 $\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.79.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{14b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

output
```
14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*x+c
)/b^2/d+14/15*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{(b\cos(c+dx))^{3/2}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}(38\sin(2(c+dx))+5\sin(4(c+dx))\right)}{180d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^3*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Cos[c + d*x]^(3/2))
```

3.79.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{9/2}dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\int (b\cos(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{7}{9}b^2\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^3}$$

input
```
Int[Cos[c + d*x]^3*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*d) + (7*b^2*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^3
```

3.79.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(107) = 214$.

Time = 4.48 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.35

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\cos^5\left(\frac{dx}{2}+\right.\right.\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}\right.}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(160*c
os(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432
*cos(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-24*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2
))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.79.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{21i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-2}{}$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/45*(21*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(3/2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b
*cos(d*x + c)^3 + 7*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

3.79.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.79.7 Maxima [F]

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

3.79.8 Giac [F]

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^3\,(b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2), x)
```

3.80 $\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{10b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

output
```
2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+10/21*b^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.80.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{(b\cos(c+dx))^{3/2}\left(20\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(23\sin(c+dx)+3\sin(3(c+d\right.}{42d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Cos[c + d*x]^(3/2))
```

3.80.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{7/2}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^2}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^2
```

3.80.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.80.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\right.\right.}}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*co
s(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*co
s(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
(2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.80.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{3}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b^{\frac{3}{2}}\,\text{weierstrassPInve}}{2}$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(3*b*cos(d*x + c)^2 + 5*b)*sqrt(b*cos(d*x + c))*sin(d
*x + c))/d
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.80.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

3.80.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2), x)
```

3.81 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.81.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{6b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/d/cos(d*x+c)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5bd\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
Sin[2*(c + d*x)]))/(5*b*d*Cos[c + d*x]^(5/2))
```

3.81.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d))/b
```

3.81.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(83) = 166$.

Time = 2.67 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.18

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\right.\right.}}$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-8*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.81.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{2\sqrt{b\cos(dx+c)}b\cos(dx+c)\sin(dx+c) + 3i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInve}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*b^(3
/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

3.81.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{3/2}\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(3/2), x)
```

3.82 $\int (b\cos(c+dx))^{3/2}\,dx$

3.82.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\cos(c+dx))^{3/2}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d
```

3.82.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int (b\cos(c+dx))^{3/2}\,dx = \frac{2(b\cos(c+dx))^{3/2}\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
Sin[c + d*x]))/(3*d*Cos[c + d*x]^(3/2))
```

3.82.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 3115$$

$$\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3121$$

$$\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3120$$

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

input
```
Int[(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d
*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(86) = 172$.

Time = 2.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{2}\left(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}\right.}{3\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\, d}}$

input
```
int((cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.82.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int (b\cos(c+dx))^{3/2}\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInvers}}{3d}$$

input
```
integrate((b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*sqrt(b*cos(d*x + c))*b*sin(d*x + c))/d
```

3.82.6 Sympy [F]

$$\int (b\cos(c+dx))^{3/2}\,dx = \int (b\cos(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((b*cos(c + d*x))**(3/2), x)
```

3.82.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2), x)
```

3.82.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\,dx = \int (b\,\cos(c+d\,x))^{3/2}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2),x)
```

output
```
int((b*cos(c + d*x))^(3/2), x)
```

3.83 $\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

3.83.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\frac{2b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

output
```
2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx=\frac{2b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
)
```

3.83.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 2030, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

output
```
(2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
)
```

3.83.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(61) = 122$.

Time = 1.80 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.69

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\, E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\, d}$
risch	$-\frac{i\sqrt{2}\, b\sqrt{\left(e^{2i(dx+c)}+1\right)b\, e^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(b\, e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\, e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{i e^{i(dx+c)}}\left(-2iE\left(\sqrt{\right.\right.}}{\sqrt{b\, e^{3i(dx+c)}+b\, e}}\right.}{}$

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.83.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \frac{i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}b^{\frac{3}{2}}}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

output
```
Timed out
```

3.83.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.83.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.84 $\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

3.84.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx=\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x
]])
```

3.84.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 2030, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x
]])
```

3.84.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.84.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 1.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\, F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\, d}$	144

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.84.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassPInverse}(-}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/d
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.84.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.84.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{3/2}}{\cos(c+dx)^2}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.85 $\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = -\frac{2b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/d/cos(d*x+c)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{2b^2\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

output
```
(2*b^2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(
d*Sqrt[b*Cos[c + d*x]])
```

3.85.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$b^3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b^3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$b^3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

output
```
b^3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

3.85.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(86) = 172$.

Time = 1.78 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.00

method	result
default	$-\frac{2b^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2(\sin}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}}$

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*b^2*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*
b)^(1/2)/d
```

3.85.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(3/2)*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*sqrt(b*cos(d*x + c))*b*sin(d*x + c))/(d*cos(d*x + c))
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.85.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.85.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{3/2}}{\cos(c+dx)^3}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.86 $\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^2*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\frac{2b^2\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

output
```
(2*b^2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d
*Sqrt[b*Cos[c + d*x]])
```

3.86.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^4\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$b^4\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b^4\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$b^4\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

output
```
b^4*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

3.86.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.

Time = 1.78 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin(}$

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b^2*((2*cos(1/2*d*x+1/2
*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*co
s(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.86.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}b^{\frac{3}{2}}\cos{(dx}}{3\,d\cos{(d}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*b^(3/2)*cos(d*x + c)^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*b*sin(d
*x + c))/(d*cos(d*x + c)^2)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.86.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

3.86.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^4}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.87 $\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx = -\frac{6b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*b^2*sin(d*x+c)/d/(b*cos(d*x+
c))^(1/2)-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx = \frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(-12*Cos[c + d*x]^(5/2)*EllipticE[(
c + d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

3.87.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^5\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^5\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3119}$$

$$b^5\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5,x]
```

output
```
b^5*((2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.87.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(110) = 220$.

Time = 2.38 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.72

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E(c}{}$

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4
*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.87.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.21

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx = \frac{-3i\sqrt{2}b^{\frac{3}{2}}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(3/2)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*b*cos(d*x + c)^2 + b)*sqrt(b*cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^3)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.87.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.87.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx=\int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx=\int \frac{(b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^5}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^5,x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^5, x)
```

3.88 $\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \frac{10b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/7*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b^3*sin(d*x+c)/d/(b*cos(d*
x+c))^(3/2)+10/21*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c
 + d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

3.88.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{2030}$$

$$b^6\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{5\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{5\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^6\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^6\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^6\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6,x]
```

output
```
b^6*((2*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*Sin[
c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

3.88.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(112) = 224$.

Time = 2.14 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.98

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos}{}}\right.}{}$

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^4+40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))*b^2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^3/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2
)/d
```

3.88.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \frac{-5i\sqrt{2}b^{\frac{3}{2}}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5i\sqrt{2}b^{\frac{3}{2}}\cos{(d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^4*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b*cos(d*x + c)^2 + 3
*b)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.88.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \int (b\cos{(dx+c)})^{\frac{3}{2}}\sec{(dx+c)}^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

3.88.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = \int \frac{(b\,\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^6}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^6,x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^6, x)
```

3.89 $\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx$

3.89.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx=-\frac{14b\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

output
```
2/9*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^4*sin(d*x+c)/d/(b*cos(d*
x+c))^(5/2)+14/15*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*b*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx=\frac{(b\cos(c+dx))^{3/2}\sec^6(c+dx)\left(-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150\sin(c+dx)+91\sin(3(\right.}{360d}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^7,x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6*(-336*Cos[c + d*x]^(9/2)*EllipticE[
(c + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d
*x)]))/(360*d)
```

3.89.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \text{2030}$$

$$b^7\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^7\left(\frac{7\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{7\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^7\left(\frac{7\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{7\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^7\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^7\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^7\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3119}$$

3.89. $\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx$

$$b^7\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^7,x]
```

output
```
b^7*((2*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + (7*((2*Sin[c + d*x]
)/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b
*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

3.89.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(134) = 268$.

Time = 2.99 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.30

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{144b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^5}-\frac{7\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin\right.\right.}}{180b\left(\cos^2\right.}$

input
```
int((cos(d*x+c)*b)^(3/2)*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-1/144
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-1
4/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)
*b*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.89.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = \frac{-21i\sqrt{2}b^{\frac{3}{2}}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(d}{}$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/45*(-21*I*sqrt(2)*b^(3/2)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*b^(3/2)*
cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(21*b*cos(d*x + c)^4 + 7*b*cos(d*x + c)^2 + 5*b
)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**7,x)
```

output
```
Timed out
```

3.89.7 Maxima [F]

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^7, x)
```

3.89.8 Giac [F]

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^7, x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^7}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^7,x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^7, x)
```

3.90 $\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{30b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^2d}$$

output
```
18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d*x+
c)/b^2/d+30/77*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+30/
77*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.90.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(240\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(290\sin(c+dx)+57\sin(3(c\right.}{616d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^3*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*C
os[c + d*x]^(5/2))
```

3.90.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{11/2}dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\int (b\cos(c+dx))^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^3}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/}}{11d}}{b^3}$$

$$\Big\downarrow \; 3121$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))}{11d}}{b^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c}{11d}}{b^3}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(}{11d}}{b^3}$$

input
```
Int[Cos[c + d*x]^3*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*d) + (9*b^2*((2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^3
```

3.90. $\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.90.4 Maple [A] (verified)

Time = 10.46 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.89

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{3}\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2040\left(\cos^{7}\left(\right.\right.\right.}{77\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}\right.\right.\right.}}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(448*c
os(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-
2040*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)
^3+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+
1/2*c)^2-1)*b)^(1/2)/d
```

3.90.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{-15i\sqrt{2}b^{\frac{5}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}b^{\frac{5}{2}}\,\text{weierstrassPI}}{}$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/77*(-15*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 15*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 2*(7*b^2*cos(d*x + c)^4 + 9*b^2*cos(d*x + c)^2 + 15*b
^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.90.7 Maxima [F]

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

3.90.8 Giac [F]

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

input
```
integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^3\,(b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2), x)
```

3.91 $\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{14b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

output
```
14/45*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*x
+c)/b/d+14/15*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}(38\sin(2(c+dx))+5\sin(4(c+dx)))\right)}{180d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Cos[c + d*x]^(5/2))
```

3.91.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{9/2}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^2}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{7}{9}b^2\int (b\cos(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{9}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{7}{9}b^2\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^2}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*d) + (7*b^2*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^2
```

3.91.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.

Time = 5.66 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{3}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\cos^{5}\left(\frac{dx}{2}+\right.\right.\right.}{45\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{d}{2}\right.}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(160*c
os(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432
*cos(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-24*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2
))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.91.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{21i\sqrt{2}b^{\frac{5}{2}}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-2}{}$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(21*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(5/2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b
^2*cos(d*x + c)^3 + 7*b^2*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x + c))
/d
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.91.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

3.91.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2), x)
```

3.92 $\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.92.1 Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{10b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+10/21*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+10/21*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.92.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\Big(20\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + \sqrt{\cos(c+dx)}(23\sin(c+dx) + 3\sin(3(c+d}{42d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x
]]*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

3.92.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{7/2}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b
```

3.92.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.92.4 Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(c\right.\right.}}$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*co
s(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*co
s(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
(2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.92.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{5}{2}}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b^{\frac{5}{2}}\,\text{weierstrassPInve}}{}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(3*b^2*cos(d*x + c)^2 + 5*b^2)*sqrt(b*cos(d*x + c))*s
in(d*x + c))/d
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c), x)
```

3.92.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(5/2), x)
```

3.93 $\int (b\cos(c+dx))^{5/2}\,dx$

3.93.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\cos(c+dx))^{5/2}\,dx=\frac{6b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

output
```
2/5*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6/5*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int (b\cos(c+dx))^{5/2}\,dx=\frac{(b\cos(c+dx))^{5/2}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\sin(2(c+dx))\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
Sin[2*(c + d*x)]))/(5*d*Cos[c + d*x]^(5/2))
```

3.93.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

input
```
Int[(b*Cos[c + d*x])^(5/2),x]
```

output
```
(6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

3.93.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.93.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(86) = 172$.

Time = 2.85 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.04

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\right.\right.}}$

input
```
int((cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-8*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.93.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.30

$$\int (b\cos(c+dx))^{5/2}\,dx = \frac{2\sqrt{b\cos(dx+c)}b^2\cos(dx+c)\sin(dx+c)+3i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInv}$$

input
```
integrate((b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*b^
(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c))) - 3*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.93.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2), x)
```

3.93.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\,dx = \int (b\,\cos(c+d\,x))^{5/2}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2),x)
```

output
```
int((b*cos(c + d*x))^(5/2), x)
```

3.94 $\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

3.94.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\frac{2b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}$$

output
```
2/3*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*sin(d*
x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.94.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx=\frac{2b(b\cos(c+dx))^{3/2}\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}\sin(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

output
```
(2*b*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(3/2))
```

3.94.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 2030, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

output
```
b*((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))
```

3.94.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.94.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(88) = 176$.

Time = 2.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^{3}\left(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{}}{3\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.94.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-}{3d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*sqrt(b*cos(d*x + c))*b^2*sin(d*x + c))/d
```

3.94.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

output
```
Timed out
```

3.94.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.94.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.95 $\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

3.95.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\frac{2b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

output
```
2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx=\frac{2b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]])
```

3.95.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 2030, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^2\int \sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

output
```
(2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]])
```

3.95.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.95.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.

Time = 6.79 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
risch	$-\frac{i\sqrt{2}\,b^2\sqrt{\left(\mathrm{e}^{2i(dx+c)}+1\right)b\,\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(b\,\mathrm{e}^{2i(dx+c)}+b\right)}{b\sqrt{\mathrm{e}^{i(dx+c)}\left(b\,\mathrm{e}^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2iE\left(\cdots\right.\right.}{\sqrt{b\,\mathrm{e}^{3i(dx+c)}+b\cdots}}\cdots\right.}{\cdots}$

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.95.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - i\sqrt{2}b^{\frac{5}{2}}\cdots}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
(I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.95.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.95.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.96 $\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=\frac{2b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*
x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx=\frac{2(b\cos(c+dx))^{5/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

output
```
(2*(b*Cos[c + d*x])^(5/2)*EllipticF[(c + d*x)/2, 2])/(d*Cos[c + d*x]^(5/2)
)
```

3.96.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 2030, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\sqrt{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{b^3\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^3\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

output
```
(2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x
]])
```

3.96.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.96.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 27.80 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^{3}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\, F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\, d}$	144

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.96.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{5}{2}}\text{weierstrassPInverse}(-\ldots}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/d
```

3.96.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.96.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.96.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{5/2}}{\cos(c+dx)^3}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.97 $\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = -\frac{2b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{2b^3\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

output
```
(2*b^3*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(
d*Sqrt[b*Cos[c + d*x]])
```

3.97.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^4\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^4\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

output
```
b^4*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c
 + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

3.97.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.

Time = 98.97 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.91

method	result
default	$-\frac{2b^3\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2(\sin \cdots}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots\right.}}$

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-2*b^3*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*
b)^(1/2)/d
```

3.97.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(5/2)*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*sqrt(b*cos(d*x + c))*b^2*sin(d*x + c))/(d*cos(d*x + c))
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

output
```
Timed out
```

3.97.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.97.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{5/2}}{\cos(c+dx)^4}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.98 $\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \frac{2b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^3*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \frac{2b^3\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

output
```
(2*b^3*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d
*Sqrt[b*Cos[c + d*x]])
```

3.98.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \; 2030$$

$$b^5\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3116$$

$$b^5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3121$$

$$b^5\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$b^5\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \; 3120$$

$$b^5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

output
```
b^5*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

3.98.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.

Time = 1.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.35

$$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\right.\right.}{3\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\left(2\left(\right.\right.}$$

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^5,x)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b^3*((2*cos(1/2*d*x+1/2
*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*co
s(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.98.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}b^{\frac{5}{2}}\cos(dx\ldots}{3\,d\cos(\ldots}$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*b^(5/2)*cos(d*x + c)^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*b^2*sin
(d*x + c))/(d*cos(d*x + c)^2)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

3.98.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{5/2}}{\cos(c+dx)^5}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^5,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

3.99 $\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx$

3.99.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx=-\frac{6b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*b^3*sin(d*x+c)/d/(b*cos(d*x+
c))^(1/2)-6/5*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx=\frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+d\ldots}{10d}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(-12*Cos[c + d*x]^(5/2)*EllipticE[(
c + d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

3.99.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^6\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$b^6\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^6\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3116}$$

$$b^6\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^6\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b^6\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^6\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^6\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6,x]
```

output
```
b^6*((2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.99.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(112) = 224$.

Time = 1.67 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.67

$$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}$$

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^6,x)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.99.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.23

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx = \frac{-3i\,\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(5/2)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)^2 + b^2)*sqrt(b*cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**6,x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

3.99.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx=\int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx=\int \frac{(b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^6}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^6,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^6, x)
```

3.100 $\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx$

3.100.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \frac{10b^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/7*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b^4*sin(d*x+c)/d/(b*cos(d*
x+c))^(3/2)+10/21*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.69

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))+6\,\text{ta}\right.}{21d}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(10*Cos[c + d*x]^(5/2)*EllipticF[(c
 + d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

3.100.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^7}dx$$

$$\Big\downarrow \ 2030$$

$$b^7\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \ 3116$$

$$b^7\left(\frac{5\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b^7\left(\frac{5\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 3116$$

$$b^7\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b^7\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \; 3121$$

$$b^7 \left(\frac{5 \left(\frac{\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^2 \sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} \right)}{7b^2} + \frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\Big\downarrow \; 3042$$

$$b^7 \left(\frac{5 \left(\frac{\sqrt{\cos(c+dx)} \int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}} dx}{3b^2 \sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} \right)}{7b^2} + \frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

$$\Big\downarrow \; 3120$$

$$b^7 \left(\frac{5 \left(\frac{2\sqrt{\cos(c+dx)}\, \text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2 d \sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} \right)}{7b^2} + \frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}} \right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7,x]
```

output
```
b^7*((2*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*Sin[
c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

3.100.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.100.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(112) = 224$.

Time = 1.06 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.98

$$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin\right.\right.}{}$$

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^7,x)
```

output
```
-2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^4+40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))*b^3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^3/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2
)/d
```

3.100.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.19

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \frac{-5i\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}b^{\frac{5}{2}}\cos(d$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^4*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b^2*cos(d*x + c)^2 +
 3*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**7,x)
```

output
```
Timed out
```

3.100.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

3.100.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^7\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{5/2}}{\cos(c+dx)^7}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^7,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^7, x)
```

3.101 $\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx$

3.101.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = -\frac{14b^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

output
```
2/9*b^7*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^5*sin(d*x+c)/d/(b*cos(d*
x+c))^(5/2)+14/15*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*b^2*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = \frac{(b\cos(c+dx))^{5/2}\sec^7(c+dx)\left(-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150\sin(c+dx)+91\sin(3(\right.}{360d}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^8,x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7*(-336*Cos[c + d*x]^(9/2)*EllipticE[
(c + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d
*x)]))/(360*d)
```

3.101.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}{\sin\left(c+dx+\frac{\pi}{2}\right)^8}dx$$

$$\Big\downarrow \text{2030}$$

$$b^8\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^8\left(\frac{7\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^8\left(\frac{7\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^8\left(\frac{7\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^8 \left(\frac{7 \left(\frac{3 \int \frac{1}{\left(b \sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}} dx}{5b^2} + \frac{2 \sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{9b^2} + \frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}} \right)$$

$$\Big\downarrow \text{3116}$$

$$b^8 \left(\frac{7 \left(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\cos(c+dx)}dx}{b^2} \right)}{5b^2} + \frac{2 \sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{9b^2} + \frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}} \right)$$

$$\Big\downarrow \text{3042}$$

$$b^8 \left(\frac{7 \left(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2} \right)}{5b^2} + \frac{2 \sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{9b^2} + \frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}} \right)$$

$$\Big\downarrow \text{3121}$$

$$b^8 \left(\frac{7 \left(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}} \right)}{5b^2} + \frac{2 \sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{9b^2} + \frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}} \right)$$

$$\Big\downarrow \text{3042}$$

$$b^8 \left(\frac{7 \left(\frac{3 \left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}} \right)}{5b^2} + \frac{2 \sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} \right)}{9b^2} + \frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}} \right)$$

$$\Big\downarrow \text{3119}$$

3.101. $\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx$

$$b^8\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

input
```
Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^8,x]
```

output
```
b^8*((2*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + (7*((2*Sin[c + d*x]
)/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b
*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

3.101.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.101.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^8,x)
```

output
```
int((cos(d*x+c)*b)^(5/2)*sec(d*x+c)^8,x)
```

3.101.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = \frac{-21i\sqrt{2}b^{\frac{5}{2}}\cos(dx+c)^5\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(d$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/45*(-21*I*sqrt(2)*b^(5/2)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*b^(5/2)*
cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(21*b^2*cos(d*x + c)^4 + 7*b^2*cos(d*x + c)^2 +
 5*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**8,x)
```

output
```
Timed out
```

3.101.7 Maxima [F]

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^8, x)
```

3.101.8 Giac [F]

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^8, x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^8}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^8,x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^8, x)
```

3.102 $\int (b\cos(c+dx))^{7/2}\,dx$

3.102.1 Optimal result

Integrand size = 12, antiderivative size = 98

$$\int (b\cos(c+dx))^{7/2}\,dx = \frac{10b^4\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^3\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

output
```
2/7*b*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+10/21*b^4*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)/d/(b*cos(d*x+c))^(1/2)+10/21*b^3*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

3.102.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int (b\cos(c+dx))^{7/2}\,dx = \frac{b^3\sqrt{b\cos(c+dx)}\left(20\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sqrt{\cos(c+dx)}(23\sin(c+dx)+3\sin(3(c+dx\right.}{42d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(7/2),x]
```

output
```
(b^3*Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x
]]*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

3.102.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \ 3121$$

$$\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

input
```
Int[(b*Cos[c + d*x])^(7/2),x]
```

output
```
(2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.102.4 Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.14

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^4\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(c\right.\right.}}$

input
```
int((cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*(48*co
s(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*co
s(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
(2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.102.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int (b\cos(c+dx))^{7/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}b^{\frac{7}{2}}\text{weierstrassPInve}}{}$$

input
```
integrate((b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*b^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 5*I*sqrt(2)*b^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(3*b^3*cos(d*x + c)^2 + 5*b^3)*sqrt(b*cos(d*x + c))*s
in(d*x + c))/d
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.102.7 Maxima [F]

$$\int (b\cos(c+dx))^{7/2}\,dx = \int (b\cos(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(7/2), x)
```

3.102.8 Giac [F]

$$\int (b\cos(c+dx))^{7/2}\,dx = \int (b\cos(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(7/2), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{7/2}\,dx = \int (b\,\cos{(c+d\,x)})^{7/2}\,dx$$

input
```
int((b*cos(c + d*x))^(7/2),x)
```

output
```
int((b*cos(c + d*x))^(7/2), x)
```

3.103 $\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77bd} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d}$$

output
```
18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/11*(b*cos(d*x+c))^(9/2)*sin(
d*x+c)/b^5/d+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+30/
77*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

3.103.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{480\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+347\sin(2(c+dx))+64\sin(4(c+dx))+7\sin(6(c+dx))}{1232d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^6/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
 64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*d*Sqrt[b*Cos[c + d*x]])
```

3.103.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{11/2}dx}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{b^6}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\int (b\cos(c+dx))^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^6}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^6}$$

$\downarrow$ 3115

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^6}$$

$\downarrow$ 3042

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/}}{11d}}{b^6}$$

$\downarrow$ 3121

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))}{11d}}{b^6}$$

$\downarrow$ 3042

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c}{11d}}{b^6}$$

$\downarrow$ 3120

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(}{11d}}{b^6}$$

input
```
Int[Cos[c + d*x]^6/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*d) + (9*b^2*((2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^6
```

3.103. $\int\frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.103.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2040\left(\cos^{7}\left(\frac{dx}{2} \cdots\right.\right.\right.}{77\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\cdots\right.\right.\right.}}$

input
```
int(cos(d*x+c)^6/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(448*cos(1
/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040
*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+1
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*
c)^2-1)*b)^(1/2)/d
```

3.103.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\left(7\cos(dx+c)^4+9\cos(dx+c)^2+15\right)\sqrt{b\cos(dx+c)}\sin(dx+c)-15i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-\ldots}{77\,b\ldots}$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*si
n(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)))/(b*d)
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^6}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^6/sqrt(b*cos(d*x + c)), x)
```

3.103.8 Giac [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^6}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^6/sqrt(b*cos(d*x + c)), x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^6}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^6/(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^6/(b*cos(c + d*x))^(1/2), x)
```

3.104 $\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.104.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

output
```
14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d
*x+c)/b^4/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{168\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + \cos(c+dx)(38\sin(2(c+dx)) + 5\sin(4(c+dx)))}{180d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2
*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*d*Sqrt[b*Cos[c + d*x]])
```

3.104.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{9/2}dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\int (b\cos(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{7}{9}b^2\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^5}$$

input
```
Int[Cos[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*d) + (7*b^2*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^5
```

3.104.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.104. $\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.104.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.20

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}\right.}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1
/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos
(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.104.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2\left(5\cos\left(dx+c\right)^3+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weiers}}{}$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
 c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

3.104.8 Giac [F]

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^5}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^5}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^5/(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^5/(b*cos(c + d*x))^(1/2), x)
```

3.105 $\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.105.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}$$

output
```
2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

3.105.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.65

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 26\sin(2(c+dx)) + 3\sin(4(c+dx))}{84d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3
*Sin[4*(c + d*x)])/(84*d*Sqrt[b*Cos[c + d*x]])
```

3.105.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^4}$$

input
```
Int[Cos[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^4
```

3.105.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.105.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.13

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\mathrm{co}\right.\right.}}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*cos(1/
2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/
2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*c
os(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.105.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^2+5\right)\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i}{21\,bd}$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqr
t(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)))/(b*d)
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.105.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

3.105.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(cos(c + d*x)^4/(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^4/(b*cos(c + d*x))^(1/2), x)
```

3.106 $\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

output
```
2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b/d/cos(d*x+c)^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)\sin(2(c+dx))}{5d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c +
d*x)])/(5*d*Sqrt[b*Cos[c + d*x]])
```

3.106.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^3}$$

input
```
Int[Cos[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d))/b^3
```

3.106.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(88) = 176$.

Time = 2.57 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.92

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\right.\right.\right.}}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/
((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.106.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\text{co}$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.106.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.106.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^3}{\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int(cos(c + d*x)^3/(b*cos(c + d*x))^(1/2),x)
```

output
```
int(cos(c + d*x)^3/(b*cos(c + d*x))^(1/2), x)
```

3.107 $\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.107.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd}$$

output
```
2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(b*
cos(d*x+c))^(1/2)/b/d
```

3.107.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*d*S
qrt[b*Cos[c + d*x]])
```

3.107.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^2}$$

input
```
Int[Cos[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/b^2
```

3.107.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(85) = 170$.

Time = 1.79 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.107.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\text{co}}{3\,bd}$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

3.107.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

3.107.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.107.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+d\,x)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos(c+d\,x)}} + \frac{2\,\sin(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{3\,b\,d}$$

input
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(
1/2)) + (2*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d)
```

3.108 $\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.108.1 Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]
)
```

3.108.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2030, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]
)
```

3.108.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(63) = 126$.

Time = 1.72 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.44

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{\left(e^{2i(dx+c)}+1\right)b\,e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\left(-2iE\left(\sqrt{-i\left(e\right.}\right.\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx}}}}{d\sqrt{\left(e^{2i(dx+c)}+1\right)b}}$

input
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.108.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\text{weierst}}{bd}$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

3.108.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(cos(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

3.108.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.108.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+d\,x)}\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\sqrt{b\,\cos(c+d\,x)}}$$

input
```
int(cos(c + d*x)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/
2))
```

3.109 $\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$

3.109.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[1/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

3.109.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}}$	54

input
```
int(1/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*
InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
```

3.109.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\text{co}}{bd}$$

input
```
integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/(b*d)
```

3.109.6 Sympy [F]

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(b*cos(c + d*x)), x)
```

3.109.7 Maxima [F]

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cos(d*x + c)), x)
```

3.109.8 Giac [F]

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cos(d*x + c)), x)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{\cos(c+d\,x)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos(c+d\,x)}}$$

input
```
int(1/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/
2))
```

3.110 $\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.110.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=-\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d
/cos(d*x+c)^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{2\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d*Sq
rt[b*Cos[c + d*x]])
```

3.110.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)$$

input
```
Int[Sec[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))
```

3.110.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.

Time = 1.90 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.00

method	result
default	$-\frac{2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2(\sin^4(}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right.}}$

input
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(
1/2)/d
```

3.110.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+}{}$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))
```

3.110.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

3.110.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.110.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)
```

3.111 $\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.111.1 Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d
/(b*cos(d*x+c))^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx=\frac{2\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*Sqr
t[b*Cos[c + d*x]])
```

3.111.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[Sec[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^2*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[
c + d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

3.111.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(83) = 166$.

Time = 1.79 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c}{2}}}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\right.}$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*((2*cos(1/2*d*x+1/2*c)^
2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x
+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(1/
2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.111.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{we}}{3\,bd\cos(dx+c)^2}$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(b*d*cos(d*x + c)^2)
```

3.111.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

3.111.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.111.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

3.112 $\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/d/(b*cos(d*x+c))^
(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+6\sin(c+dx)+2\sec(c+dx)\tan(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[
c + d*x]*Tan[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

3.112.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^3\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[Sec[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^3*((2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.112.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(109) = 218$.

Time = 2.42 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.78

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E(\mathrm{co}\cdots}{\cdots}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2
*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4
*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.112.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3 \operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))}{}$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*
x + c))/(b*d*cos(d*x + c)^3)
```

3.112.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**3/sqrt(b*cos(c + d*x)), x)
```

3.112.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.112.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

3.113 $\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.113.1 Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/7*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b*sin(d*x+c)/d/(b*cos(d*x+
c))^(3/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 2(5+3\sec^2(c+dx))\tan(c+dx)}{21d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2
)*Tan[c + d*x])/(21*d*Sqrt[b*Cos[c + d*x]])
```

3.113.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^4\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$b^4\left(\frac{5\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{5\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3116}$$

$$b^4\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3121}$$

$$b^4\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$b^4\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$b^4\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[Sec[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^4*((2*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*Sin[
c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

3.113. $\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(107) = 214$.

Time = 2.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.81

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{28b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^4}-\frac{5\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{d}{}\right.\right.\right.}}{21b\left(\cos^2\left(\frac{dx}{}\right.\right.}\right.}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/28*cos(1/
2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^4-5/21*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+10/21*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2)))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.113.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.21

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-5i\,\sqrt{2}\sqrt{b}\cos{(dx+c)}^4\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+5i\,\sqrt{2}\sqrt{b}\cos{(dx+c)}^4 \cdots}{21\,bd\,c\cdots}$$

input
```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5
*cos(d*x + c)^2 + 3)*sin(d*x + c))/(b*d*cos(d*x + c)^4)
```

3.113.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec^4{(c+dx)}}{\sqrt{b\cos{(c+dx)}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**4/sqrt(b*cos(c + d*x)), x)
```

3.113.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

3.113.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)), x)
```

3.114 $\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = -\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

output
```
2/9*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^2*sin(d*x+c)/d/(b*cos(d*
x+c))^(5/2)+14/15*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*c
os(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{-42\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+42\sin(c+dx)+2\sec(c+dx)\left(7+5\sec^2(c+dx)\right)\tan(c+dx)}{45d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Se
c[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*d*Sqrt[b*Cos[c + d*x]]
)
```

3.114.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^5\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^5\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{7\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{7\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^5\left(\frac{7\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^5\left(\frac{7\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3116

$$b^5\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3042

$$b^5\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3121

$$b^5\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3042

$$b^5\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3119

3.114. $\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

$$b^5\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

input
```
Int[Sec[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

output
```
b^5*((2*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + (7*((2*Sin[c + d*x]
)/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b
*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

3.114.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(133) = 266$.

Time = 3.18 (sec) , antiderivative size = 413, normalized size of antiderivative = 3.30

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{72b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^5}-\frac{7\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{d}{2}\right.\right.\right.}}{90b\left(\cos^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

input
```
int(sec(d*x+c)^5/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/72*cos(1/
2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^5-7/90*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-28/15*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1
/2*d*x+1/2*c)^2)^(1/2)+14/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-14/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1
/2)/d
```

3.114.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{-21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))}{}$$

input
```
integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*
cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt
(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)
```

3.114.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec^5\left(c+dx\right)}{\sqrt{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sec(c + d*x)**5/sqrt(b*cos(c + d*x)), x)
```

3.114.7 Maxima [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

3.114.8 Giac [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)), x)
```

3.115 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{77bd\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^6d}$$

output
```
18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+2/11*(b*cos(d*x+c))^(9/2)*sin(
d*x+c)/b^6/d+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+3
0/77*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d
```

3.115.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.59

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{480\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+347\sin(2(c+dx))+64\sin(4(c+dx)}{1232bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^7/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
 64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*b*d*Sqrt[b*Cos[c + d*x]])
```

3.115.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{11/2}dx}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{b^7}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{9}{11}b^2\int (b\cos(c+dx))^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{9}{11}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^7}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^7}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^7}$$

$\big\downarrow$ 3042

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/}}{11d}}{b^7}$$

$\big\downarrow$ 3121

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))}{11d}}{b^7}$$

$\big\downarrow$ 3042

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c}{11d}}{b^7}$$

$\big\downarrow$ 3120

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(}{11d}}{b^7}$$

input
```
Int[Cos[c + d*x]^7/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*d) + (9*b^2*((2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^7
```

3.115. $\int\frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.115.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.115.4 Maple [A] (verified)

Time = 5.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.84

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2040\left(\cos^{7}\left(\frac{dx}{2}\right.\right.\right.}{77b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$

input
```
int(cos(d*x+c)^7/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(448*cos
(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3
+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/
2*c)^2-1)*b)^(1/2)/d
```

3.115.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(7\cos\left(dx+c\right)^4+9\cos\left(dx+c\right)^2+15\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)-15i\sqrt{2}\sqrt{}}{}$$

input
```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*si
n(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)))/(b^2*d)
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.115.7 Maxima [F]

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(3/2), x)
```

3.115.8 Giac [F]

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(3/2), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^7}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^7/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^7/(b*cos(c + d*x))^(3/2), x)
```

3.116 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

output
```
14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d
*x+c)/b^5/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{168\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + \cos(c+dx)(38\sin(2(c+dx)) + 5\sin(4(c+}{180bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^6/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2
*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*b*d*Sqrt[b*Cos[c + d*x]])
```

3.116.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{9/2}dx}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^6}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\int (b\cos(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{7}{9}b^2\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^6}$$

input
```
Int[Cos[c + d*x]^6/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*d) + (7*b^2*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^6
```

3.116.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.116. $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.116.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{45b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}\right.}$

input
```
int(cos(d*x+c)^6/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(160*cos
(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-24*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))
^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.116.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(5\cos\left(dx+c\right)^3+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\mathrm{weier}}{}$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
 c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.116.7 Maxima [F]

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^6}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(3/2), x)
```

3.116.8 Giac [F]

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^6}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(3/2), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input

```
int(cos(c + d*x)^6/(b*cos(c + d*x))^(3/2),x)
```

output

```
int(cos(c + d*x)^6/(b*cos(c + d*x))^(3/2), x)
```

3.117 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.117.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

output
```
2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d
```

3.117.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))}{84bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^5/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3
*Sin[4*(c + d*x)])/(84*b*d*Sqrt[b*Cos[c + d*x]])
```

3.117.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{7/2}dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

$$\Big\downarrow \; 3120$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^5}$$

input
```
Int[Cos[c + d*x]^5/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^5
```

3.117.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.117.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{21b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(c\right.\right.}}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*cos(
1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2
*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.117.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\,\sqrt{b\cos\left(dx+c\right)}\left(3\,\cos\left(dx+c\right)^2+5\right)\sin\left(dx+c\right)-5i\,\sqrt{2}\sqrt{b}\text{weierstrassPInvers}}{}$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqr
t(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)))/(b^2*d)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.117.7 Maxima [F]

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^5}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(3/2), x)
```

3.117.8 Giac [F]

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^5}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(3/2), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^5}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^5/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^5/(b*cos(c + d*x))^(3/2), x)
```

3.118 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.118.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

output
```
2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)\sin(2(c+dx))}{5bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c +
d*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

3.118.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}dx}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^4}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^4}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^4}$$

input
```
Int[Cos[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d))/b^4
```

3.118.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.

Time = 2.85 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.96

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\right.\right.\right.}}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-8*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/
(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c
)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.118.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weier}$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.118.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.118.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^4}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^4/(b*cos(c + d*x))^(3/2), x)
```

3.119 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.119.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}$$

output
```
2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/b^2/d
```

3.119.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))}{3bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*b*d
*Sqrt[b*Cos[c + d*x]])
```

3.119.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}dx}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b^3}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^3}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^3}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^3}$$

input
```
Int[Cos[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/b^3
```

3.119.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(88) = 176$.

Time = 2.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}}{3b\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.119.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weier}}{3}$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.119.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^3}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^3/(b*cos(c + d*x))^(3/2), x)
```

3.120 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x
]])
```

3.120.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x
]])
```

3.120.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 2.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,e^{-i(dx+c)}}{db\sqrt{\left(e^{2i(dx+c)}+1\right)b\,e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{i e^{i(dx+c)}}\left(-2iE\left(\sqrt{-i(\cdots}\right.\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(d\cdots}}}\cdots\right.}{db\sqrt{\left(e^{2i(dx+c)}+1\cdots\right.}}$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))/b/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/si
n(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.120.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

3.120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.120.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.120.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)^2}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(3/2), x)
```

3.121 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.121.1 Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]]
)
```

3.121.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2030, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]]
)
```

3.121.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 1.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$	144

input
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/s
in(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.121.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weier}}{b^2d}$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/(b^2*d)
```

3.121.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(cos(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

3.121.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.121.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(c+d\,x)}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(cos(c + d*x)/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)/(b*cos(c + d*x))^(3/2), x)
```

3.122 $\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$

3.122.1 Optimal result

Integrand size = 12, antiderivative size = 68

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
2*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b
^2/d/cos(d*x+c)^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(-3/2),x]
```

output
```
(2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(b*d*
Sqrt[b*Cos[c + d*x]])
```

3.122.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(-3/2),x]
```

output
```
(-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

3.122.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 1.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.91

method	result
default	$-\frac{2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\right.\right.}}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

input
```
int(1/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)
^(1/2)/d
```

3.122.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+$$

input
```
integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))
```

3.122.6 Sympy [F]

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((b*cos(c + d*x))**(-3/2), x)
```

3.122.7 Maxima [F]

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(-3/2), x)
```

3.122.8 Giac [F]

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(-3/2), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(1/(b*cos(c + d*x))^(3/2), x)
```

3.123 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.123.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

output
```
2/3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d
/(b*cos(d*x+c))^(1/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{2\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*b*d*S
qrt[b*Cos[c + d*x]])
```

3.123.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 2030, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)$$

input
```
Int[Sec[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b*((2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c
+ d*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)))
```

3.123.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(85) = 170$.

Time = 2.15 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.49

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\cdots\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\cdots\right.}$

input
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/b*((2*cos(1/2*d*x+1/2*c
)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d
*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*cos(
1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.123.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.49

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos{(dx+c)}^2 \,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(b^2*d*cos(d*x + c)^2)
```

3.123.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

3.123.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.123.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

3.124 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.124.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

output
```
2/5*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/b/d/(b*cos(d*x+c))^
(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 6\sin(c+dx) + 2\sec(c+dx)\tan(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[
c + d*x]*Tan[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

3.124.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b^2\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b^2*((2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[
c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.124.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(110) = 220$.

Time = 2.65 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.74

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\mathrm{co}\right.\right.}{}$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.124.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\cos{(dx+c)}^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(dx}$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*
x + c))/(b^2*d*cos(d*x + c)^3)
```

3.124.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^2{(c+dx)}}{(b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x)
```

3.124.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec{(dx+c)}^2}{(b\cos{(dx+c)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.124.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

3.125 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

output
```
2/7*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*sin(d*x+c)/d/(b*cos(d*x+c)
)^(3/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+2(5+3\sec^2(c+dx))\tan(c+dx)}{21bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2
)*Tan[c + d*x])/(21*b*d*Sqrt[b*Cos[c + d*x]])
```

3.125.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{5\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{5\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^3\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^3\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b^3*((2*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*Sin[
c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

3.125.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(109) = 218$.

Time = 2.34 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.10

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\frac{\cos}{}}\right.}{}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^4+40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))/b*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^
(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)^3/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/
d
```

3.125.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))}{}$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5
*cos(d*x + c)^2 + 3)*sin(d*x + c))/(b^2*d*cos(d*x + c)^4)
```

3.125.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(3/2), x)
```

3.125.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.125.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)), x)
```

3.126 $\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.126.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = -\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}$$

output
```
2/9*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b*sin(d*x+c)/d/(b*cos(d*x+
c))^(5/2)+14/15*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*c
os(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-42\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 42\sin(c+dx) + 2\sec(c+dx)\,(7+5\sec^2(c}{45bd\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Se
c[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*b*d*Sqrt[b*Cos[c + d*x
]])
```

3.126.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^4\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{7\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{7\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{7\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^4\left(\frac{7\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^4\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3042

$$b^4\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3121

$$b^4\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3042

$$b^4\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3119

$$b^4\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

3.126. $\int\frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

input
```
Int[Sec[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

output
```
b^4*((2*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + (7*((2*Sin[c + d*x]
)/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b
*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

3.126.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(134) = 268$.

Time = 3.12 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.30

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{144b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^5}-\frac{7\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\right.\right.\right.}}{180b\left(\cos^2\left(\right.\right.}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1/144*c
os(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/
15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b
*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b
)^(1/2)/d
```

3.126.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{-21i\sqrt{2}\sqrt{b}\cos{(dx+c)}^5\ \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(d}}{}$$

input
```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*
cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt
(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5)
```

3.126.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)**4/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sec(c + d*x)**4/(b*cos(c + d*x))**(3/2), x)
```

3.126.7 Maxima [F]

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.126.8 Giac [F]

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2)), x)
```

3.127 $\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{77b^2d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^7d}$$

output
```
18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+2/11*(b*cos(d*x+c))^(9/2)*sin(
d*x+c)/b^7/d+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
+30/77*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d
```

3.127.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.59

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{480\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 347\sin(2(c+dx)) + 64\sin(4(c+dx}{1232b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^8/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
 64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*b^2*d*Sqrt[b*Cos[c + d*x]
])
```

3.127.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{11/2}dx}{b^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{11/2}dx}{b^8}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\int (b\cos(c+dx))^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{1}{3}b^2\int\frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{9}{11}b^2\left(\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{9/2}}{11d}}{b^8}$$

input
```
Int[Cos[c + d*x]^8/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*d) + (9*b^2*((2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sqrt[b*Cos[c + d*
x]]*Sin[c + d*x])/(3*d)))/7))/11)/b^8
```

3.127. $\int\frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.127.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.127.4 Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.84

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2040\left(\cos^{7}\left(\frac{dx}{2}\right.\right.\right.}{77b^{2}\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^{2}\left(\frac{dx}{2}\right.\right.\right.}}$

input
```
int(cos(d*x+c)^8/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(448*c
os(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-
2040*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)
^3+15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x
+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+
1/2*c)^2-1)*b)^(1/2)/d
```

3.127.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(7\cos\left(dx+c\right)^4+9\cos\left(dx+c\right)^2+15\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)-15i\sqrt{2}\sqrt{}$$

input
```
integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*si
n(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)))/(b^3*d)
```

3.127.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**8/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.127.7 Maxima [F]

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^8}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^8/(b*cos(d*x + c))^(5/2), x)
```

3.127.8 Giac [F]

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^8}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^8/(b*cos(d*x + c))^(5/2), x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^8}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^8/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^8/(b*cos(c + d*x))^(5/2), x)
```

3.128 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.128.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

output
```
14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d
*x+c)/b^6/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{168\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)(38\sin(2(c+dx))+5\sin(4(c+d}{180b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^7/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2
*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.128.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{9/2}dx}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{9/2}dx}{b^7}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{7}{9}b^2\int (b\cos(c+dx))^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{9}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7}{9}b^2\left(\frac{3b^2\sqrt{b\cos(c+dx)}\int\sqrt{\sin(c+dx+\frac{\pi}{2})}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{7}{9}b^2\left(\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{7/2}}{9d}}{b^7}$$

input
```
Int[Cos[c + d*x]^7/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*d) + (7*b^2*((6*b^2*Sqrt[b*C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)))/9)/b^7
```

3.128.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.128. $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.128.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-432\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{45b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{d}{2}\right.}$

input
```
int(cos(d*x+c)^7/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(160*c
os(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432
*cos(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-24*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2
))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.128.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(5\cos\left(dx+c\right)^3+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\mathrm{weier}}{}$$

input
```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
 c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**7/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(5/2), x)
```

3.128.8 Giac [F]

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^7}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(5/2), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^7}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^7/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^7/(b*cos(c + d*x))^(5/2), x)
```

3.129 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

output
```
2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/b^2/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d
```

3.129.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))}{84b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^6/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3
*Sin[4*(c + d*x)])/(84*b^2d*Sqrt[b*Cos[c + d*x]])
```

3.129.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2030, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 2030$$

$$\frac{\int (b\cos(c+dx))^{7/2}dx}{b^6}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}dx}{b^6}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{5}{7}b^2\int (b\cos(c+dx))^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{7}b^2\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{7}b^2\left(\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

$$\Big\downarrow \ 3121$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{7}b^2\left(\frac{b^2\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{5}{7}b^2\left(\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}\right)+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}}{b^6}$$

input
```
Int[Cos[c + d*x]^6/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (5*b^2*((2*b^2*Sqrt[Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)))/7)/b^6
```

3.129.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.129.4 Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{21b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(c\right.\right.}}$

input
```
int(cos(d*x+c)^6/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*co
s(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*co
s(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
(2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.129.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^2+5\right)\sin\left(dx+c\right)-5i\sqrt{2}\sqrt{b}\text{weierstrassPInvers}}{}$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqr
t(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)))/(b^3*d)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^6}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(5/2), x)
```

3.129.8 Giac [F]

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^6}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(5/2), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^6/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^6/(b*cos(c + d*x))^(5/2), x)
```

3.130 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

output
```
2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)\sin(2(c+dx))}{5b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^5/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c +
d*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.130.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/2}dx}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}dx}{b^5}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\cos(c+dx)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}b^2\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3b^2\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^5}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6b^2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}}{b^5}$$

input
```
Int[Cos[c + d*x]^5/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d))/b^5
```

3.130.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.

Time = 2.81 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.96

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{(2(\cos^2}}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-8*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2
*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.130.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c) + 3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weier}$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.130.7 Maxima [F]

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

3.130.8 Giac [F]

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^5}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^5/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^5/(b*cos(c + d*x))^(5/2), x)
```

3.131 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.131.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}$$

output
```
2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/b^3/d
```

3.131.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^4/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*b^2
*d*Sqrt[b*Cos[c + d*x]])
```

3.131.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{3/2}dx}{b^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}dx}{b^4}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{3}b^2\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^4}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{b^2\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^4}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2b^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}}{b^4}$$

input
```
Int[Cos[c + d*x]^4/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d))/b^4
```

3.131.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.131.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(88) = 176$.

Time = 2.44 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.64

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2}}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\,d}}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.131.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weier}}{3}$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.131.7 Maxima [F]

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

3.131.8 Giac [F]

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)^4}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^4/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^4/(b*cos(c + d*x))^(5/2), x)
```

3.132 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.132.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x
]])
```

3.132.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt{b\cos(c+dx)}dx}{b^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3}$$

$$\Big\downarrow \text{ 3121}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^3\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^3\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x
]])
```

3.132.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.132.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 1.91 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,e^{-i(dx+c)}}{d\,b^2\sqrt{\left(e^{2i(dx+c)}+1\right)b\,e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{i e^{i(dx+c)}}\,\left(-2iE\left(\sqrt{-\cdots}\right.\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(\cdots}}}\cdots}{d\,b^2\sqrt{\left(e^{2i(dx+c)}\cdots}}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))/b^2/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.132.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.132.8 Giac [F]

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^3/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^3/(b*cos(c + d*x))^(5/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.133.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(d*(b*Cos[c + d*x])^(5/2)
)
```

3.133.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x
]])
```

3.133.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 1.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.51

method	result	size
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)b}\,d}$	144

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1
/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.133.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\text{weier}}{b^3d}$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*
x + c)))/(b^3*d)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.133.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.133.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(5/2), x)
```

3.134 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.134.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/b^3/d/cos(d*x+c)^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(-\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(b^2*
d*Sqrt[b*Cos[c + d*x]])
```

3.134.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2030, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}}{b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}}{b}$$

input
```
Int[Cos[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

output
```
((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]))/b
```

3.134.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.134.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.

Time = 1.74 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.91

method	result
default	$-\frac{2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\cdots\right.\right.}\right.}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\cdots\right.}}$

input
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+b*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*
b)^(1/2)/d
```

3.134.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
 + c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.134.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.134.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(c+d\,x)}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(cos(c + d*x)/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)/(b*cos(c + d*x))^(5/2), x)
```

3.135 $\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$

3.135.1 Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

output
```
2/3*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b
^2/d/(b*cos(d*x+c))^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\left(\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(-5/2),x]
```

output
```
(2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*b^2*d
*Sqrt[b*Cos[c + d*x]])
```

3.135.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

input
```
Int[(b*Cos[c + d*x])^(-5/2),x]
```

output
```
(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d
*x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

3.135.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.135.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.

Time = 1.66 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\cdots\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\cdots\right.}$

input
```
int(1/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/b^2*((2*cos(1/2*d*x+1/2
*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2
*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/((2*co
s(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.135.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\cos{(dx+c)}^2\,\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+}{}$$

input
```
integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x
 + c))/(b^3*d*cos(d*x + c)^2)
```

3.135.6 Sympy [F]

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))**(5/2),x)
```

output
```
Integral((b*cos(c + d*x))**(-5/2), x)
```

3.135.7 Maxima [F]

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(-5/2), x)
```

3.135.8 Giac [F]

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(-5/2), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(1/(b*cos(c + d*x))^(5/2), x)
```

3.136 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.136.1 Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^
(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.70

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+6\sin(c+dx)+2\sec(c+dx)\tan(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[
c + d*x]*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.136.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 2030$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/2}}dx$$

$$\Big\downarrow \ 3116$$

$$b\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3116$$

$$b\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3042$$

$$b\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \ 3121$$

$$b\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$b\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)$$

input
```
Int[Sec[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

output
```
b*((2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c +
 d*x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2))
```

3.136.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.136.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(109) = 218$.

Time = 2.78 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.78

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\mathrm{co}\right.\right.}{}$

input
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.136.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*
x + c))/(b^3*d*cos(d*x + c)^3)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.136.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.137.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}$$

output
```
2/7*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*sin(d*x+c)/b/d/(b*cos(d*x+c)
)^(3/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right) + 2(5+3\sec^2(c+dx))\tan(c+dx)}{21b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2
)*Tan[c + d*x])/(21*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.137.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{9/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(\frac{5\int \frac{1}{(b\cos(c+dx))^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{5\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^2\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3121}$$

$$b^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^2\left(\frac{5\left(\frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{3b^2\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$b^2\left(\frac{5\left(\frac{2\sqrt{\cos(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}\right)}{7b^2}+\frac{2\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}\right)$$

input
```
Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

output
```
b^2*((2*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (5*((2*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*Sin[
c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))))/(7*b^2))
```

3.137.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

3.137. $\int\frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.137.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(110) = 220$.

Time = 2.23 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.06

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+60\sqrt{\frac{1}{2}-\cos}\right.}{}$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^4+40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*sin(1/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))/b^2*((2*cos(1/2*d*x+1/2*c)^2-1)*b*sin(1/2*d*x+1/2*c)^2
)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/
2*d*x+1/2*c)^2-1)^3/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2
)/d
```

3.137.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))}{}$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5
*cos(d*x + c)^2 + 3)*sin(d*x + c))/(b^3*d*cos(d*x + c)^4)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.137.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.137.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)), x)
```

3.138 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.138.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = -\frac{14\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}}$$

output
```
2/9*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*sin(d*x+c)/d/(b*cos(d*x+c)
)^(5/2)+14/15*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*c
os(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.64

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-42\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+42\sin(c+dx)+2\sec(c+dx)\left(7+5\sec^2(c\right.}{45b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Se
c[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*b^2*d*Sqrt[b*Cos[c + d
*x]])
```

3.138.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 2030, 3116, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{7\int \frac{1}{(b\cos(c+dx))^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{7\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{7\left(\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$b^3\left(\frac{7\left(\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$$\Big\downarrow \text{3116}$$

$$b^3\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3042

$$b^3\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int\sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3121

$$b^3\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3042

$$b^3\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int\sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

$\downarrow$ 3119

$$b^3\left(\frac{7\left(\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}\right)}{9b^2}+\frac{2\sin(c+dx)}{9bd(b\cos(c+dx))^{9/2}}\right)$$

3.138. $\int\frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

input
```
Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

output
```
b^3*((2*Sin[c + d*x])/(9*b*d*(b*Cos[c + d*x])^(9/2)) + (7*((2*Sin[c + d*x]
)/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b
*Cos[c + d*x]])))/(5*b^2)))/(9*b^2))
```

3.138.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.138.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(133) = 266$.

Time = 3.17 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.33

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}{144b\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^5}-\frac{7\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\right.\right.\right.}}{180b\left(\cos^2\left(\frac{d}{}\right.\right.}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-1/144
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(
1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-1
4/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)
*b*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/((2*cos(1/2*d*x+1/2*c)^2-1)
*b)^(1/2)/d
```

3.138.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{-21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\ \text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(d}{}$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*
cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt
(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5)
```

3.138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.138.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.138.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2)), x)
```

3.139 $\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$

3.139.1 Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = -\frac{6\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

output
```
2/5*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/b^3/d/(b*cos(d*x+c)
)^(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \frac{-6\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+6\sin(c+dx)+2\sec(c+dx)\tan(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(-7/2),x]
```

output
```
(-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[
c + d*x]*Tan[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

3.139.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\cos(c+dx)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\int \sqrt{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\int \sqrt{\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b^2\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3\left(\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}\right)}{5b^2}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

input
```
Int[(b*Cos[c + d*x])^(-7/2),x]
```

output
```
(2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (3*((-2*Sqrt[b*Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*Sin[c + d*
x])/(b*d*Sqrt[b*Cos[c + d*x]])))/(5*b^2)
```

3.139.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

3.139.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(112) = 224$.

Time = 2.59 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.67

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)b\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,E\left(\cos\right.\right.}{}$

input
```
int(1/(cos(d*x+c)*b)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*b*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1
/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*
d*x+1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)
^4*b+b*sin(1/2*d*x+1/2*c)^2)^(1/2)/((2*cos(1/2*d*x+1/2*c)^2-1)*b)^(1/2)/d
```

3.139.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx}{}$$

input
```
integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*
x + c))/(b^4*d*cos(d*x + c)^3)
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.139.7 Maxima [F]

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(-7/2), x)
```

3.139.8 Giac [F]

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(-7/2), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(b\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(b*cos(c + d*x))^(7/2),x)
```

output
```
int(1/(b*cos(c + d*x))^(7/2), x)
```

3.140 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.140.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx=\frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/4*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*x*(b*cos(d*x+c)
)^(1/2)/cos(d*x+c)^(1/2)+3/8*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1
/2)/d
```

3.140.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(32*d*Sqrt[Cos[c + d*x]])
```

3.140.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3115$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 3115$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Co
s[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Cos[c + d*x]]
```

3.140.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.140.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

method	result
default	$\frac{\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}x}{4(\mathrm{e}^{2i(dx+c)}+1)}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{5i(dx+c)}}{32(\mathrm{e}^{2i(dx+c)}+1)d}+\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{-i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}-$

input
```
int(cos(d*x+c)^(7/2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(cos(d*x+c)*b)^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*cos(d*x+c)*sin(d*x
+c)+3*d*x+3*c)/cos(d*x+c)^(1/2)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.80

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \left[\frac{2\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)+3\sqrt{-b}\log\left(2\,b\cos(dx+c)^2-2\sqrt{b}\right.}{16\,d}\right.$$

input
```
integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*si
n(d*x + c) + 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sq
rt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*(sqrt(b*cos(d*x + c))*
(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.50

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*sqrt(b)/d
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(80) = 160$.

Time = 2.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.07

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{3\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+12\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-10\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+18\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{8\left(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+4\,d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}$$

input
```
integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 12*sqrt(b)*d*x*tan(1/2*d*x + 1
/2*c)^6 - 10*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 18*sqrt(b)*d*x*tan(1/2*d*x +
 1/2*c)^4 + 6*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 12*sqrt(b)*d*x*tan(1/2*d*x
+ 1/2*c)^2 - 6*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 3*sqrt(b)*d*x + 10*sqrt(b)
*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c
)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 14.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\right.\right.}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3
*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) +
 1))
```

3.140. $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.141 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)^3*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}(5+\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[Cos[c
 + d*x]])
```

3.141.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3113}$$

$$-\frac{\sqrt{b\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]],x]
```

output
```
-((Sqrt[b*Cos[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[Cos[c
+ d*x]]))
```

3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.141.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result
default	$\frac{\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\sqrt{\cos\left(dx+c\right)b}}{3d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{i\sqrt{\cos\left(dx+c\right)b}\,\left(\sqrt{\cos}\left(dx+c\right)\right)\mathrm{e}^{4i\left(dx+c\right)}}{12\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)d}-\frac{3i\sqrt{\cos\left(dx+c\right)b}\,\left(\sqrt{\cos}\left(dx+c\right)\right)\mathrm{e}^{2i\left(dx+c\right)}}{4\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)d}+\frac{3i\sqrt{\cos\left(dx+c\right)b}\,\left(\sqrt{\cos}\left(dx+c\right)\right)}{4\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)d}+\frac{i\sqrt{\cos}}{}$

input
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3\,d\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(d*sqrt(cos(d*x
 + c)))
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b}\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c)\right)\right)\right)}{12\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))/d
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71047 vs. $2(60) = 120$.

Time = 9.09 (sec) , antiderivative size = 71047, normalized size of antiderivative = 1014.96

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-1/96*(3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^6*tan(c) - 24*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan
(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^4*tan(c)^2 - 18*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^6*tan(c)^2 + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c)^2 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 + 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4
*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*...
```

3.141.9 Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(10\,\sin(2\,c+2\,d\,x)+\sin(4\,c+4\,d\,x))}{12\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c
+ 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.141. $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.142 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*sin(d*x+c)*cos(d*x+c)^(1/2
)*(b*cos(d*x+c))^(1/2)/d
```

3.142.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[Cos[c +
d*x]])
```

3.142.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[Cos[
c + d*x]]
```

3.142.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.142.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result	si
default	$\frac{\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$	42
risch	$\frac{\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}x}{\mathrm{e}^{2i(dx+c)}+1}-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{3i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}+\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{-i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}$	13

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(cos(d*x+c)*b)^(1/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^(1/2)
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.38

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{-b}\log\left(2\,b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos}\right.}{4\,d}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)*lo
g(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*
sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
+ c)^(3/2))))/d]
```

3.142.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(56) = 112.

Time = 32.91 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \begin{cases} x\sqrt{b\cos(c)}\cos^{\frac{3}{2}}(c) & \text{for } d=0 \\ 0 & \text{for } c=-dx+\frac{\pi}{2}\vee c=- \\ \frac{x\sqrt{b\cos(c+dx)}\sin^2(c+dx)}{2\sqrt{\cos(c+dx)}}+\frac{x\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}{2}+\frac{\sqrt{b\cos(c+dx)}\sin(c+dx)\sqrt{\cos(c+dx)}}{2d} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2),x)
```

output
```
Piecewise((x*sqrt(b*cos(c))*cos(c)**(3/2), Eq(d, 0)), (0, Eq(c, -d*x + pi/
2) | Eq(c, -d*x + 3*pi/2)), (x*sqrt(b*cos(c + d*x))*sin(c + d*x)**2/(2*sqr
t(cos(c + d*x))) + x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)/2 + sqrt(b*c
os(c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), True))
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})\sqrt{b}}{4\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*sqrt(b)/d
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(51) = 102.

Time = 1.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.67

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\sqrt{b}dx+2\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{2\left(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+d\right)}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 2*sqrt(b)*d*x*tan(1/2*d*x + 1/2*
c)^2 - 2*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + sqrt(b)*d*x + 2*sqrt(b)*tan(1/2*
d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 14.57 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\sqrt{\cos{(c+d\,x)}}\,\sqrt{b\,\cos{(c+d\,x)}}\,(\sin{(c+d\,x)}+\sin{(3\,c+3\,d\,x)}+4\,d\,x\,\cos{(c+d\,x)})}{4\,d\,(\cos{(2\,c+2\,d\,x)}+1)}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x
) + 4*d*x*cos(c + d*x)))/(4*d*(cos(2*c + 2*d*x) + 1))
```

3.142. $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

3.143 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx$

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.143.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.143.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.143.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	29
risch	$-\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\,\mathrm{e}^{2i(dx+c)}}{(\mathrm{e}^{2i(dx+c)}+1)d}+\frac{i\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))}{(\mathrm{e}^{2i(dx+c)}+1)d}$	85

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/d/cos(d*x+c)^(1/2)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

3.143.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(29) = 58.

Time = 1.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx = \begin{cases} x\sqrt{b\cos(c)}\sqrt{\cos(c)} & \text{for } d=0 \\ 0 & \text{for } c=-dx+\frac{\pi}{2}\vee c=-dx+\frac{3\pi}{2} \\ \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)
```

output
```
Piecewise((x*sqrt(b*cos(c))*sqrt(cos(c)), Eq(d, 0)), (0, Eq(c, -d*x + pi/2
) | Eq(c, -d*x + 3*pi/2)), (sqrt(b*cos(c + d*x))*sin(c + d*x)/(d*sqrt(cos(
c + d*x))), True))
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b}\sin\left(dx+c\right)}{d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b)*sin(d*x + c)/d
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx = \frac{2\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b)*tan(1/2*d*x + 1/2*c)/(d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sin(2\,c+2\,d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(d*(cos(2*c +
 2*d*x) + 1))
```

3.144 $\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

3.144.1 Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

output
```
x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx=\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

3.144.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int 1dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

output
```
(x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

3.144.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.144.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{x\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}$	21
default	$\frac{\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	28

input
```
int((cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.92

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{2\,d}, \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\cos(d}}{\sqrt{b}c}\right.}{d}\right.$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/d, sqrt(b)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{x\sqrt{b\cos{(c+dx)}}}{\sqrt{\cos{(c+dx)}}}$$

input
```
integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

output
```
x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

3.144.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{\sqrt{b\cos{(dx+c)}}}{\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = \frac{x\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
```

output
```
(x*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2)
```

3.145 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

3.145.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

3.145.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.145.4 Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{2\sqrt{\cos(dx+c)b}\,\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	40
risch	$-\frac{\sqrt{\cos(dx+c)b}\,\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}+\frac{\sqrt{\cos(dx+c)b}\,\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}$	73

input
```
int((cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)*arctanh(cot(d*x+c)-csc(d*x+c))
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.42

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{\sqrt{b}\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,d},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)))
)/d]
```

3.145.6 Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

3.145.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(d\right.\right.\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

3.145.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos\left(dx+c\right)}}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.146 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

3.146.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{2031}$$
$$\frac{\sqrt{b\cos(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\sqrt{b\cos(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{24}$$
$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

3.146.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.146.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}$	29
risch	$\frac{2i\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	38

input
```
int((cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/d/cos(d*x+c)^(3/2)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

3.146.6 Sympy [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(b*cos(c + d*x))/cos(c + d*x)**(5/2), x)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*d)
```

3.146.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 14.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\,\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)\,1\mathrm{i}+\sin(2\,c+2\,d\,x)+1\mathrm{i})}{d\,\sqrt{\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(d*
cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.147 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*arctanh(sin(d*x
+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx=\frac{\sqrt{b\cos(c+dx)}(\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx))}{2d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x]
))/(2*d*Cos[c + d*x]^(5/2))
```

3.147.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c +
 d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

3.147.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.147.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{\left(-\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)-1\right)+\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)+1\right)+\sin\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)b}}{2d\cos\left(dx+c\right)^{\frac{5}{2}}}$	84
risch	$-\frac{i\sqrt{\cos\left(dx+c\right)b}\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{\sqrt{\cos\left(dx+c\right)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{\sqrt{\cos\left(dx+c\right)b}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos\left(dx+c\right)}\,d}-\frac{\sqrt{\cos\left(dx+c\right)b}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos\left(dx+c\right)}\,d}$	132

input
```
int((cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+cos(d*x+c)^2*ln(-cot(d*x
+c)+csc(d*x+c)+1)+sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\sqrt{\cos}}{4d\cos(dx+c)^3}\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^3}\right]$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(60) = 120$.

Time = 0.44 (sec) , antiderivative size = 661, normalized size of antiderivative = 9.18

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{\left(4\left(\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)\right)\cos\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)-4\left(\sin\left(4\,dx\right.\right.\right.}{}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)
^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

3.147.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(c+dx)}}{\cos(c+dx)^{7/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2), x)
```

3.148 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*sin(d*x+c)^3*(b*cos
(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[Cos[c + d
*x]])
```

3.148.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{b\cos(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

output
```
-((Sqrt[b*Cos[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[Cos[c
+ d*x]]))
```

3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.148.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{(2(\cos^2(dx+c))+1)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}$	42
risch	$\frac{4i\sqrt{\cos(dx+c)b}\,(3\,e^{2i(dx+c)}+1)}{3\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^3}$	51

input
```
int((cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*cos(d*x+c)^2+1)*(cos(d*x+c)*b)^(1/2)*sin(d*x+c)/cos(d*x+c)^(7/2)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+1\right)\sin(dx+c)}{3\,d\cos(dx+c)^{\frac{7}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(d*cos(d*x +
c)^(7/2))
```

3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(60) = 120.

Time = 0.45 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{4\left((3\cos(2d\right.}{3\left(2\left(3\cos(4dx+4c)+3\cos(2dx+2c)+1\right)\cos(6dx+6c)+\cos(6dx+6c)^2+6\left(3\cos(2dx+2c)\right.\right.}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1
)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c)
 + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*c
os(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d
*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*si
n(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*
c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

3.148.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 16.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\sqrt{b\,\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)\,15\mathrm{i}+\cos(4\,c+4\,d\,x)\,6\mathrm{i}+\cos(6\,c+6\,d\,x)\,1\mathrm{i}+9\,\sin(2\,c+2\,d\,x)+6}{3\,d\,\sqrt{\cos(c+d\,x)}\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4\,c+4\,d\,x)+\cos(6\,c+6}$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2),x)
```

output
```
(2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + co
s(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*
d*x) + 10i))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*
d*x) + cos(6*c + 6*d*x) + 10))
```

3.149 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.149.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{3\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/4*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+3/8*sin(d*x+c)*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+3/8*arctanh(sin(d*x+c))*(b*cos(d*x+c))
^(1/2)/d/cos(d*x+c)^(1/2)
```

3.149.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\left(3\operatorname{arctanh}(\sin(c+dx))\cos^4(c+dx)+\left(2+3\cos^2(c+dx)\right)\sin(c+dx)\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(11/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos
[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

3.149.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2031, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\downarrow 4257$$

$$\frac{\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(11/2),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[S
in[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqrt[Cos[c +
d*x]]
```

3.149.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.149.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+3\left(\cos^2(dx+c)\right)\sin(dx+c)+2\sin}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{i\sqrt{\cos(dx+c)b}\left(3\,\mathrm{e}^{7i(dx+c)}+11\,\mathrm{e}^{5i(dx+c)}-11\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{4\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{3\sqrt{\cos(dx+c)b}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8\sqrt{\cos(dx+c)}\,d}-\frac{3\sqrt{\cos(dx+c)b}\,\ln(\mathrm{e}^{i(c}}{8\sqrt{\cos(dx+c)}}$

input
```
int((cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(-3*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*cos(d*x+c)^4*ln(-cot
(d*x+c)+csc(d*x+c)+1)+3*cos(d*x+c)^2*sin(d*x+c)+2*sin(d*x+c))*(cos(d*x+c)*
b)^(1/2)/cos(d*x+c)^(9/2)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$
$$= \left[\frac{3\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}(3\,\mathrm{c}}{16\,d\cos(dx+c)^5}\right.$$
$$-\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\sqrt{b\cos(dx+c)}\left(3\cos(dx+c)^2+2\right)\sqrt{\cos(dx}}{8\,d\cos(dx+c)^5}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
+ c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x
+ c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(
b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^5)]
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(89) = 178$.

Time = 0.48 (sec) , antiderivative size = 1656, normalized size of antiderivative = 15.48

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
4*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8
*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8
*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*
x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) +
 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8
*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16
*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x +...
```

3.149.8 Giac [F]

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{11/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(11/2),x)
```

output
```
int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(11/2), x)
```

3.150 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.150.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx=\frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{3b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{b\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/4*b*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*x*(b*cos(d*
x+c))^(1/2)/cos(d*x+c)^(1/2)+3/8*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+
c))^(1/2)/d
```

3.150.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx=\frac{(b\cos(c+dx))^{3/2}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*
x)]))/(32*d*Cos[c + d*x]^(3/2))
```

3.150.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (
Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Cos[c + d*x]]
```

3.150.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.150.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\,(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3b\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}x}{4(\mathrm{e}^{2i(dx+c)}+1)} - \frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{5i(dx+c)}}{32(\mathrm{e}^{2i(dx+c)}+1)d} + \frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{-i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*b*(cos(d*x+c)*b)^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*cos(d*x+c)*sin(d
*x+c)+3*d*x+3*c)/cos(d*x+c)^(1/2)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.81

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \left[\frac{2\left(2\,b\cos\left(dx+c\right)^{2}+3\,b\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\,\sqrt{-b}b\log\left(2\,b\cos\left(d\right.\right.}{16\,d}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(2*b*cos(d*x + c)^2 + 3*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c) + 3*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((2*b*cos(d*x +
c)^2 + 3*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*b^(3/
2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))
/d]
```

3.150.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.52

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\left(12\,(dx+c)b+b\sin\left(4\,dx+4\,c\right)+8\,b\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/32*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(83) = 166.

Time = 2.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\left(3\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+12\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-10\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+18\sqrt{b}dx\,\mathrm{t}}{8\left(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+4\,d\,\mathrm{t}\right.}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*(3*sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^8 + 12*sqrt(b)*d*x*tan(1/2*d*x + 1
/2*c)^6 - 10*sqrt(b)*tan(1/2*d*x + 1/2*c)^7 + 18*sqrt(b)*d*x*tan(1/2*d*x +
 1/2*c)^4 + 6*sqrt(b)*tan(1/2*d*x + 1/2*c)^5 + 12*sqrt(b)*d*x*tan(1/2*d*x
+ 1/2*c)^2 - 6*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + 3*sqrt(b)*d*x + 10*sqrt(b)
*tan(1/2*d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2
*c)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 14.93 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\right.}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c +
 3*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x)
 + 1))
```

3.150. $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.151 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx=\frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*sin(d*x+c)^3*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx=\frac{(b\cos(c+dx))^{3/2}(5+\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Cos[c +
d*x]^(3/2))
```

3.151.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2),x]
```

output
```
-((b*Sqrt[b*Cos[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[Cos[
c + d*x]]))
```

3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.151.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.57

method	result
default	$\frac{b\left(2+\cos^{2}(dx+c)\right)\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{4i(dx+c)}}{12\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{3ib\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)\mathrm{e}^{2i(dx+c)}}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{3ib\sqrt{\cos(dx+c)b}\,\left(\sqrt{\cos}(dx+c)\right)}{4\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{ib\sqrt{}}{}$

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*b*(2+cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)*sin(d*x+c)/cos(d*x+c)^(1/2)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\left(b\cos\left(dx+c\right)^2+2\,b\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos
(d*x + c)))
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))))*sqrt(b)/d
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71048 vs. $2(62) = 124$.

Time = 11.24 (sec) , antiderivative size = 71048, normalized size of antiderivative = 986.78

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/96*(3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^6*tan(c) - 24*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan
(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^4*tan(c)^2 - 18*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*sqrt(b
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^6*tan(c)^2 + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c)^2 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*sqrt(b)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 + 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4
*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*...
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*
c + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.151. $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

3.152 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx$

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*sin(d*x+c)*cos(d*x+c)^
(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.152.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \frac{(b\cos(c+dx))^{3/2}(2(c+dx)+\sin(2(c+dx)))}{4d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x
]^(3/2))
```

3.152.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[Co
s[c + d*x]]
```

3.152.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.152.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

method	result
default	$\frac{b\sqrt{\cos(dx+c)b}\,(\cos(dx+c)\sin(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$
risch	$\frac{b\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{i(dx+c)}x}{\mathrm{e}^{2i(dx+c)}+1} - \frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{3i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d} + \frac{ib\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{-i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}$

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*b*(cos(d*x+c)*b)^(1/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^(1/2
)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.35

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \left[\frac{2\sqrt{b\cos(dx+c)}b\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{-b}b\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+\right.}{4d}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)*
b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x +
c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*s
in(d*x + c) + b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*co
s(d*x + c)^(3/2))))/d]
```

3.152.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \int (b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)), x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.43

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \frac{(2(dx+c)b+b\sin(2dx+2c))\sqrt{b}}{4d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*sqrt(b)/d
```

3.152.8 Giac [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \frac{\left(\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,\sqrt{b}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\sqrt{b}dx+2\,\sqrt{b}}{2\left(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+d\right)}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(b)*d*x*tan(1/2*d*x + 1/2*c)^4 + 2*sqrt(b)*d*x*tan(1/2*d*x + 1/2*
c)^2 - 2*sqrt(b)*tan(1/2*d*x + 1/2*c)^3 + sqrt(b)*d*x + 2*sqrt(b)*tan(1/2*
d*x + 1/2*c))*b/(d*tan(1/2*d*x + 1/2*c)^4 + 2*d*tan(1/2*d*x + 1/2*c)^2 + d
)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 14.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}\,dx = \frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2),x)
```

output
```
(b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d
*x) + 4*d*x*cos(c + d*x)))/(4*d*(cos(2*c + 2*d*x) + 1))
```

3.153 $\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

3.153.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \cos(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.153.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.153.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	30
risch	$\frac{b\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\sqrt{\cos(dx+c)}}$	30

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
b*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/d/cos(d*x+c)^(1/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(dx+c)}b\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*b*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

3.153.6 Sympy [A] (verification not implemented)

Time = 15.89 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \begin{cases} \frac{(b\cos(c+dx))^{\frac{3}{2}}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} & \text{for } d\neq 0 \\ \frac{x(b\cos(c))^{\frac{3}{2}}}{\sqrt{\cos(c)}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

output
```
Piecewise(((b*cos(c + d*x))**(3/2)*sin(c + d*x)/(d*cos(c + d*x)**(3/2)), N
e(d, 0)), (x*(b*cos(c))**(3/2)/sqrt(cos(c)), True))
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.39

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^{\frac{3}{2}}\sin(dx+c)}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
b^(3/2)*sin(d*x + c)/d
```

3.153.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\sin(c+d\,x)\,\sqrt{b\cos(c+d\,x)}}{d\,\sqrt{\cos(c+d\,x)}}$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

output
```
(b*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(d*cos(c + d*x)^(1/2))
```

3.154 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

output
```
b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

3.154.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int 1dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

3.154.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.154.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{bx\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}}$	22
default	$\frac{b\sqrt{\cos(dx+c)b}\,(dx+c)}{d\sqrt{\cos(dx+c)}}$	29

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b*x*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.80

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left[\frac{\sqrt{-b}b\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right.}{2\,d}\right.$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*s
qrt(cos(d*x + c))*sin(d*x + c) - b)/d, b^(3/2)*arctan(sqrt(b*cos(d*x + c))
*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

3.154.6 Sympy [A] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{x(b\cos\left(c+dx\right))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}\left(c+dx\right)}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

output
```
x*(b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
2*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

3.154.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b\,x\,\sqrt{b\,\cos(c+d\,x)}}{\sqrt{\cos(c+d\,x)}}$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

output
```
(b*x*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2)
```

3.155 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b\,\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

output
```
b*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\text{arctanh}(\sin(c+dx))(b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]^(3/2))
```

3.155.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\,\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

3.155.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.155.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{2\sqrt{\cos(dx+c)b}\, b\, \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	41
risch	$-\frac{b\sqrt{\cos(dx+c)b}\, \ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\, d}+\frac{b\sqrt{\cos(dx+c)b}\, \ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\, d}$	75

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)*b*arctanh(cot(d*x+c)-csc(d*x+c)
)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.35

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{b^{\frac{3}{2}}\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,d}, -\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/2*b^(3/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b
)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))/d]
```

3.155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\left(b\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-b\log\left(\cos\left(dx+c\right)^2+\right.\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

3.155.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

3.156 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

3.156.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2))
```

3.156.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 2031$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4254$$

$$-\frac{b\sqrt{b\cos(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

3.156.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.156.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b\sin(dx+c)\sqrt{\cos(dx+c)b}}{d\cos(dx+c)^{\frac{3}{2}}}$	30
risch	$\frac{2ib\sqrt{\cos(dx+c)b}}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	39

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
b*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/d/cos(d*x+c)^(3/2)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(dx+c)}b\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*b*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
2*b^(3/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*d)
```

3.156.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}\,(\cos(2c+2dx)\ 1\mathrm{i}+\sin(2c+2dx)+1\mathrm{i})}{d\sqrt{\cos(c+dx)}\ (\cos(2c+2dx)+1)}$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
```

output
```
(b*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(
d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.157 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b\,\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/2*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b*arctanh(sin
(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{(b\cos(c+dx))^{3/2}\left(\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*
x]))/(2*d*Cos[c + d*x]^(7/2))
```

3.157.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c
 + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

3.157.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.157.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{b\left(-\left(\cos^{2}(dx+c)\right)\ln\left(-\cot(dx+c)+\csc(dx+c)-1\right)+\left(\cos^{2}(dx+c)\right)\ln\left(-\cot(dx+c)+\csc(dx+c)+1\right)+\sin(dx+c)\right)\sqrt{\cos(dx+c)b}}{2d\cos(dx+c)^{\frac{5}{2}}}$	85
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\left(\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}-\frac{b\sqrt{\cos(dx+c)b}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)}\,d}+\frac{b\sqrt{\cos(dx+c)b}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}$	135

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*b*(-cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+cos(d*x+c)^2*ln(-cot(d
*x+c)+csc(d*x+c)+1)+sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.76

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left[\frac{b^{\frac{3}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+\cdots}{4\,d\cos(dx+c)^3}\right.$$
$$\left.-\frac{\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}b\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,d\cos(dx+c)^3}\right]$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/4*(b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^3), -1/2*(sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x +
 c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*b*sqrt(c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(62) = 124.

Time = 0.44 (sec) , antiderivative size = 691, normalized size of antiderivative = 9.34

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2
*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x +
 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) +
 b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)
^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2
*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) +
 b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x
+ 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos
(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d
)
```

3.157.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2), x)
```

3.158 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.158.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b*sin(d*x+c)^3*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{3/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

output
```
((b*Cos[c + d*x])^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Cos[c + d*x]
^(3/2))
```

3.158.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{b\sqrt{b\cos(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

output
```
-((b*Sqrt[b*Cos[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[Cos[
c + d*x]]))
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.158.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{b\left(2\left(\cos^{2}\left(dx+c\right)\right)+1\right)\sqrt{\cos\left(dx+c\right)b}\,\sin\left(dx+c\right)}{3d\cos\left(dx+c\right)^{\frac{7}{2}}}$	43
risch	$\frac{4ib\sqrt{\cos\left(dx+c\right)b}\,\left(3\,\mathrm{e}^{2i\left(dx+c\right)}+1\right)}{3\sqrt{\cos\left(dx+c\right)}\,d\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)^{3}}$	52

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*b*(2*cos(d*x+c)^2+1)*(cos(d*x+c)*b)^(1/2)*sin(d*x+c)/cos(d*x+c)^(7/2
)
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*cos(d*x + c)^2 + b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
+ c)^(7/2))
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(62) = 124.

Time = 0.41 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.15

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx =$$

$$-\frac{4\,(3\,b\cos\left(6\,dx\right.}{3\,\left(2\,\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^{2}+6\,\left(3\,\cos\left(2\,dx+2\,c\right)\right.\right.}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
-4/3*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d
*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x
 + 2*c) + b)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6
*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)
^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2
*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

3.158.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2), x)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 15.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2\,b\,\sqrt{b\,\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)\,15\mathrm{i}+\cos(4\,c+4\,d\,x)\,6\mathrm{i}+\cos(6\,c+6\,d\,x)\,1}{3\,d\,\sqrt{\cos(c+d\,x)}\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4}$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2),x)
```

output
```
(2*b*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i +
cos(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c +
6*d*x) + 10i))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c +
4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.159 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 110

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{3b\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/4*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+3/8*b*sin(d*x+c)*
(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+3/8*b*arctanh(sin(d*x+c))*(b*cos(d
*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b\sqrt{b\cos(c+dx)}(3\operatorname{arctanh}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(c}{8d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(13/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*C
os[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

3.159.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2031, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(13/2),x]
```

output
```
(b*Sqrt[b*Cos[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh
[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqrt[Cos[c
+ d*x]]
```

3.159.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.159.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

method	result
default	$\frac{b\left(-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+3\left(\cos^2(dx+c)\right)\sin(dx+c)+2\sin\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib\sqrt{\cos(dx+c)b}\left(3\,\mathrm{e}^{7i(dx+c)}+11\,\mathrm{e}^{5i(dx+c)}-11\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{4\sqrt{\cos(dx+c)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{3b\sqrt{\cos(dx+c)b}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8\sqrt{\cos(dx+c)}\,d}-\frac{3b\sqrt{\cos(dx+c)b}\,\ln(}{8\sqrt{\cos(dx+}}$

input
```
int((cos(d*x+c)*b)^(3/2)/cos(d*x+c)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*b*(-3*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*cos(d*x+c)^4*ln(-c
ot(d*x+c)+csc(d*x+c)+1)+3*cos(d*x+c)^2*sin(d*x+c)+2*sin(d*x+c))*(cos(d*x+c
)*b)^(1/2)/cos(d*x+c)^(9/2)
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.13

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \left[\frac{3\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{16\,d\cos\left(dx+\right.}\right.$$

$$-\frac{3\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{5}-\left(3\,b\cos\left(dx+c\right)^{2}+2\,b\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx\right.}}{8\,d\cos\left(dx+c\right)^{5}}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

output
```
[1/16*(3*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
+ c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(3*b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*b*arctan(sqrt(b*co
s(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 -
 (3*b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^5)]
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. 2(92) = 184.

Time = 0.48 (sec) , antiderivative size = 1742, normalized size of antiderivative = 15.84

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x
+ 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x
 + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4
*c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c)
 + 4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) +
 b)*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*
b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(
4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36...
```

3.159.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2), x)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^{13/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(13/2),x)
```

output
```
int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(13/2), x)
```

3.160 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2b^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{b\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

output
```
b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-2/3*b^2*sin(d*x+c)^
3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*b^2*sin(d*x+c)^5*(b*cos(d*x+
c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.49

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{(b\cos(c+dx))^{5/2}\sin(c+dx)\left(15-10\sin^2(c+dx)+3\sin^4(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sin[c + d*x]*(15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d
*x]^4))/(15*d*Cos[c + d*x]^(5/2))
```

3.160.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2),x]
```

output
```
-((b^2*Sqrt[b*Cos[c + d*x]]*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c
+ d*x]^5/5))/(d*Sqrt[Cos[c + d*x]]))
```

3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.160.4 Maple [A] (verified)

Time = 3.74 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result
default	$\frac{b^{2}\left(3\left(\cos^{4}\left(dx+c\right)\right)+4\left(\cos^{2}\left(dx+c\right)\right)+8\right)\sqrt{\cos\left(dx+c\right)b}\,\sin\left(dx+c\right)}{15d\sqrt{\cos\left(dx+c\right)}}$
risch	$-\frac{i b^{2}\sqrt{\cos\left(dx+c\right)b}\,\left(\sqrt{\cos}\left(dx+c\right)\right)\mathrm{e}^{6i\left(dx+c\right)}}{80\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)d}-\frac{5i b^{2}\sqrt{\cos\left(dx+c\right)b}\,\left(\sqrt{\cos}\left(dx+c\right)\right)\mathrm{e}^{2i\left(dx+c\right)}}{8\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)d}+\frac{5i b^{2}\sqrt{\cos\left(dx+c\right)b}\,\left(\sqrt{\cos}\left(dx+c\right)\right)}{8\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)d}+$

input
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d*b^2*(3*cos(d*x+c)^4+4*cos(d*x+c)^2+8)*(cos(d*x+c)*b)^(1/2)*sin(d*x+
c)/cos(d*x+c)^(1/2)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\left(3\,b^2\cos\left(dx+c\right)^4+4\,b^2\cos\left(dx+c\right)^2+8\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*b^2*cos(d*x + c)^4 + 4*b^2*cos(d*x + c)^2 + 8*b^2)*sqrt(b*cos(d*x
+ c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\left(3\,b^2\sin\left(5\,dx+5\,c\right)+25\,b^2\sin\left(\frac{3}{5}\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)+150\,b^2\sin\left(\frac{1}{5}\ \text{ar}\right.\right.}{240\,d}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/240*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), c
os(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x +
5*c))))*sqrt(b)/d
```

3.160.8 Giac [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.160.9 Mupad [B] (verification not implemented)

Time = 15.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(175\,\sin\left(2\,c+2\,d\,x\right)+28\,\sin\left(4\,c+4\,d\,x\right)+3\,\sin\left(6\,c+6\,d\,x\right)\right)}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2),x)
```

output
```
(b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(175*sin(2*c + 2*d*x) + 28*
sin(4*c + 4*d*x) + 3*sin(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.161 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{b^2\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

output
```
1/4*b^2*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b^2*x*(b*co
s(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+3/8*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*co
s(d*x+c))^(1/2)/d
```

3.161.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{(b\cos(c+dx))^{5/2}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*
x)]))/(32*d*Cos[c + d*x]^(5/2))
```

3.161.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 +
 (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[Cos[c + d*x]]
```

3.161.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.161.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result
default	$\frac{b^{2} \sqrt{\cos \left(d x +c \right) b}\, \left(2 \sin \left(d x +c \right) \left(\cos^{3}\left(d x +c \right)\right)+3 \cos \left(d x +c \right) \sin \left(d x +c \right)+3 d x +3 c \right)}{8 d \sqrt{\cos \left(d x +c \right)}}$
risch	$\frac{3 b^{2} \sqrt{\cos \left(d x +c \right) b}\, \left(\sqrt{\cos}\left(d x +c \right)\right) {\mathrm e}^{i \left(d x +c \right)} x}{4 \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}-\frac{i b^{2} \sqrt{\cos \left(d x +c \right) b}\, \left(\sqrt{\cos}\left(d x +c \right)\right) {\mathrm e}^{5 i \left(d x +c \right)}}{32 \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right) d}+\frac{i b^{2} \sqrt{\cos \left(d x +c \right) b}\, \left(\sqrt{\cos}\left(d x +c \right)\right) {\mathrm e}^{-i \left(d x +c \right.}}{4 \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right) d}$

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*b^2*(cos(d*x+c)*b)^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*cos(d*x+c)*sin
(d*x+c)+3*d*x+3*c)/cos(d*x+c)^(1/2)
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.80

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \left[\frac{3\sqrt{-b}b^2\log\left(2\,b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\,(}{16\,d}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*b^2*cos(d*x + c)^2 + 3*b^
2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1/8*(3*b^(5/2)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
(2*b^2*cos(d*x + c)^2 + 3*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/d]
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\left(12\,(dx+c)b^2+b^2\sin(4\,dx+4\,c)+8\,b^2\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)\sqrt{b}}{32\,d}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(89) = 178$.

Time = 2.48 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.90

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{3\,b^{\frac{5}{2}}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+12\,b^{\frac{5}{2}}dx\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-10\,b^{\frac{5}{2}}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+18\,b^{\frac{5}{2}}dx\tan\left(\right.}{8\left(d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+4\,d\tan\left(\frac{1}{2}\right.\right.}$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
1/8*(3*b^(5/2)*d*x*tan(1/2*d*x + 1/2*c)^8 + 12*b^(5/2)*d*x*tan(1/2*d*x + 1
/2*c)^6 - 10*b^(5/2)*tan(1/2*d*x + 1/2*c)^7 + 18*b^(5/2)*d*x*tan(1/2*d*x +
 1/2*c)^4 + 6*b^(5/2)*tan(1/2*d*x + 1/2*c)^5 + 12*b^(5/2)*d*x*tan(1/2*d*x
+ 1/2*c)^2 - 6*b^(5/2)*tan(1/2*d*x + 1/2*c)^3 + 3*b^(5/2)*d*x + 10*b^(5/2)
*tan(1/2*d*x + 1/2*c))/(d*tan(1/2*d*x + 1/2*c)^8 + 4*d*tan(1/2*d*x + 1/2*c
)^6 + 6*d*tan(1/2*d*x + 1/2*c)^4 + 4*d*tan(1/2*d*x + 1/2*c)^2 + d)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\right.}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2),x)
```

output
```
(b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c
 + 3*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*
x) + 1))
```

3.161. $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

3.162 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

output
```
b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*sin(d*x+c)^
3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \frac{(b\cos(c+dx))^{5/2}(5+\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Cos[c +
d*x]^(5/2))
```

3.162.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2),x]
```

output
```
-((b^2*Sqrt[b*Cos[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[Co
s[c + d*x]]))
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.162.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result
default	$\frac{b^2(2+\cos^2(dx+c))\sqrt{\cos(dx+c)b}\,\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{4i(dx+c)}}{12(\mathrm{e}^{2i(dx+c)}+1)d}-\frac{3ib^2\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))\mathrm{e}^{2i(dx+c)}}{4(\mathrm{e}^{2i(dx+c)}+1)d}+\frac{3ib^2\sqrt{\cos(dx+c)b}\,(\sqrt{\cos}(dx+c))}{4(\mathrm{e}^{2i(dx+c)}+1)d}+$

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*b^2*(2+cos(d*x+c)^2)*(cos(d*x+c)*b)^(1/2)*sin(d*x+c)/cos(d*x+c)^(1/2
)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \frac{\left(b^2\cos\left(dx+c\right)^2+2\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt
(cos(d*x + c)))
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \frac{\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/12*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))))*sqrt(b)/d
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71047 vs. $2(66) = 132$.

Time = 8.37 (sec) , antiderivative size = 71047, normalized size of antiderivative = 934.83

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/96*(3*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-
1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*b^(5/2)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*b
^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^6*tan(c) - 24*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan
(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*b^(5/2
)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^4*tan(1/3*c)^4*tan(c)^2 - 18*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/
2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*b^(5/2
)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^6*tan(c)^2 + 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*b^(5/2)
*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)
^4*tan(1/3*c)^6*tan(c)^2 - 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*b^(5/2)*d*x^4*tan
(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 + 48*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4
*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*b^(5/2)*
d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*...
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx = \frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2),x)
```

output
```
(b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(
4*c + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.162. $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}\,dx$

3.163 $\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

output
```
1/2*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)*cos(d*x
+c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

3.163.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(b\cos(c+dx))^{5/2}(2(c+dx)+\sin(2(c+dx)))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x
]^(5/2))
```

3.163.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[
Cos[c + d*x]]
```

3.163.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.163.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{b^{2} \sqrt{\cos \left(d x +c \right) b}\, \left(\cos \left(d x +c \right) \sin \left(d x +c \right)+d x +c \right)}{2 d \sqrt{\cos \left(d x +c \right)}}$	45
risch	$\frac{b^{2} x \sqrt{\cos \left(d x +c \right) b}}{2 \sqrt{\cos \left(d x +c \right)}}+\frac{b^{2} \sqrt{\cos \left(d x +c \right) b}\, \sin \left(2 d x +2 c \right)}{4 \sqrt{\cos \left(d x +c \right)}\, d}$	61

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*b^2*(cos(d*x+c)*b)^(1/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/cos(d*x+c)^(1
/2)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.30

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \left[\frac{2\sqrt{b\cos(dx+c)}b^2\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{-b}b^2\log\left(2\,b\cos(dx+c)^2-\right.}{4\,d}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b
)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*
x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x +
 c))*sin(d*x + c) + b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2))))/d]
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{(2\,(dx+c)b^2+b^2\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*sqrt(b)/d
```

3.163.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\sqrt{b\,\cos(c+d\,x)}\,(\sin(2\,c+2\,d\,x)+2\,d\,x)}{4\,d\,\sqrt{\cos(c+d\,x)}}$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(sin(2*c + 2*d*x) + 2*d*x))/(4*d*cos(c + d*x)^
(1/2))
```

3.164 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.164.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

output
```
b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2))
```

3.164.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}dx$$
$$\Big\downarrow \text{ 2031}$$
$$\frac{b^2\sqrt{b\cos(c+dx)}\int \cos(c+dx)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\cos(c+dx)}}$$
$$\Big\downarrow \text{ 3117}$$
$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

3.164.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.164.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b^{2} \sin(dx+c)\sqrt{\cos(dx+c) b}}{d \sqrt{\cos(dx+c)}}$	32
risch	$\frac{b^{2} \sin(dx+c)\sqrt{\cos(dx+c) b}}{d \sqrt{\cos(dx+c)}}$	32

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b^2*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/d/cos(d*x+c)^(1/2)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos\left(dx+c\right)}b^{2}\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*b^2*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^{\frac{5}{2}}\sin(dx+c)}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
b^(5/2)*sin(d*x + c)/d
```

3.164.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{b^2\sin(c+d\,x)\,\sqrt{b\cos(c+d\,x)}}{d\,\sqrt{\cos(c+d\,x)}}$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
```

output
```
(b^2*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(d*cos(c + d*x)^(1/2))
```

3.165 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

output
```
b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{x(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(x*(b*Cos[c + d*x])^(5/2))/Cos[c + d*x]^(5/2)
```

3.165.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int 1dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

output
```
(b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

3.165.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.165.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{b^{2} x \sqrt{\cos(dx+c) b}}{\sqrt{\cos(dx+c)}}$	24
default	$\frac{b^{2} \sqrt{\cos(dx+c) b}\,(dx+c)}{d \sqrt{\cos(dx+c)}}$	31

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
b^2*x*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left[\frac{\sqrt{-b}b^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right.}{2\,d}\right.$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)
*sqrt(cos(d*x + c))*sin(d*x + c) - b)/d, b^(5/2)*arctan(sqrt(b*cos(d*x + c
))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

output
```
Timed out
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
2*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

3.165.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{b^2\,x\,\sqrt{b\,\cos(c+d\,x)}}{\sqrt{\cos(c+d\,x)}}$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
```

output
```
(b^2*x*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2)
```

3.166 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{b^2\operatorname{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

output
```
b^2*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))(b\cos(c+dx))^{5/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]^(5/2))
```

3.166.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$\Big\downarrow \; 2031$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sec(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{b^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

output
```
(b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

3.166.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.166.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{2\sqrt{\cos(dx+c)b}\, b^{2} \operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)}}$	43
risch	$\frac{b^{2}\sqrt{\cos(dx+c)b}\, \ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\, d}-\frac{b^{2}\sqrt{\cos(dx+c)b}\, \ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\, d}$	79

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(1/2)*b^2*arctanh(cot(d*x+c)-csc(d*x+
c))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\, dx = \left[\frac{b^{\frac{5}{2}}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2\,d}, -\frac{\sqrt{-b}b^{2}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/2*b^(5/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b
)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x +
c))))/d]
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.166.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.00

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b^2\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-b^2\log\left(\cos\left(dx+c\right)^2\right.}{2\,d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

3.166.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2), x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos{(c+d\,x)})^{5/2}}{\cos{(c+d\,x)}^{7/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2), x)
```

3.167 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.167.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

output
```
b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\sin(c+dx)}{d\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(7/2))
```

3.167.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

3.167.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.167.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b^{2} \sin \left(d x +c \right) \sqrt{\cos \left(d x +c \right) b}}{d \cos \left(d x +c \right)^{\frac{3}{2}}}$	32
risch	$\frac{2 i b^{2} \sqrt{\cos \left(d x +c \right) b}}{\sqrt{\cos \left(d x +c \right)}\, d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}$	41

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
b^2*sin(d*x+c)*(cos(d*x+c)*b)^(1/2)/d/cos(d*x+c)^(3/2)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(dx+c)}b^2\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*b^2*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\,b^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
2*b^(5/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*d)
```

3.167.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2), x)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 14.49 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.77

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx=\frac{b^2\sqrt{b\,\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)\,\,1\mathrm{i}+\sin(2\,c+2\,d\,x)+1\mathrm{i})}{d\,\sqrt{\cos(c+d\,x)}\,\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)
```

output
```
(b^2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))
/(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.168 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{b^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/2*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b^2*arctanh
(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(\text{arctanh}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*
x]))/(2*d*Cos[c + d*x]^(9/2))
```

3.168.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2031, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan
[c + d*x])/(2*d)))/Sqrt[Cos[c + d*x]]
```

3.168.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.168.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	siz
default	$\frac{b^{2}\left(-\left(\cos^{2}\left(d x+c\right)\right)\ln\left(-\cot\left(d x+c\right)+\csc\left(d x+c\right)-1\right)+\left(\cos^{2}\left(d x+c\right)\right)\ln\left(-\cot\left(d x+c\right)+\csc\left(d x+c\right)+1\right)+\sin\left(d x+c\right)\right)\sqrt{\cos\left(d x+c\right)b}}{2d\cos\left(d x+c\right)^{\frac{5}{2}}}$	87
risch	$-\frac{i b^{2}\sqrt{\cos\left(d x+c\right)b}\,\left({\mathrm e}^{3 i\left(d x+c\right)}-{\mathrm e}^{i\left(d x+c\right)}\right)}{\sqrt{\cos\left(d x+c\right)}\,d\left({\mathrm e}^{2 i\left(d x+c\right)}+1\right)^{2}}-\frac{b^{2}\sqrt{\cos\left(d x+c\right)b}\,\ln\left({\mathrm e}^{i\left(d x+c\right)}-i\right)}{2\sqrt{\cos\left(d x+c\right)}\,d}+\frac{b^{2}\sqrt{\cos\left(d x+c\right)b}\,\ln\left({\mathrm e}^{i\left(d x+c\right)}+i\right)}{2\sqrt{\cos\left(d x+c\right)}\,d}$	14

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*b^2*(-cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+cos(d*x+c)^2*ln(-cot
(d*x+c)+csc(d*x+c)+1)+sin(d*x+c))*(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.69

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \left[\frac{b^{\frac{5}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+ \cdots}{4d\cos(dx+c)^3}\right.$$
$$\left.-\frac{\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}b^2\sqrt{\cos(dx+c)}\sin(dx+c)}{2d\cos(dx+c)^3}\right]$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

output
```
[1/4*(b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3), -1/2*(sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d
*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*b^2*
sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

output
```
Timed out
```

3.168.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(66) = 132.

Time = 0.40 (sec) , antiderivative size = 747, normalized size of antiderivative = 9.58

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2
*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*c
os(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*c
os(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x
 + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*co
s(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*
(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d
*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqr
t(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

3.168.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2), x)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^{11/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2), x)
```

3.169 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

output
```
b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b^2*sin(d*x+c)^
3*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Cos[c + d*x]
^(5/2))
```

3.169.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{b^2\sqrt{b\cos(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

output
```
-((b^2*Sqrt[b*Cos[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[Co
s[c + d*x]]))
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.169.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{b^{2}\left(2\left(\cos^{2}\left(dx+c\right)\right)+1\right)\sqrt{\cos\left(dx+c\right)b}\,\sin\left(dx+c\right)}{3d\cos\left(dx+c\right)^{\frac{7}{2}}}$	45
risch	$\frac{4ib^{2}\sqrt{\cos\left(dx+c\right)b}\left(3\,\mathrm{e}^{2i\left(dx+c\right)}+1\right)}{3\sqrt{\cos\left(dx+c\right)}\,d\left(\mathrm{e}^{2i\left(dx+c\right)}+1\right)^{3}}$	54

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*b^2*(2*cos(d*x+c)^2+1)*(cos(d*x+c)*b)^(1/2)*sin(d*x+c)/cos(d*x+c)^(7
/2)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(2\,b^2\cos\left(dx+c\right)^2+b^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

output
```
1/3*(2*b^2*cos(d*x + c)^2 + b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(
d*x + c)^(7/2))
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2),x)
```

output
```
Timed out
```

3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(66) = 132.

Time = 0.38 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.09

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx =$$

$$-\frac{4\left(3\,b^2\cos\left(6\,dx+\right.\right.}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^2+6\left(3\,\cos\left(2\,dx+2\,c\right)\right.\right.}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

output
```
-4/3*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2
*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c)
 + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*co
s(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
+ 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

3.169.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2), x)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.72

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2\,b^2\,\sqrt{b\,\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)\,15\mathrm{i}+\cos(4\,c+4\,d\,x)\,6\mathrm{i}+\cos(6\,c+6\,d\,x)}{3\,d\,\sqrt{\cos(c+d\,x)}\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4}$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2),x)
```

output
```
(2*b^2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i
+ cos(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c
+ 6*d*x) + 10i))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c
+ 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.170 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

3.170.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{3b^2\text{arctanh}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

output
```
1/4*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+3/8*b^2*sin(d*x
+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+3/8*b^2*arctanh(sin(d*x+c))*(b
*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{(b\cos(c+dx))^{5/2}\left(3\text{arctanh}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(\cdots\right.}{8d\cos^{\frac{13}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(15/2),x]
```

output
```
((b*Cos[c + d*x])^(5/2)*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*C
os[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/2))
```

3.170.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2031, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}dx$$

$$\Big\downarrow \; 2031$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \sec^5(c+dx)dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4255$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4255$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(15/2),x]
```

output
```
(b^2*Sqrt[b*Cos[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTa
nh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqrt[Cos[
c + d*x]]
```

3.170.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.170.4 Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\left(3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)-3\left(\cos^4(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)-3\left(\cos^2(dx+c)\right)\sin(dx+c)-2\sin}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^2\sqrt{\cos(dx+c)b}\left(3\,e^{7i(dx+c)}+11\,e^{5i(dx+c)}-11\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{4\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^4}+\frac{3b^2\sqrt{\cos(dx+c)b}\,\ln\left(e^{i(dx+c)}+i\right)}{8\sqrt{\cos(dx+c)}\,d}-\frac{3b^2\sqrt{\cos(dx+c)b}}{8\sqrt{\cos(d}}$

input
```
int((cos(d*x+c)*b)^(5/2)/cos(d*x+c)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(3*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-3*cos(d*x+c)^4*ln(-cot
(d*x+c)+csc(d*x+c)+1)-3*cos(d*x+c)^2*sin(d*x+c)-2*sin(d*x+c))*(cos(d*x+c)*
b)^(1/2)*b^2/cos(d*x+c)^(9/2)
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.10

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \left[\frac{3\,b^{\frac{5}{2}}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{16\,d\cos\left(dx\right.}\right.$$
$$-\frac{3\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\left(3\,b^2\cos\left(dx+c\right)^2+2\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(\right.}}{8\,d\cos\left(dx+c\right)^5}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="fricas")
```

output
```
[1/16*(3*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
+ c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(3*b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*b^2*arctan(sqr
t(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x +
c)^5 - (3*b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(15/2),x)
```

output
```
Timed out
```

3.170.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1914 vs. 2(98) = 196.

Time = 0.51 (sec) , antiderivative size = 1914, normalized size of antiderivative = 16.50

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x
 + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*s
in(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) +
 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x
+ 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*
cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2
 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x +
 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*
c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*co
s(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2
*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4
*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c)
+ 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(
3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c)...
```

3.170.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{15}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{15/2}}\,dx$$

input
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(15/2),x)
```

output
```
int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(15/2), x)
```

3.171 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.171.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sin^5(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/3*sin(d*x+c)^3*cos(d*
x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+1/5*sin(d*x+c)^5*cos(d*x+c)^(1/2)/d/(b*c
os(d*x+c))^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.53

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)\left(15-10\sin^2(c+dx)+3\sin^4(c+dx)\right)}{15d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(11/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(15 - 10*Sin[c + d*x]^2 + 3*Sin[c + d*x]^
4))/(15*d*Sqrt[b*Cos[c + d*x]])
```

3.171.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^5(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(\sin^4(c+dx)-2\sin^2(c+dx)+1\right)d(-\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-\frac{1}{5}\sin^5(c+dx)+\frac{2}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(11/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Sin[c + d*x] + (2*Sin[c + d*x]^3)/3 - Sin[c + d*x]
^5/5))/(d*Sqrt[b*Cos[c + d*x]]))
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.171.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\left(3\left(\cos^{4}(dx+c)\right)+4\left(\cos^{2}(dx+c)\right)+8\right)\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{15d\sqrt{\cos(dx+c)b}}$	52
risch	$\frac{5\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{8d\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(5dx+5c)}{80\sqrt{\cos(dx+c)b}\,d}+\frac{5\left(\sqrt{\cos}(dx+c)\right)\sin(3dx+3c)}{48\sqrt{\cos(dx+c)b}\,d}$	95

input
```
int(cos(d*x+c)^(11/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15/d*(3*cos(d*x+c)^4+4*cos(d*x+c)^2+8)*sin(d*x+c)*cos(d*x+c)^(1/2)/(cos(
d*x+c)*b)^(1/2)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\left(3\,\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{2}+8\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,bd\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sqrt(b*cos(d*x + c))*sin(d*
x + c)/(b*d*sqrt(cos(d*x + c)))
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{3\,\sin\left(5\,dx+5\,c\right)+25\,\sin\left(\frac{3}{5}\,\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)+150\,\sin\left(\frac{1}{5}\,\arctan\left(\sin\left(5\,dx+5\right.\right.\right.}{240\,\sqrt{b}d}$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/240*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
 + 5*c))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))/(sqr
t(b)*d)
```

3.171.8 Giac [F]

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{11}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(11/2)/sqrt(b*cos(d*x + c)), x)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 14.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(175\,\sin(2\,c+2\,d\,x)+28\,\sin(4\,c+4\,d\,x)+3\,\sin(6\,c+6\,d\,x))}{240\,b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(11/2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(175*sin(2*c + 2*d*x) + 28*sin(
4*c + 4*d*x) + 3*sin(6*c + 6*d*x)))/(240*b*d*(cos(2*c + 2*d*x) + 1))
```

3.172 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.172.1 Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

output
```
3/8*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*cos(d*x+c)^(7/2
)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+3/8*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^
(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])
)/(32*d*Sqrt[b*Cos[c + d*x]])
```

3.172.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(9/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[
c + d*x]*Sin[c + d*x])/(2*d)))/4))/Sqrt[b*Cos[c + d*x]]
```

3.172.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.172.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{(2\sin(dx+c)(\cos^3(dx+c))+3\cos(dx+c)\sin(dx+c)+3dx+3c)(\sqrt{\cos}(dx+c))}{8d\sqrt{\cos(dx+c)b}}$	62
risch	$\frac{3x(\sqrt{\cos}(dx+c))}{8\sqrt{\cos(dx+c)b}}+\frac{(\sqrt{\cos}(dx+c))\sin(4dx+4c)}{32\sqrt{\cos(dx+c)b}\,d}+\frac{(\sqrt{\cos}(dx+c))\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	87

input
```
int(cos(d*x+c)^(9/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(2*sin(d*x+c)*cos(d*x+c)^3+3*cos(d*x+c)*sin(d*x+c)+3*d*x+3*c)*cos(d*
x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.86

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{2\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)-3\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b}\right.}{16\,bd}\right.$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*si
n(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sq
rt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/8*(sqrt(b*cos(d*x +
c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)
]
```

3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.50

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)}{32\,\sqrt{b}d}$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))/(sqrt(b)*d)
```

3.172.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(9/2)/sqrt(b*cos(d*x + c)), x)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 14.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\right.\right.}{32\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(9/2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3
*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*b*d*(cos(2*c + 2*d*x)
 + 1))
```

3.173 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.173.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*cos(d*
x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(5+\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[b*Cos[c
 + d*x]])
```

3.173.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \textbf{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \textbf{3113}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(7/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(d*Sqrt[b*Cos[c
+ d*x]]))
```

3.173.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.173.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\left(2+\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{3d\sqrt{\cos\left(dx+c\right)b}}$	40
risch	$\frac{3\sin\left(dx+c\right)\left(\sqrt{\cos}\left(dx+c\right)\right)}{4d\sqrt{\cos\left(dx+c\right)b}}+\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\sin\left(3dx+3c\right)}{12\sqrt{\cos\left(dx+c\right)b}\,d}$	63

input
```
int(cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3\,bd\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b*d*sqrt(cos(d
*x + c)))
```

3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)}{12\,\sqrt{b}d}$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))))/(sqrt(b)*d)
```

3.173.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{7}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/sqrt(b*cos(d*x + c)), x)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(10\,\sin(2\,c+2\,d\,x)+\sin(4\,c+4\,d\,x))}{12\,b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(7/2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c
+ 4*d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.174 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.174.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

output
```
1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*x*cos(d*x+c)^(1
/2)/(b*cos(d*x+c))^(1/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Cos[c +
d*x]])
```

3.174.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/Sqrt[b*Cos[
c + d*x]]
```

3.174.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.174.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{(\cos(dx+c)\sin(dx+c)+dx+c)\left(\sqrt{\cos}(dx+c)\right)}{2d\sqrt{\cos(dx+c)b}}$	42
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{2\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(2dx+2c)}{4\sqrt{\cos(dx+c)b}\,d}$	55

input
```
int(cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(cos(d*x+c)*sin(d*x+c)+d*x+c)*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.49

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c) - \sqrt{-b}\log\left(2\,b\cos(dx+c)^2 + 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos\right.}{4\,bd}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*lo
g(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*
sin(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(
d*x + c)^(3/2))))/(b*d)]
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\,dx + 2\,c + \sin(2\,dx + 2\,c)}{4\,\sqrt{b}d}$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(sqrt(b)*d)
```

3.174.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 14.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+4\,d\,x\,\cos(c+d\,x))}{4\,b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(5/2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x
) + 4*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*d*x) + 1))
```

3.175 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.175.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

3.175.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

3.175.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.175.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	29
risch	$\frac{\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{d\sqrt{\cos(dx+c)b}}$	29

input
```
int(cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/d/(cos(d*x+c)*b)^(1/2)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{bd\sqrt{\cos\left(dx+c\right)}}$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

3.175.6 Sympy [A] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \begin{cases} \frac{\sin\left(c+dx\right)\sqrt{\cos\left(c+dx\right)}}{d\sqrt{b\cos\left(c+dx\right)}} & \text{for}\ d \neq 0 \\ \frac{x\cos^{\frac{3}{2}}\left(c\right)}{\sqrt{b\cos\left(c\right)}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Piecewise((sin(c + d*x)*sqrt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))), Ne(d,
 0)), (x*cos(c)**(3/2)/sqrt(b*cos(c)), True))
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(dx+c)}{\sqrt{b}d}$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(sqrt(b)*d)
```

3.175.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sin(2\,c+2\,d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{b\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

input
```
int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(b*d*(cos(2*c
 + 2*d*x) + 1))
```

3.175. $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

3.176 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx$

3.176.1 Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx = \frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

output
```
x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx = \frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

3.176.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \; 2031$$

$$\frac{\sqrt{\cos(c+dx)}\int 1dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \; 24$$

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/Sqrt[b*Cos[c + d*x]],x]
```

output
```
(x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

3.176.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.176.4 Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{\sqrt{\cos(dx+c)b}}$	21
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\sqrt{\cos(dx+c)b}}$	28

input
```
int(cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.04

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[-\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{2\,bd},\ \frac{\arctan\left(\frac{\sqrt{b\cos(dx+\ldots}}{\sqrt{b}\cos\ldots}\right.}{\sqrt{b}d\ldots}\right.$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sq
rt(cos(d*x + c))*sin(d*x + c) - b)/(b*d), arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(sqrt(b)*d)]
```

3.176.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx = \frac{x\sqrt{\cos{(c+dx)}}}{\sqrt{b\cos{(c+dx)}}}$$

input
```
integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x))
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(b)*d)
```

3.176.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx = \int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{b\cos{(dx+c)}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx = \frac{2\,x\cos\left(c+d\,x\right)^{3/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(1/2),x)
```

output
```
(2*x*cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2))/(b*(cos(2*c + 2*d*x) + 1))
```

3.177 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

3.177.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

3.177.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2032, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

3.177.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.177.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
default	$-\frac{2\left(\sqrt{\cos}(dx+c)\right)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}}$	40
risch	$\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos(dx+c)b}\,d}$	73

input
```
int(1/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^(1/2)*arctanh(cot(d*x+c)-csc(d*x+c))/(cos(d*x+c)*b)^(1/2)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.52

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \left[\frac{\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\sqrt{b}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{bd}\right]$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(sqrt(b)*d), -sqrt(
-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))/(b*d)]
```

3.177.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

3.177.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time $= 0.39$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 1.97$

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \frac{\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+\right.\right.}{2\,\sqrt{b}d}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(sqrt(b)*d)
```

3.177.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.178 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.178.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

3.178.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2032, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\cos(c+dx)}\int 1d(-\tan(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

3.178.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.178.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)}{d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}$	29
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}$	34

input
```
int(1/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)/d/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{bd\cos(dx+c)^{\frac{3}{2}}}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^(3/2))
```

3.178.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{2\sqrt{b}\sin(2dx+2c)}{\left(b\cos(2dx+2c)^2+b\sin(2dx+2c)^2+2b\cos(2dx+2c)+b\right)d}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 +
 2*b*cos(2*d*x + 2*c) + b)*d)
```

3.178.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)), x)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\,\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)\,1\mathrm{i}+\sin(2\,c+2\,d\,x)+1\mathrm{i})}{b\,d\,\sqrt{\cos(c+d\,x)}\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(b*
d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.179 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.179.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*arctanh(sin(d*x
+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])/(2*d*Cos[c + d*x]^(3
/2)*Sqrt[b*Cos[c + d*x]])
```

3.179.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2032, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/Sqrt[b*Cos[c + d*x]]
```

3.179.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.179.4 Maple [A] (verified)

Time = 3.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{-\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^{2}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)}{2d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$	84
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}-1\right)}{2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}-\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)b}\,d}$	122

input
```
int(1/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+cos(d*x+c)^2*ln(-cot(d*x
+c)+csc(d*x+c)+1)+sin(d*x+c))/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.88

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}\sqrt{\cos\ldots}}{4bd\cos(dx+c)^3}\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2bd\cos(dx+c)^3}\right]$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*
x + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

3.179.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.179.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(60) = 120$.

Time = 0.41 (sec) , antiderivative size = 661, normalized size of antiderivative = 9.18

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx =$$

$$-\frac{4\left(\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)\right)\cos\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)-4\left(\sin\left(4\,dx+\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((2*(2*cos(2*d*x + 2*c) + 1)*
cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*c
os(2*d*x + 2*c) + 1)*sqrt(b)*d)
```

3.179.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.180 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.180.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/3*sin(d*x+c)^3/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+sin(d*x+c)/d/cos(
d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[b*Cos[c + d
*x]])
```

3.180.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2032, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\cos(c+dx)}\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(d*Sqrt[b*Cos[c
+ d*x]]))
```

3.180.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.180.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{\left(2\left(\cos^{2}\left(dx+c\right)\right)+1\right)\sin\left(dx+c\right)}{3d\sqrt{\cos\left(dx+c\right)b}\,\cos\left(dx+c\right)^{\frac{5}{2}}}$	42
risch	$\frac{2i\left(4\cos\left(dx+c\right)+2i\sin\left(dx+c\right)\right)}{3\sqrt{\cos\left(dx+c\right)b}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i\left(dx+c\right)}+1\right)^{2}d}$	56

input
```
int(1/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*cos(d*x+c)^2+1)*sin(d*x+c)/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2)
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+1\right)\sin(dx+c)}{3\,bd\cos(dx+c)^{\frac{7}{2}}}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b*d*cos(d*x
+ c)^(7/2))
```

3.180.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.180.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(60) = 120.

Time = 0.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.20

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{4\,((3\cos(}{3\left(2\left(3\cos(4\,dx+4\,c)+3\cos(2\,dx+2\,c)+1\right)\cos(6\,dx+6\,c)+\cos(6\,dx+6\,c)^2+6\left(3\cos(2\,dx+2\,c)\right.\right.}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1
)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*co
s(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c
) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x
+ 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6
*cos(2*d*x + 2*c) + 1)*sqrt(b)*d)
```

3.180.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 15.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.87

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$=\frac{2\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6}{3\,b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\right.}$$

input
```
int(1/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
(2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + co
s(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*
d*x) + 10i))/(3*b*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c +
4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.181 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

3.181.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/4*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+3/8*sin(d*x+c)/d/co
s(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+3/8*arctanh(sin(d*x+c))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.62

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\cos^4(c+dx)+(2+3\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[1/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d
*x])/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

3.181.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2032, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin
[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/Sqrt[b*Cos[c +
d*x]]
```

3.181.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.181.4 Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

method	result
default	$\frac{-3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)-1)+3(\cos^4(dx+c))\ln(-\cot(dx+c)+\csc(dx+c)+1)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(}{8d\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3)}{8\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^3 d}+\frac{3(\sqrt{\cos}(dx+c))\ln(e^{i(dx+c)}+i)}{8\sqrt{\cos(dx+c)b}\,d}-\frac{3(\sqrt{\cos}(dx+c))\ln(e^{i(dx+c)}-i)}{8\sqrt{\cos(dx+c)b}\,d}$

input
```
int(1/cos(d*x+c)^(9/2)/(cos(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(-3*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*cos(d*x+c)^4*ln(-cot
(d*x+c)+csc(d*x+c)+1)+3*cos(d*x+c)^2*sin(d*x+c)+2*sin(d*x+c))/(cos(d*x+c)*
b)^(1/2)/cos(d*x+c)^(7/2)
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$$
$$= \left[\frac{3\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}(3\ c}{16\,bd\cos(dx+c)^5}\right.$$
$$-\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\sqrt{b\cos(dx+c)}(3\cos(dx+c)^2+2)\sqrt{\cos(dx-}{8\,bd\cos(dx+c)^5}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
+ c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))
*sin(d*x + c))/(b*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*
x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqr
t(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/
(b*d*cos(d*x + c)^5)]
```

3.181.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

output
```
Timed out
```

3.181.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(89) = 178$.

Time = 0.41 (sec) , antiderivative size = 1656, normalized size of antiderivative = 15.48

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
4*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8
*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8
*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*
x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) +
 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8
*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16
*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x +...
```

3.181.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)}\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(9/2)), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \int \frac{1}{{\cos\left(c+d\,x\right)}^{9/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.182 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.182.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

output
```
3/8*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*cos(d*x+c)^(7
/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+3/8*x*cos(d*x+c)^(1/2)/b/(b*cos(d*
x+c))^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])
)/(32*d*(b*Cos[c + d*x])^(3/2))
```

3.182.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[
c + d*x]*Sin[c + d*x])/(2*d)))/4))/(b*Sqrt[b*Cos[c + d*x]])
```

3.182.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.182.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2\sin(dx+c)\left(\cos^{3}(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)}{8db\sqrt{\cos(dx+c)b}}$	65
risch	$\frac{3x\left(\sqrt{\cos}(dx+c)\right)}{8b\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(4dx+4c)}{32b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	96

input
```
int(cos(d*x+c)^(11/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*cos(d*x+c)^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*cos(d*x+c)*sin(d*x+c)+
3*d*x+3*c)/b/(cos(d*x+c)*b)^(1/2)
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{2\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)-3\sqrt{-b}\log\left(\right.}{16\,b^2d}\right.$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*si
n(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sq
rt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/8*(sqrt(b*cos(d*x
+ c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^
2*d)]
```

3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c)\right)}{32\,b^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))/(b^(3/2)*d)
```

3.182.8 Giac [F]

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(11/2)/(b*cos(d*x + c))^(3/2), x)
```

3.182.9 Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\right.}{32\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(11/2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3
*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*b^2*d*(cos(2*c + 2*d*
x) + 1))
```

3.183 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.183.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*cos(
d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(5+\cos(2(c+dx)))\sin(c+dx)}{6d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b*Cos[c + d
*x])^(3/2))
```

3.183.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(b*d*Sqrt[b*Cos[
c + d*x]]))
```

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.183.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{(2+\cos^2(dx+c))\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{3db\sqrt{\cos(dx+c)b}}$	43
risch	$\frac{3\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{4bd\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(3dx+3c)}{12b\sqrt{\cos(dx+c)b}\,d}$	69

input
```
int(cos(d*x+c)^(9/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(cos(d*x+c)*b)^(1/2)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3\,b^2d\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b^2*d*sqrt(cos
(d*x + c)))
```

3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c)\right)\right)}{12\,b^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))))/(b^(3/2)*d)
```

3.183.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{9}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(3/2), x)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(10\,\sin(2\,c+2\,d\,x)+\sin(4\,c+4\,d\,x))}{12\,b^2\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(9/2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c
+ 4*d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.184 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.184.1 Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

output
```
1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*x*cos(d*x+c)^
(1/2)/b/(b*cos(d*x+c))^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)(2(c+dx)+\sin(2(c+dx)))}{4d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*(b*Cos[c + d*x]
)^(3/2))
```

3.184.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(b*Sqrt[b*C
os[c + d*x]])
```

3.184.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.184.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2db\sqrt{\cos(dx+c)b}}$	45
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{2b\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(2dx+2c)}{4b\sqrt{\cos(dx+c)b}\,d}$	61

input
```
int(cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*cos(d*x+c)^(1/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/b/(cos(d*x+c)*b)^(1/2
)
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.28

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{\;}\right.}{4b^2d}\right.$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*lo
g(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*
sin(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*co
s(d*x + c)^(3/2))))/(b^2*d)]
```

3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\,dx+2\,c+\sin(2\,dx+2\,c)}{4\,b^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(3/2)*d)
```

3.184.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x)
```

3.184.9 Mupad [B] (verification not implemented)

Time = 14.87 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+4\,d\,x\,\cos(c+d}{4\,b^2\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(7/2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x
) + 4*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.185 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.185.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(3/2))
```

3.185.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{ 2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

3.185.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.185.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}$	32
risch	$\frac{\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{bd\sqrt{\cos(dx+c)b}}$	32

input
```
int(cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(cos(d*x+c)*b)^(1/2)
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^2d\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))
```

3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\sin(dx+c)}{b^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(b^(3/2)*d)
```

3.185.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx=\frac{\sqrt{\cos(c+d\,x)}\,\sin(2\,c+2\,d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{b^2\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(5/2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
(cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(b^2*d*(cos(2
*c + 2*d*x) + 1))
```

3.186 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

3.186.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

output
```
x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{x\sqrt{b\cos(c+dx)}}{b^2\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(x*Sqrt[b*Cos[c + d*x]])/(b^2*Sqrt[Cos[c + d*x]])
```

3.186.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int 1dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]])
```

3.186.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.186.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{b\sqrt{\cos(dx+c)b}}$	24
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{db\sqrt{\cos(dx+c)b}}$	31

input
```
int(cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*cos(d*x+c)^(1/2)/b/(cos(d*x+c)*b)^(1/2)
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right.}{2\,b^{2}d}\right.$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sq
rt(cos(d*x + c))*sin(d*x + c) - b)/(b^2*d), arctan(sqrt(b*cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(b^(3/2)*d)]
```

3.186.6 Sympy [A] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{x\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
x*cos(c + d*x)**(3/2)/(b*cos(c + d*x))**(3/2)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(3/2)*d)
```

3.186.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{2\,x\cos(c+d\,x)^{3/2}\,\sqrt{b\,\cos(c+d\,x)}}{b^2\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
(2*x*cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2))/(b^2*(cos(2*c + 2*d*x) + 1
))
```

3.187 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx$

3.187.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.187.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])^(3/2))
```

3.187.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \; 2031$$
$$\frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \; 3042$$
$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b\sqrt{b\cos(c+dx)}}$$
$$\Big\downarrow \; 4257$$
$$\frac{\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(3/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

3.187.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.187.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{2\left(\sqrt{\cos}(dx+c)\right)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}\,b}$	43
risch	$-\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b\sqrt{\cos(dx+c)b}\,d}$	79

input
```
int(cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^(1/2)*arctanh(cot(d*x+c)-csc(d*x+c))/(cos(d*x+c)*b)^(1/2)/
b
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,b^{\frac{3}{2}}d},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{b^2d}\right]$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(b^(3/2)*d), -sqrt(
-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))/(b^2*d)]
```

3.187.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(b*cos(c + d*x))**(3/2), x)
```

3.187.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin\right.}{2b^{\frac{3}{2}}d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(3/2)*d)
```

3.187.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{\cos\left(c+d\,x\right)}}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(3/2),x)
```

output
```
int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(3/2), x)
```

3.188 $\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

3.188.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(3/2))
```

3.188.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2032, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\cos(c+dx)}\int 1d(-\tan(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

3.188.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.188.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)}{bd\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}$	32
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{b\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,d}$	37

input
```
int(1/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{b^2d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^(3/2))
```

3.188.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(c+dx)})^{\frac{3}{2}}\sqrt{\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(1/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

3.188.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(31) = 62$.

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b}\sin{(2\,dx+2\,c)}}{(b^2\cos{(2\,dx+2\,c)}^2 + b^2\sin{(2\,dx+2\,c)}^2 + 2\,b^2\cos{(2\,dx+2\,c)} + b^2}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)
^2 + 2*b^2*cos(2*d*x + 2*c) + b^2)*d)
```

3.188.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)})^{\frac{3}{2}}\sqrt{\cos{(dx+c)}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))), x)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1\mathrm{i}\right)}{b^2\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
((b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(b^
2*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.189 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

3.189.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/2*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*arctanh(sin(d
*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*
x]]*(b*Cos[c + d*x])^(3/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2032, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/(b*Sqrt[b*Cos[c + d*x]])
```

3.189.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.189.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{-\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)-1\right)+\left(\cos^{2}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)+1\right)+\sin\left(dx+c\right)}{2db\sqrt{\cos\left(dx+c\right)b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}$	87
risch	$-\frac{i\left(\mathrm{e}^{2i(dx+c)}-1\right)}{2b\sqrt{\cos\left(dx+c\right)b}\,\sqrt{\cos\left(dx+c\right)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2b\sqrt{\cos\left(dx+c\right)b}\,d}-\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2b\sqrt{\cos\left(dx+c\right)b}\,d}$	131

input
```
int(1/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+cos(d*x+c)^2*ln(-cot(d*x
+c)+csc(d*x+c)+1)+sin(d*x+c))/b/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.65

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2}{\cos(dx+c)^{3}}\right.}{4\,b^{2}d\cos\left(dx-\right.}\right.$$

$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^{2}d\cos\left(dx+c\right)^{3}}\right]$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(
d*x + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x +
 c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

3.189.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\cos\left(c+dx\right)\right)^{\frac{3}{2}}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Integral(1/((b*cos(c + d*x))**(3/2)*cos(c + d*x)**(3/2)), x)
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(66) = 132$.

Time = 0.39 (sec) , antiderivative size = 670, normalized size of antiderivative = 8.59

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx =$$

$$-\frac{4\left(\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)\right)\cos\left(\frac{3}{2}\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)\right)\right)-4\left(\sin\left(4\,dx+4\,c\right.\right.}{}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((b*cos(4*d*x + 4*c)^2 + 4*b*
cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)*d)
```

3.189.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.190 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

3.190.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/3*sin(d*x+c)^3/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+sin(d*x+c)/b/d/
cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Cos[c + d*x]^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*(b*Cos[c + d*x])
^(3/2))
```

3.190.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2032, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\cos(c+dx)}\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(b*d*Sqrt[b*Cos[
c + d*x]]))
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.190.4 Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\left(2\left(\cos^{2}\left(d x +c \right)\right)+1\right) \sin \left(d x +c \right)}{3 d b \sqrt{\cos \left(d x +c \right) b}\, \cos \left(d x +c \right)^{\frac{5}{2}}}$	45
risch	$\frac{2 i \left(4 \cos \left(d x +c \right)+2 i \sin \left(d x +c \right)\right)}{3 b \sqrt{\cos \left(d x +c \right) b}\, \sqrt{\cos \left(d x +c \right)}\, \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{2} d}$	59

input
```
int(1/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*cos(d*x+c)^2+1)*sin(d*x+c)/b/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5/2
)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.58

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+1\right)\sin(dx+c)}{3\,b^2d\cos(dx+c)^{\frac{7}{2}}}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b^2*d*cos(d*
x + c)^(7/2))
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.09

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{}{3\left(b\cos(6\,dx+6\,c)^2+9\,b\cos(4\,dx+4\,c)^2+9\,b\cos(2\,dx+2\,c)^2+b\right.}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1
)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*
b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*
b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(
4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d
*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x +
4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)*d)
```

3.190.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 16.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.72

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{2\sqrt{b\cos(c+dx)}\,(\cos(2c+2dx)\,15\mathrm{i}+\cos(4c+4dx)\,6\mathrm{i}+\cos(6}{3b^2d\sqrt{\cos(c+dx)}\,(15\cos(2c+2d}$$

input
```
int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
(2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + co
s(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*
d*x) + 10i))/(3*b^2*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c
+ 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.191 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

3.191.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/4*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+3/8*sin(d*x+c)/b/
d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+3/8*arctanh(sin(d*x+c))*cos(d*x+c)
^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}$$

input
```
Integrate[1/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d
*x])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

3.191.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2032, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin
[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/(b*Sqrt[b*Cos[c
 + d*x]])
```

3.191.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.191.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$-\frac{3\left(\cos^{4}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)-1\right)-3\left(\cos^{4}\left(dx+c\right)\right)\ln\left(-\cot\left(dx+c\right)+\csc\left(dx+c\right)+1\right)-3\left(\cos^{2}\left(dx+c\right)\right)\sin\left(dx+c\right)-2\sin(}{8db\sqrt{\cos\left(dx+c\right)b}\,\cos\left(dx+c\right)^{\frac{7}{2}}}$
risch	$-\frac{i\left(3\,\mathrm{e}^{6i(dx+c)}+11\,\mathrm{e}^{4i(dx+c)}-11\,\mathrm{e}^{2i(dx+c)}-3\right)}{8b\sqrt{\cos\left(dx+c\right)b}\,\sqrt{\cos\left(dx+c\right)}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}d}+\frac{3\left(\sqrt{\cos}\left(dx+c\right)\right)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8b\sqrt{\cos\left(dx+c\right)b}\,d}-\frac{3\left(\sqrt{\cos}\left(dx+c\right)\right)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8b\sqrt{\cos\left(dx+c\right)b}\,d}$

input
```
int(1/cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(3*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)-3*cos(d*x+c)^4*ln(-cot
(d*x+c)+csc(d*x+c)+1)-3*cos(d*x+c)^2*sin(d*x+c)-2*sin(d*x+c))/b/(cos(d*x+c
)*b)^(1/2)/cos(d*x+c)^(7/2)
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.01

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \left[\frac{3\sqrt{b}\cos(dx+c)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)}{\cos(dx+c)^3}\right.}{16} \right.$$
$$-\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^5-\sqrt{b\cos(dx+c)}(3\cos(dx+c)^2+2)\sqrt{\cos(dx+c)}}{8b^2d\cos(dx+c)^5}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
+ c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))
*sin(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(
d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - s
qrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c)
)/(b^2*d*cos(d*x + c)^5)]
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.191.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(98) = 196$.

Time = 0.42 (sec) , antiderivative size = 1679, normalized size of antiderivative = 14.47

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
4*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8
*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8
*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*
x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) +
 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8
*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16
*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x +...
```

3.191.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^{7/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)),x)
```

output
```
int(1/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.192 $\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.192.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

output
```
3/8*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*cos(d*x+c)^
(7/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+3/8*x*cos(d*x+c)^(1/2)/b^2/(b*
cos(d*x+c))^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(13/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])
)/(32*b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.192.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2031, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(13/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (3*(x/2 + (Cos[
c + d*x]*Sin[c + d*x])/(2*d)))/4))/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.192.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.192.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(2\sin(dx+c)\left(\cos^3(dx+c)\right)+3\cos(dx+c)\sin(dx+c)+3dx+3c\right)}{8d\,b^2\sqrt{\cos(dx+c)b}}$	65
risch	$\frac{3x\left(\sqrt{\cos}(dx+c)\right)}{8b^2\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(4dx+4c)}{32b^2\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(2dx+2c)}{4b^2\sqrt{\cos(dx+c)b}\,d}$	96

input
```
int(cos(d*x+c)^(13/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*cos(d*x+c)^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*cos(d*x+c)*sin(d*x+c)+
3*d*x+3*c)/b^2/(cos(d*x+c)*b)^(1/2)
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{2\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+3\right)\sqrt{\cos(dx+c)}\sin(dx+c) - 3\sqrt{-b}\log\left(\right.}{16\,b^3d}\right.$$

input
```
integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*si
n(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sq
rt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/8*(sqrt(b*cos(d*x
+ c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^
3*d)]
```

3.192.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(13/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\arctan\left(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c)\right)\right.}{32\,b^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))/(b^(5/2)*d)
```

3.192.8 Giac [F]

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{13}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(13/2)/(b*cos(d*x + c))^(5/2), x)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 15.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(8\,\sin(c+d\,x)+9\,\sin(3\,c+3\,d\,x)+\sin(5\,c+5}{32\,b^3\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(13/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3
*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*b^3*d*(cos(2*c + 2*d*
x) + 1))
```

3.193 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.193.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{\cos(c+dx)}\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*co
s(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.63

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}(5+\cos(2(c+dx)))\sin(c+dx)}{6b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*d*Sqrt[b*C
os[c + d*x]])
```

3.193.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/(b^2*d*Sqrt[b*Co
s[c + d*x]]))
```

3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

3.193.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{(2+\cos^2(dx+c))\sin(dx+c)(\sqrt{\cos}(dx+c))}{3d\,b^2\sqrt{\cos(dx+c)b}}$	43
risch	$\frac{3\sin(dx+c)(\sqrt{\cos}(dx+c))}{4b^2d\sqrt{\cos(dx+c)b}}+\frac{(\sqrt{\cos}(dx+c))\sin(3dx+3c)}{12b^2\sqrt{\cos(dx+c)b}\,d}$	69

input
```
int(cos(d*x+c)^(11/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2+cos(d*x+c)^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/(cos(d*x+c)*b)^(1/2
)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3\,b^3d\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b^3*d*sqrt(cos
(d*x + c)))
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c)\right)\right)}{12\,b^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))))/(b^(5/2)*d)
```

3.193.8 Giac [F]

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{11}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(11/2)/(b*cos(d*x + c))^(5/2), x)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(11/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c
+ 4*d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.194 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.194.1 Optimal result

Integrand size = 23, antiderivative size = 69

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

output
```
1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*x*cos(d*x+c
)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos(c+dx)}(2(c+dx)+\sin(2(c+dx)))}{4b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Cos[
c + d*x]])
```

3.194.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(b^2*Sqrt[b
*Cos[c + d*x]])
```

3.194.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.194.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)\left(\cos(dx+c)\sin(dx+c)+dx+c\right)}{2d\,b^{2}\sqrt{\cos(dx+c)b}}$	45
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{2b^{2}\sqrt{\cos(dx+c)b}}+\frac{\left(\sqrt{\cos}(dx+c)\right)\sin(2dx+2c)}{4b^{2}\sqrt{\cos(dx+c)b}\,d}$	61

input
```
int(cos(d*x+c)^(9/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*cos(d*x+c)^(1/2)*(cos(d*x+c)*sin(d*x+c)+d*x+c)/b^2/(cos(d*x+c)*b)^(1
/2)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.28

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c) - \sqrt{-b}\log\left(2\,b\cos(dx+c)^2 + 2\sqrt{\right.}{4\,b^3d}\right.$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*lo
g(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*
sin(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*co
s(d*x + c)^(3/2))))/(b^3*d)]
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.36

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\,dx+2\,c+\sin(2\,dx+2\,c)}{4\,b^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(5/2)*d)
```

3.194.8 Giac [F]

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 14.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\right.\right.}{4\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(9/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x
) + 4*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.195 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.195.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.195.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.195.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

3.195.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^2 d\sqrt{\cos(dx+c)b}}$	32
risch	$\frac{\sin(dx+c)\left(\sqrt{\cos}(dx+c)\right)}{b^2 d\sqrt{\cos(dx+c)b}}$	32

input
```
int(cos(d*x+c)^(7/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(cos(d*x+c)*b)^(1/2)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^3 d\sqrt{\cos(dx+c)}}$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sin(dx+c)}{b^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
sin(d*x + c)/(b^(5/2)*d)
```

3.195.8 Giac [F]

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 13.90 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+d\,x)}\,\sin(2\,c+2\,d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{b^3\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

input
```
int(cos(c + d*x)^(7/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
(cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(b^3*d*(cos(2
*c + 2*d*x) + 1))
```

3.196 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.196.1 Optimal result

Integrand size = 23, antiderivative size = 27

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

output
```
x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{x\sqrt{b\cos(c+dx)}}{b^3\sqrt{\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(x*Sqrt[b*Cos[c + d*x]])/(b^3*Sqrt[Cos[c + d*x]])
```

3.196.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2031, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int 1dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.196.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

3.196.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{b^2\sqrt{\cos(dx+c)b}}$	24
default	$\frac{\left(\sqrt{\cos}(dx+c)\right)(dx+c)}{d\,b^2\sqrt{\cos(dx+c)b}}$	31

input
```
int(cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
x*cos(d*x+c)^(1/2)/b^2/(cos(d*x+c)*b)^(1/2)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right.}{2\,b^3d}\right.$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sq
rt(cos(d*x + c))*sin(d*x + c) - b)/(b^3*d), arctan(sqrt(b*cos(d*x + c))*si
n(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(b^(5/2)*d)]
```

3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\ \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(5/2)*d)
```

3.196.8 Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\,x\cos\left(c+d\,x\right)^{3/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b^3\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

input
```
int(cos(c + d*x)^(5/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
(2*x*cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2))/(b^3*(cos(2*c + 2*d*x) + 1
))
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

3.197.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2 d\sqrt{b\cos(c+dx)}}$$

output
```
arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\cos^{\frac{5}{2}}(c+dx)}{d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^(5/2))/(d*(b*Cos[c + d*x])^(5/2))
```

3.197.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2031}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\text{arctanh}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

3.197.3.1 Defintions of rubi rules used

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.197.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{2\left(\sqrt{\cos}(dx+c)\right)\operatorname{arctanh}(\cot(dx+c)-\csc(dx+c))}{d\sqrt{\cos(dx+c)b}\,b^{2}}$	43
risch	$-\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{b^{2}\sqrt{\cos(dx+c)b}\,d}+\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{b^{2}\sqrt{\cos(dx+c)b}\,d}$	79

input
```
int(cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*cos(d*x+c)^(1/2)*arctanh(cot(d*x+c)-csc(d*x+c))/(cos(d*x+c)*b)^(1/2)/
b^2
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.22

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx=\left[\frac{\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2\,b^{\frac{5}{2}}d},\right.$$
$$\left.-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{b^{3}d}\right]$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(b^(5/2)*d), -sqrt(
-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))/(b^3*d)]
```

3.197.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.

Time $= 0.38$ (sec) , antiderivative size $= 65$, normalized size of antiderivative $= 1.81$

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\right.}{2\,b^{\frac{5}{2}}d}$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d
*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(5/2)*d)
```

3.197.8 Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^{3/2}}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(5/2), x)
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx$

3.198.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

output
```
sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx=\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(5/2),x]
```

output
```
(Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(5/2))
```

3.198.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2031, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \ 2031$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 4254$$

$$-\frac{\sqrt{\cos(c+dx)}\int 1d(-\tan(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(5/2),x]
```

output
```
Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

3.198.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2031
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m + 1/
2)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.198.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\sin(dx+c)}{b^2 d\sqrt{\cos(dx+c)}\,\sqrt{\cos(dx+c)b}}$	32
risch	$\frac{2i\left(\sqrt{\cos}(dx+c)\right)}{b^2\sqrt{\cos(dx+c)b}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	41

input
```
int(cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(1/2)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^3d\cos(dx+c)^{\frac{3}{2}}}$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^(3/2))
```

3.198.6 Sympy [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Integral(sqrt(cos(c + d*x))/(b*cos(c + d*x))**(5/2), x)
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b}\sin(2\,dx+2\,c)}{(b^3\cos(2\,dx+2\,c)^2+b^3\sin(2\,dx+2\,c)^2+2\,b^3\cos(2\,dx+2\,c)+b^3)d}$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2*sqrt(b)*sin(2*d*x + 2*c)/((b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)
^2 + 2*b^3*cos(2*d*x + 2*c) + b^3)*d)
```

3.198.8 Giac [F]

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,3}{b^3\,d\,\left(4\,\cos(2\,c+2\,d\,x)+\cos(4\,c+4\,d\,x)+3\right)}$$

input
```
int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(5/2),x)
```

output
```
(2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(cos(c + d*x)*3i + sin(c + d*
x) + cos(3*c + 3*d*x)*1i + sin(3*c + 3*d*x)))/(b^3*d*(4*cos(2*c + 2*d*x) +
 cos(4*c + 4*d*x) + 3))
```

3.199 $\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

3.199.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/2*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*arctanh(sin
(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\cos(c+dx)}(\operatorname{arctanh}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx))}{2b^3d\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x]
))/(2*b^3*d*Cos[c + d*x]^(5/2))
```

3.199.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2032, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d
*x])/(2*d)))/(b^2*Sqrt[b*Cos[c + d*x]])
```

3.199.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.199.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{-\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+\left(\cos^2(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+\sin(dx+c)}{2d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{3}{2}}}$	87
risch	$-\frac{i\left(e^{2i(dx+c)}-1\right)}{2b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)d}+\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(e^{i(dx+c)}+i\right)}{2b^2\sqrt{\cos(dx+c)b}\,d}-\frac{\left(\sqrt{\cos}(dx+c)\right)\ln\left(e^{i(dx+c)}-i\right)}{2b^2\sqrt{\cos(dx+c)b}\,d}$	131

input
```
int(1/cos(d*x+c)^(1/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-cos(d*x+c)^2*ln(-cot(d*x+c)+csc(d*x+c)-1)+cos(d*x+c)^2*ln(-cot(d*x
+c)+csc(d*x+c)+1)+sin(d*x+c))/b^2/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(3/2)
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.65

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-}{\cos(dx+c)^3}\right.}{4\,b^3d\cos(dx}\right.$$
$$\left. -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c)^3-\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\,b^3d\cos(dx+c)^3}\right]$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c
))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(
d*x + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x +
 c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(66) = 132$.

Time = 0.45 (sec) , antiderivative size = 688, normalized size of antiderivative = 8.82

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4
*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x
 + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4
*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))/((b^2*cos(4*d*x + 4*c)^2 + 4*
b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2
+ 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)*d)
```

3.199.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))), x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.200 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

3.200.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/3*sin(d*x+c)^3/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+sin(d*x+c)/b^
2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\cos^{\frac{5}{2}}(c+dx)\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Cos[c + d*x]^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*(b*Cos[c + d*x])
^(5/2))
```

3.200.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2032, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\sqrt{\cos(c+dx)}\int \left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\cos(c+dx)}\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(-Tan[c + d*x] - Tan[c + d*x]^3/3))/(b^2*d*Sqrt[b*Co
s[c + d*x]]))
```

3.200.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.200.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(2(\cos^2(dx+c))+1)\sin(dx+c)}{3d\,b^2\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{5}{2}}}$	45
risch	$\frac{2i(4\cos(dx+c)+2i\sin(dx+c))}{3b^2\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,(e^{2i(dx+c)}+1)^2 d}$	59

input
```
int(1/cos(d*x+c)^(3/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(2*cos(d*x+c)^2+1)*sin(d*x+c)/b^2/(cos(d*x+c)*b)^(1/2)/cos(d*x+c)^(5
/2)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.58

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\cos(dx+c)}\left(2\cos(dx+c)^2+1\right)\sin(dx+c)}{3\,b^3d\cos(dx+c)^{\frac{7}{2}}}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b^3*d*cos(d*
x + c)^(7/2))
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(66) = 132.

Time = 0.39 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.51

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{}{3\left(b^2\cos(6\,dx+6\,c)^2+9\,b^2\cos(4\,dx+4\,c)^2+9\,b^2\cos(2\,dx+2\,c)^2\right.}$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1
)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4
*c)*sin(2*d*x + 2*c))/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2
+ 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*
c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2
 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*
d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(
4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c))*sqrt(b)*d)
```

3.200.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 15.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.72

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sqrt{b\cos(c+dx)}\,(\cos(2c+2dx)\,15\mathrm{i}+\cos(4c+4dx)\,6\mathrm{i}+\cos(6}{3b^3 d\sqrt{\cos(c+dx)}\,(15\cos(2c+2d}$$

input
```
int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
(2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + co
s(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*
d*x) + 10i))/(3*b^3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c
+ 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.201 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

3.201.1 Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

output
```
1/4*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+3/8*sin(d*x+c)/
b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+3/8*arctanh(sin(d*x+c))*cos(d*
x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.57

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{3\text{arctanh}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

input
```
Integrate[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d
*x])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

3.201.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2032, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{2032}$$

$$\frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2\sqrt{b\cos(c+dx)}}$$

input
```
Int[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

output
```
(Sqrt[Cos[c + d*x]]*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(ArcTanh[Sin
[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/(b^2*Sqrt[b*Cos
[c + d*x]])
```

3.201.3.1 Defintions of rubi rules used

rule 2032
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[a^(m - 1/
2)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v])   Int[v^(m + n)*Fx, x], x] /; FreeQ[{a
, b, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.201.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
default	$\frac{-3\left(\cos^{4}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)-1)+3\left(\cos^{4}(dx+c)\right)\ln(-\cot(dx+c)+\csc(dx+c)+1)+3\left(\cos^{2}(dx+c)\right)\sin(dx+c)+2\sin(}{8d\,b^{2}\sqrt{\cos(dx+c)b}\,\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i\left(3\,e^{6i(dx+c)}+11\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}-3\right)}{8b^{2}\sqrt{\cos(dx+c)b}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)^{3}d}-\frac{3\left(\sqrt{\cos}(dx+c)\right)\ln\left(e^{i(dx+c)}-i\right)}{8b^{2}\sqrt{\cos(dx+c)b}\,d}+\frac{3\left(\sqrt{\cos}(dx+c)\right)\ln\left(e^{i(dx+c)}+i\right)}{8b^{2}\sqrt{\cos(dx+c)b}\,d}$

input
```
int(1/cos(d*x+c)^(5/2)/(cos(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d*(-3*cos(d*x+c)^4*ln(-cot(d*x+c)+csc(d*x+c)-1)+3*cos(d*x+c)^4*ln(-cot
(d*x+c)+csc(d*x+c)+1)+3*cos(d*x+c)^2*sin(d*x+c)+2*sin(d*x+c))/b^2/(cos(d*x
+c)*b)^(1/2)/cos(d*x+c)^(7/2)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.01

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \left[\frac{3\sqrt{b}\cos\left(dx+c\right)^5\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)}{\cos(dx+c)^3}\right.}{16}\right.$$

$$-\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos\left(dx+c\right)^5-\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^2+2\right)\sqrt{\cos\left(dx+c\right)}}{8\,b^3d\cos\left(dx+c\right)^5}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
+ c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))
*sin(d*x + c))/(b^3*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(
d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - s
qrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c)
)/(b^3*d*cos(d*x + c)^5)]
```

3.201.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. $2(98) = 196$.

Time = 0.44 (sec) , antiderivative size = 1729, normalized size of antiderivative = 14.91

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*
sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
4*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8
*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8
*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*
x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) +
 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8
*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16
*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x +...
```

3.201.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.201.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)),x)
```

output
```
int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.202 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

3.202.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+}{d(4+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m],[5/3+1
/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(4+3*m)/(sin(d*x+c)^2)^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = -\frac{\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{4}{3}+m\right),\frac{1}{2}\left(\frac{10}{3}+m\right),\cos^2(c+dx)}{d\left(\frac{4}{3}+m\right)}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*Hypergeometric
2F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/
(d*(4/3 + m)))
```

3.202.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\cos^m(c+dx)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{1}{3}}(c+dx)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{1}{3}}dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+4),\frac{1}{6}(3m+10),\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4
+ 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Si
n[c + d*x]^2])
```

3.202.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.202.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right) \left(\cos\left(dx+c\right) b\right)^{\frac{1}{3}} dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(1/3),x)
```

3.202.5 Fricas [F]

$$\int \cos^{m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}} \cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.202.6 Sympy [F]

$$\int \cos^{m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int \sqrt[3]{b\cos\left(c+dx\right)}\cos^{m}\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*cos(c + d*x)**m, x)
```

3.202.7 Maxima [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.202.8 Giac [F]

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{1/3}\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3), x)
```

3.203 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

3.203.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/10*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d
*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3\cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{10d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1
/2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(10*d)
```

3.203.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2])
```

3.203.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.203.4 Maple [F]

$$\int \left(\cos^2(dx+c)\right) (\cos(dx+c)\,b)^{\frac{1}{3}}\,dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(1/3),x)
```

3.203.5 Fricas [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.203.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3),x)
```

output
```
Timed out
```

3.203.7 Maxima [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.203.8 Giac [F]

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{1/3}\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3), x)
```

3.204 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

3.204.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/7*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*
x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*b*d)
```

3.204.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{4/3}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.204.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.204.4 Maple [F]

$$\int \cos (dx+c) (\cos (dx+c) b)^{\frac{1}{3}} dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(1/3),x)
```

3.204.5 Fricas [F]

$$\int \cos(c+dx) \sqrt[3]{b\cos(c+dx)}\, dx = \int (b\cos (dx+c))^{\frac{1}{3}} \cos (dx+c)\, dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.204.6 Sympy [F]

$$\int \cos(c+dx) \sqrt[3]{b\cos(c+dx)}\, dx = \int \sqrt[3]{b\cos (c+dx)} \cos (c+dx)\, dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*cos(c + d*x), x)
```

3.204.7 Maxima [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.204.8 Giac [F]

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \int \cos(c+d\,x)\,(b\,\cos(c+d\,x))^{1/3}\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(1/3), x)
```

3.205 $\int \sqrt[3]{b\cos(c+dx)}\,dx$

3.205.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*d)
```

3.205.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

3.205.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.205.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{1}{3}}\, dx$$

input
```
int((cos(d*x+c)*b)^(1/3),x)
```

output
```
int((cos(d*x+c)*b)^(1/3),x)
```

3.205.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\, dx = \int (b\cos (dx+c))^{\frac{1}{3}}\, dx$$

input
```
integrate((b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3), x)
```

3.205.6 Sympy [F]

$$\int \sqrt[3]{b\cos(c+dx)}\, dx = \int \sqrt[3]{b\cos (c+dx)}\, dx$$

input
```
integrate((b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3), x)
```

3.205.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3), x)
```

3.205.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3), x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = \int (b\,\cos(c+d\,x))^{1/3}\,dx$$

input
```
int((b*cos(c + d*x))^(1/3),x)
```

output
```
int((b*cos(c + d*x))^(1/3), x)
```

3.206 $\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx$

3.206.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

output
```
-3*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c
)/d/(sin(d*x+c)^2)^(1/2)
```

3.206.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = -\frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x],x]
```

output
```
(-3*b*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

3.206.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 2030$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \; 3122$$

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x],x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

3.206.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.206.4 Maple [F]

$$\int (\cos (dx+c) b)^{\frac{1}{3}} \sec (dx+c) dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*sec(d*x+c),x)
```

3.206.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}} \sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.206.6 Sympy [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = \int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x), x)
```

3.206.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.206.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^{1/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^(1/3)/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

3.207 $\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx$

3.207.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{3b\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*b*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c)
)^(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{3b\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^2,x]
```

output
```
(3*b*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

3.207.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3b\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^2,x]
```

output
```
(3*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*
(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

3.207.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.207.4 Maple [F]

$$\int (\cos (dx + c) b)^{\frac{1}{3}} \left(\sec^2 (dx + c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*sec(d*x+c)^2,x)
```

3.207.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int (b\cos (dx + c))^{\frac{1}{3}} \sec (dx + c)^2 \,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.207.6 Sympy [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \sqrt[3]{b\cos (c + dx)} \sec^2 (c + dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c)**2,x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x)**2, x)
```

3.207.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.207.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{1/3}}{\cos(c+dx)^2}\,dx$$

input
```
int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^2, x)
```

3.208 $\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx$

3.208.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{3b^2 \operatorname{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*b^2*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{3\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\operatorname{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{5d}$$

input
```
Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^3,x]
```

output
```
(3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, C
os[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(5*d)
```

3.208.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \; 2030$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3b^2\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

input
```
Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^3,x]
```

output
```
(3*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

3.208.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.208.4 Maple [F]

$$\int (\cos (dx+c) b)^{\frac{1}{3}} \left(\sec^3 (dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(1/3)*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^(1/3)*sec(d*x+c)^3,x)
```

3.208.5 Fricas [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int (b\cos (dx+c))^{\frac{1}{3}} \sec (dx+c)^3 \, dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.208.6 Sympy [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int \sqrt[3]{b\cos (c+dx)} \sec^3 (c+dx)\, dx$$

input
```
integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c)**3,x)
```

output
```
Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x)**3, x)
```

3.208.7 Maxima [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.208.8 Giac [F]

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{1/3}}{\cos(c+dx)^3}\,dx$$

input
```
int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^3, x)
```

3.209 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx$

3.209.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx =$$
$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+}{d(5+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m],[11/6+
1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(5+3*m)/(sin(d*x+c)^2)^(1/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx =$$
$$-\frac{\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{5}{3}+m\right),\frac{1}{2}\left(\frac{11}{3}+m\right),\cos^2(c+dx)\right.}{d\left(\frac{5}{3}+m\right)}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*Hypergeometric
2F1[1/2, (5/3 + m)/2, (11/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/
(d*(5/3 + m)))
```

3.209.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\cos^m(c+dx)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \cos^{m+\frac{2}{3}}(c+dx)dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(b\cos(c+dx))^{2/3}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{2}{3}}dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+5),\frac{1}{6}(3m+11),\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5
+ 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Si
n[c + d*x]^2])
```

3.209.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.209.4 Maple [F]

$$\int \left(\cos^{m}\left(dx+c\right)\right) \left(\cos\left(dx+c\right) b\right)^{\frac{2}{3}} dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(2/3),x)
```

3.209.5 Fricas [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int \left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}} \cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.209.6 Sympy [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int \left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}} \cos^{m}\left(c+dx\right) dx$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((b*cos(c + d*x))**(2/3)*cos(c + d*x)**m, x)
```

3.209.7 Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.209.8 Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.209.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{2/3}\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3), x)
```

3.210 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx$

3.210.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx = -\frac{3(b\cos(c+dx))^{11/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/11*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*x+c)^2)*sin
(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.210.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx = -\frac{3\cos^2(c+dx)(b\cos(c+dx))^{2/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{11d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1
/2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(11*d)
```

3.210.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{8/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{8/3}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x
]^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[Sin[c + d*x]^2])
```

3.210.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.210.4 Maple [F]

$$\int \left(\cos^{2}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(2/3),x)
```

3.210.5 Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx=\int \left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\cos\left(dx+c\right)^{2}\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

3.210.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.210.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

3.210.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{2/3}\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3), x)
```

3.211 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx$

3.211.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = -\frac{3(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/8*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x
+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = -\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{8bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(8*b*d)
```

3.211.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{5/3}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b}$$

$$\Big\downarrow \; 3122$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.211.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.211.4 Maple [F]

$$\int \cos(dx+c)(\cos(dx+c)\,b)^{\frac{2}{3}}\,dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(2/3),x)
```

3.211.5 Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

3.211.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.211.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

3.211.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx = \int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{2/3}\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(2/3), x)
```

3.212 $\int (b\cos(c+dx))^{2/3}\,dx$

3.212.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (b\cos(c+dx))^{2/3}\,dx = -\frac{3(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

output
```
-3/5*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*
x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (b\cos(c+dx))^{2/3}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5d}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*d)
```

3.212.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

3.212.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.212.4 Maple [F]

$$\int (\cos (dx + c) b)^{\frac{2}{3}} dx$$

input
```
int((cos(d*x+c)*b)^(2/3),x)
```

output
```
int((cos(d*x+c)*b)^(2/3),x)
```

3.212.5 Fricas [F]

$$\int (b\cos(c + dx))^{2/3} dx = \int (b\cos (dx + c))^{\frac{2}{3}} dx$$

input
```
integrate((b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3), x)
```

3.212.6 Sympy [F]

$$\int (b\cos(c + dx))^{2/3} dx = \int (b\cos (c + dx))^{\frac{2}{3}} dx$$

input
```
integrate((b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((b*cos(c + d*x))**(2/3), x)
```

3.212.7 Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3), x)
```

3.212.8 Giac [F]

$$\int (b\cos(c+dx))^{2/3}\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\,dx = \int (b\,\cos(c+d\,x))^{2/3}\,dx$$

input
```
int((b*cos(c + d*x))^(2/3),x)
```

output
```
int((b*cos(c + d*x))^(2/3), x)
```

3.213 $\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx$

3.213.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = -\frac{3(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/2*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x
+c)/d/(sin(d*x+c)^2)^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = -\frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x],x]
```

output
```
(-3*b*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(1/3))
```

3.213.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \frac{1}{\sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x],x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2])
```

3.213.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.213.4 Maple [F]

$$\int (\cos (dx+c) b)^{\frac{2}{3}} \sec (dx+c) \, dx$$

input
```
int((cos(d*x+c)*b)^(2/3)*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^(2/3)*sec(d*x+c),x)
```

3.213.5 Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = \int (b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c) \,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

3.213.6 Sympy [F]

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = \int (b\cos (c+dx))^{\frac{2}{3}} \sec (c+dx) \,dx$$

input
```
integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**(2/3)*sec(c + d*x), x)
```

3.213.7 Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

3.213.8 Giac [F]

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx = \int \frac{(b\,\cos(c+d\,x))^{2/3}}{\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

3.214 $\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx$

3.214.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \frac{3b\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*b*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^
(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \frac{3b\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^2,x]
```

output
```
(3*b*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(d*(b*Cos[c + d*x])^(1/3))
```

3.214.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \ 2030$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \ 3122$$

$$\frac{3b\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^2,x]
```

output
```
(3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b
*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

3.214.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.214.4 Maple [F]

$$\int \left(\cos \left(d x +c \right) b \right)^{\frac{2}{3}} \left(\sec^{2}\left(d x +c \right)\right)d x$$

input
```
int((cos(d*x+c)*b)^(2/3)*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^(2/3)*sec(d*x+c)^2,x)
```

3.214.5 Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \int \left(b \cos \left(d x + c\right)\right)^{\frac{2}{3}} \sec \left(d x + c\right)^{2}\,{d x}$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.214.7 Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

3.214.8 Giac [F]

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx = \int \frac{(b\,\cos(c+d\,x))^{2/3}}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

3.215 $\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx$

3.215.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx=\frac{3b^2\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*b^2*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(4/3)/(sin(d*x+c)^2)^(1/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx=\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{4d}$$

input
```
Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^3,x]
```

output
```
(3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, C
os[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

3.215.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{2/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b^2\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

input
```
Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^3,x]
```

output
```
(3*b^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*
d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

3.215.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.215.4 Maple [F]

$$\int (\cos (dx+c) b)^{\frac{2}{3}} \left(\sec^3 (dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(2/3)*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^(2/3)*sec(d*x+c)^3,x)
```

3.215.5 Fricas [F]

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx = \int (b\cos (dx+c))^{\frac{2}{3}} \sec (dx+c)^3 \,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.215.7 Maxima [F]

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

3.215.8 Giac [F]

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

3.215.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{2/3}}{\cos(c+dx)^3}\,dx$$

input
```
int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

3.216 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx$

3.216.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx = -\frac{3b\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

output
```
-3*b*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/
6+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx = -\frac{\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{7}{3}+m\right),\frac{1}{2}\left(\frac{13}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{7}{3}+m\right)}$$

input
```
Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*Hypergeometric
2F1[1/2, (7/3 + m)/2, (13/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/
(d*(7/3 + m)))
```

3.216.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\cos^m(c+dx)\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \cos^{m+\frac{4}{3}}(c+dx)dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sqrt[3]{b\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m+\frac{4}{3}}dx}{\sqrt[3]{\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+7),\frac{1}{6}(3m+13),\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (
7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[
Sin[c + d*x]^2])
```

3.216.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.216.4 Maple [F]

$$\int (\cos^m (dx + c)) (\cos (dx + c) b)^{\frac{4}{3}} dx$$

input
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)^m*(cos(d*x+c)*b)^(4/3),x)
```

3.216.5 Fricas [F]

$$\int \cos^m(c + dx)(b\cos(c + dx))^{4/3} dx = \int (b\cos (dx + c))^{\frac{4}{3}} \cos (dx + c)^m dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^m*cos(d*x + c), x)
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \cos^m(c + dx)(b\cos(c + dx))^{4/3} dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.216.7 Maxima [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

3.216.8 Giac [F]

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int \cos(c+d\,x)^m\,(b\,\cos(c+d\,x))^{4/3}\,dx$$

input
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3), x)
```

3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx$

3.217.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/13*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin
(d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx = -\frac{3\cos^2(c+dx)(b\cos(c+dx))^{4/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{13d}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1
/2, 13/6, 19/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(13*d)
```

3.217.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{10/3}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{10/3}dx}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{13/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x
]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])
```

3.217.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.217.4 Maple [F]

$$\int \left(\cos^{2}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^(4/3),x)
```

3.217.5 Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx=\int \left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}\cos\left(dx+c\right)^{2}\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^3, x)
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.217.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx=\int (b\cos{(dx+c)})^{\frac{4}{3}}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.217.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx=\int (b\cos{(dx+c)})^{\frac{4}{3}}\cos{(dx+c)}^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx=\int \cos{(c+d\,x)}^2\,(b\,\cos{(c+d\,x)})^{4/3}\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3), x)
```

3.218 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx$

3.218.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/10*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d
*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{10bd}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(10*b*d)
```

3.218.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{7/3}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{7/3}dx}{b}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.218.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.218.4 Maple [F]

$$\int \cos(dx+c)\,(\cos(dx+c)\,b)^{\frac{4}{3}}\,dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^(4/3),x)
```

3.218.5 Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^2, x)
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.218.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.218.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx = \int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{4/3}\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^(4/3), x)
```

3.219 $\int (b\cos(c+dx))^{4/3}\,dx$

3.219.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

output
```
-3/7*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*
x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*d)
```

3.219.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

3.219.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.219.4 Maple [F]

$$\int (\cos (dx + c) b)^{\frac{4}{3}} dx$$

input
```
int((cos(d*x+c)*b)^(4/3),x)
```

output
```
int((cos(d*x+c)*b)^(4/3),x)
```

3.219.5 Fricas [F]

$$\int (b\cos(c + dx))^{4/3} dx = \int (b\cos (dx + c))^{\frac{4}{3}} dx$$

input
```
integrate((b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c), x)
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c + dx))^{4/3} dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.219.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3), x)
```

3.219.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\,dx = \int (b\,\cos(c+d\,x))^{4/3}\,dx$$

input
```
int((b*cos(c + d*x))^(4/3),x)
```

output
```
int((b*cos(c + d*x))^(4/3), x)
```

3.220 $\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx$

3.220.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx = -\frac{3(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x
+c)/d/(sin(d*x+c)^2)^(1/2)
```

3.220.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx = -\frac{3b\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x],x]
```

output
```
(-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3,
 Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*d)
```

3.220.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \sqrt[3]{b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x],x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

3.220.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.220.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{4}{3}} \sec (dx+c)\, dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*sec(d*x+c),x)
```

3.220.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\, dx = \int (b\cos (dx+c))^{\frac{4}{3}} \sec (dx+c)\, dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c), x)
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\, dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c),x)
```

output
```
Timed out
```

3.220.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx = \int (b\cos{(dx+c)})^{\frac{4}{3}}\sec{(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.220.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx = \int (b\cos{(dx+c)})^{\frac{4}{3}}\sec{(dx+c)}\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx = \int \frac{(b\,\cos{(c+d\,x)})^{4/3}}{\cos{(c+d\,x)}}\,dx$$

input
```
int((b*cos(c + d*x))^(4/3)/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

3.221 $\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx$

3.221.1 Optimal result

Integrand size = 21, antiderivative size = 54

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = -\frac{3b\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

output
```
-3*b*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x
+c)/d/(sin(d*x+c)^2)^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = -\frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^2,x]
```

output
```
(-3*b^2*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

3.221.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^2,x]
```

output
```
(-3*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

3.221.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.221.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{4}{3}} \left(\sec ^2(dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*sec(d*x+c)^2,x)
```

3.221.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c)^2, x)
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c)**2,x)
```

output
```
Timed out
```

3.221.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.221.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.221.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{4/3}}{\cos(c+dx)^2}\,dx$$

input
```
int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^2, x)
```

3.222 $\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx$

3.222.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx=\frac{3b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*b^2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.222.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx=\frac{3b^2\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^3,x]
```

output
```
(3*b^2*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

3.222.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b^2\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

input
```
Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^3,x]
```

output
```
(3*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*
d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

3.222.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.222.4 Maple [F]

$$\int (\cos (dx+c)\, b)^{\frac{4}{3}} \left(\sec^{3}(dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^(4/3)*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^(4/3)*sec(d*x+c)^3,x)
```

3.222.5 Fricas [F]

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c)^3, x)
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c)**3,x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

3.222.8 Giac [F]

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx = \int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx = \int \frac{(b\cos(c+dx))^{4/3}}{\cos(c+dx)^3}\,dx$$

input
```
int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^3, x)
```

3.223 $\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.223.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*s
in(d*x+c)/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{\cos^{1+m}(c+dx)\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{3}+m\right),\frac{1}{2}\left(\frac{8}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{2}{3}+m\right)\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(1/3),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (
8/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(2/3 + m)*(b*Cos[c +
d*x])^(1/3)))
```

3.223.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{1}{3}}(c+dx)dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{1}{3}}dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+2),\frac{1}{6}(3m+8),\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

3.223.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.223.4 Maple [F]

$$\int \frac{\cos^{m}(dx+c)}{(\cos(dx+c)\, b)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^m/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)^m/(cos(d*x+c)*b)^(1/3),x)
```

3.223.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\, dx = \int \frac{\cos(dx+c)^{m}}{(b\cos(dx+c))^{\frac{1}{3}}}\, dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)
```

3.223.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos^m{(c+dx)}}{\sqrt[3]{b\cos{(c+dx)}}}\,dx$$

input
```
integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

3.223.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^m}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

3.223.8 Giac [F]

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^m}{(b\cos{(dx+c)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^m}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int(cos(c + d*x)^m/(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^m/(b*cos(c + d*x))^(1/3), x)
```

3.224 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.224.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/8*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x
+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.224.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cos^2(c+dx)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{8d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d
*x]^2]*Sqrt[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

3.224.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{5/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{5/3}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])
```

3.224.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.224.4 Maple [F]

$$\int \frac{\cos^2(dx+c)}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

3.224.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)/b, x)
```

3.224.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)**2/(b*cos(c + d*x))**(1/3), x)
```

3.224.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

3.224.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^2}{(b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(1/3), x)
```

3.225 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.225.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/5*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*
x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5bd}$$

input
```
Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*b*d)
```

3.225.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (b\cos(c+dx))^{2/3}dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b}$$

$$\Big\downarrow \; 3122$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.225.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.225.4 Maple [F]

$$\int \frac{\cos \left(d x +c \right)}{\left(\cos \left(d x +c \right) b \right)^{\frac{1}{3}}}d x$$

input
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

3.225.5 Fricas [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos \left(dx+c\right)}{\left(b\cos \left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)/b, x)
```

3.225.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos \left(c+dx\right)}{\sqrt[3]{b\cos \left(c+dx\right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral(cos(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

3.225.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

3.225.8 Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

input
```
int(cos(c + d*x)/(b*cos(c + d*x))^(1/3),x)
```

output
```
int(cos(c + d*x)/(b*cos(c + d*x))^(1/3), x)
```

3.226 $\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.226.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

output
```
-3/2*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^(-1/3),x]
```

output
```
(-3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Sin
[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(1/3))
```

3.226.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(-1/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

3.226.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.226.4 Maple [F]

$$\int \frac{1}{\left(\cos \left(dx+c\right) b\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(1/(cos(d*x+c)*b)^(1/3),x)
```

3.226.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)
```

3.226.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral((b*cos(c + d*x))**(-1/3), x)
```

3.226.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{1}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(-1/3), x)
```

3.226.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{1}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(-1/3), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{1}{(b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int(1/(b*cos(c + d*x))^(1/3),x)
```

output
```
int(1/(b*cos(c + d*x))^(1/3), x)
```

3.227 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.227.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1
/3)/(sin(d*x+c)^2)^(1/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*b*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(d*(b*Cos[c + d*x])^(4/3))
```

3.227.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[Sec[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*C
os[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

3.227.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.227.4 Maple [F]

$$\int \frac{\sec \left(d x +c \right)}{\left(\cos \left(d x +c \right) b \right)^{\frac{1}{3}}}d x$$

input
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(1/3),x)
```

3.227.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(d*x + c)), x)
```

3.227.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

3.227.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

3.227.8 Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)\ (b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)), x)
```

3.228 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.228.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3b\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*b*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c)
)^(4/3)/(sin(d*x+c)^2)^(1/2)
```

3.228.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{7/3}}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*b^2*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

3.228.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3b\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

input
```
Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*b*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*
(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

3.228.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.228.4 Maple [F]

$$\int \frac{\sec^{2}\left(dx+c\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(1/3),x)
```

3.228.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\sec\left(dx+c\right)^{2}}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

3.228.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx=\int \frac{\sec^{2}\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(1/3), x)
```

3.228.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

3.228.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)), x)
```

3.229 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

3.229.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3b^2\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*b^2*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x
+c))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

3.229.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{3b^2\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{7/3}}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*b^2*Csc[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sqr
t[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(7/3))
```

3.229.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b^2\sin(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

input
```
Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(1/3),x]
```

output
```
(3*b^2*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7
*d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

3.229.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.229.4 Maple [F]

$$\int \frac{\sec^3(dx+c)}{(\cos(dx+c)\,b)^{\frac{1}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(1/3),x)
```

output
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(1/3),x)
```

3.229.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

3.229.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)
```

output
```
Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(1/3), x)
```

3.229.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

3.229.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(b\,\cos(c+d\,x))^{1/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)), x)
```

3.230 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.230.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cos^{1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
-3*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c)^2)*s
in(d*x+c)/d/(1+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.230.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{\cos^{1+m}(c+dx)\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{3}+m\right),\frac{1}{2}\left(\frac{7}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{1}{3}+m\right)(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(2/3),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (
7/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1/3 + m)*(b*Cos[c +
d*x])^(2/3)))
```

3.230.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{m-\frac{2}{3}}(c+dx)dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{2}{3}}(c+dx)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{2}{3}}dx}{(b\cos(c+dx))^{2/3}}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m+1),\frac{1}{6}(3m+7),\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

input
```
Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin
[c + d*x]^2])
```

3.230.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.230.4 Maple [F]

$$\int \frac{\cos^{m}\left(dx+c\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^m/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)^m/(cos(d*x+c)*b)^(2/3),x)
```

3.230.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos\left(dx+c\right)^{m}}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)
```

3.230.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos^{m}\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)
```

3.230.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

3.230.8 Giac [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(c+d\,x)^m}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int(cos(c + d*x)^m/(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^m/(b*cos(c + d*x))^(2/3), x)
```

3.231 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.231.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3(b\cos(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/7*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*
x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cos^2(c+dx)\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c +
d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

3.231.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{4/3}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}dx}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2])
```

3.231.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.231.4 Maple [F]

$$\int \frac{\cos^2(dx+c)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

3.231.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)/b, x)
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

output
```
Timed out
```

3.231.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.231.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(c+d\,x)^2}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(2/3), x)
```

3.232 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.232.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/4*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x
+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.232.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4bd}$$

input
```
Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*b*d)
```

3.232.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int \sqrt[3]{b\cos(c+dx)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.232.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.232.4 Maple [F]

$$\int \frac{\cos \left(d x +c \right)}{\left(\cos \left(d x +c \right) b \right)^{\frac{2}{3}}}d x$$

input
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

3.232.5 Fricas [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)/b, x)
```

3.232.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx=\int \frac{\cos\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral(cos(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

3.232.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.232.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\cos(c+d\,x)}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int(cos(c + d*x)/(b*cos(c + d*x))^(2/3),x)
```

output
```
int(cos(c + d*x)/(b*cos(c + d*x))^(2/3), x)
```

3.233 $\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$

3.233.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

output
```
-3*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c
)/b/d/(sin(d*x+c)^2)^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{2/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(-2/3),x]
```

output
```
(-3*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[Sin
[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

3.233.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(-2/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2])
```

3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.233.4 Maple [F]

$$\int \frac{1}{\left(\cos \left(d x +c \right) b \right)^{\frac{2}{3}}}d x$$

input
```
int(1/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(1/(cos(d*x+c)*b)^(2/3),x)
```

3.233.5 Fricas [F]

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b \cos \left(d x + c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

3.233.6 Sympy [F]

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b \cos{\left(c + d x \right)}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral((b*cos(c + d*x))**(-2/3), x)
```

3.233.7 Maxima [F]

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(-2/3), x)
```

3.233.8 Giac [F]

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(-2/3), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\,\cos(c+d\,x))^{2/3}}\,dx$$

input
```
int(1/(b*cos(c + d*x))^(2/3),x)
```

output
```
int(1/(b*cos(c + d*x))^(2/3), x)
```

3.234 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.234.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^
(2/3)/(sin(d*x+c)^2)^(1/2)
```

3.234.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d(b\cos(c+dx))^{5/3}}$$

input
```
Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(5/3))
```

3.234.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \; 2030$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{5/3}}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

input
```
Int[Sec[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b
*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

3.234.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.234.4 Maple [F]

$$\int \frac{\sec (dx+c)}{(\cos (dx+c)\, b)^{\frac{2}{3}}}dx$$

input
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(2/3),x)
```

3.234.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\, dx = \int \frac{\sec (dx+c)}{(b\cos (dx+c))^{\frac{2}{3}}}\, dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*x + c)), x)
```

3.234.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\, dx = \int \frac{\sec (c+dx)}{(b\cos (c+dx))^{\frac{2}{3}}}\, dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral(sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

3.234.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.234.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

3.235 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.235.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/5*b*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c)
)^(5/3)/(sin(d*x+c)^2)^(1/2)
```

3.235.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5d(b\cos(c+dx))^{8/3}}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b^2*Cot[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(8/3))
```

3.235.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{8/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3b\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

input
```
Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*
(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

3.235.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.235.4 Maple [F]

$$\int \frac{\sec^2(dx+c)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(2/3),x)
```

3.235.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

3.235.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x)
```

3.235.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.235.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^2\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

3.236 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

3.236.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b^2\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/8*b^2*hypergeom([-4/3, 1/2],[-1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x
+c))^(8/3)/(sin(d*x+c)^2)^(1/2)
```

3.236.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{3b^2\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{8d(b\cos(c+dx))^{8/3}}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b^2*Csc[c + d*x]*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sqr
t[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(8/3))
```

3.236.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{11/3}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3b^2\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{8/3}}$$

input
```
Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(2/3),x]
```

output
```
(3*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8
*d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

3.236.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.236.4 Maple [F]

$$\int \frac{\sec^3(dx+c)}{(\cos(dx+c)\,b)^{\frac{2}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3),x)
```

output
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(2/3),x)
```

3.236.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

3.236.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

output
```
Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(2/3), x)
```

3.236.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

3.236.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{\sec\left(dx+c\right)^3}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^3\,\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

3.237 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.237.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)\sin}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2)*sin(d
*x+c)/b/d/(1-3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{\cos^{1+m}(c+dx)\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{3}+m\right),\frac{1}{2}\left(\frac{5}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx}}{d\left(-\frac{1}{3}+m\right)(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(4/3),x]
```

output
```
-((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2,
(5/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1/3 + m)*(b*Cos[c
+ d*x])^(4/3)))
```

3.237.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{2034}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \cos^{m-\frac{4}{3}}(c+dx)dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{\cos(c+dx)}\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-\frac{4}{3}}dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\sin(c+dx)\cos^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(3m-1),\frac{1}{6}(3m+5),\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c
+ d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
 d*x]^2])
```

3.237.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.237.4 Maple [F]

$$\int \frac{\cos^{m}\left(dx+c\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^m/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)^m/(cos(d*x+c)*b)^(4/3),x)
```

3.237.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(dx+c\right)^{m}}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)
```

3.237.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos^{m}\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x)
```

3.237.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

3.237.8 Giac [F]

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(c+d\,x)^m}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int(cos(c + d*x)^m/(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^m/(b*cos(c + d*x))^(4/3), x)
```

3.238 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.238.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3(b\cos(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/5*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*
x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

3.238.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5b^2d}$$

input
```
Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6,
Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*b^2*d)
```

3.238.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{2/3}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{2/3}dx}{b^2}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

3.238.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.238.4 Maple [F]

$$\int \frac{\cos^2(dx+c)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

3.238.5 Fricas [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)/b^2, x)
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.238.7 Maxima [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.238.8 Giac [F]

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(c+d\,x)^2}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)^2/(b*cos(c + d*x))^(4/3), x)
```

3.239 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.239.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

output
```
-3/2*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x
+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = -\frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2bd\sqrt[3]{b\cos(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Sin
[c + d*x]^2])/(2*b*d*(b*Cos[c + d*x])^(1/3))
```

3.239.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt[3]{b\sin\left(c+dx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2])
```

3.239.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.239.4 Maple [F]

$$\int \frac{\cos(dx+c)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(cos(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

3.239.5 Fricas [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Timed out
```

3.239.7 Maxima [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.239.8 Giac [F]

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

input
```
int(cos(c + d*x)/(b*cos(c + d*x))^(4/3),x)
```

output
```
int(cos(c + d*x)/(b*cos(c + d*x))^(4/3), x)
```

3.240 $\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$

3.240.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
3*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^
(1/3)/(sin(d*x+c)^2)^(1/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{4/3}}$$

input
```
Integrate[(b*Cos[c + d*x])^(-4/3),x]
```

output
```
(3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Sin
[c + d*x]^2])/(d*(b*Cos[c + d*x])^(4/3))
```

3.240.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$
$$\Big\downarrow \text{3122}$$
$$\frac{3\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^(-4/3),x]
```

output
```
(3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b
*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

3.240.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.240.4 Maple [F]

$$\int \frac{1}{\left(\cos \left(d x +c \right) b \right)^{\frac{4}{3}}}d x$$

input
```
int(1/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(1/(cos(d*x+c)*b)^(4/3),x)
```

3.240.5 Fricas [F]

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2), x)
```

3.240.6 Sympy [F]

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(b\cos\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral((b*cos(c + d*x))**(-4/3), x)
```

3.240.7 Maxima [F]

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^(-4/3), x)
```

3.240.8 Giac [F]

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^(-4/3), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int(1/(b*cos(c + d*x))^(4/3),x)
```

output
```
int(1/(b*cos(c + d*x))^(4/3), x)
```

3.241 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.241.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/4*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^
(4/3)/(sin(d*x+c)^2)^(1/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{7/3}}$$

input
```
Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sqrt[S
in[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

3.241.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \text{ 2030}$$

$$b\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{7/3}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

input
```
Int[Sec[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b
*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

3.241.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.241.4 Maple [F]

$$\int \frac{\sec (dx+c)}{(\cos (dx+c)\, b)^{\frac{4}{3}}}dx$$

input
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(sec(d*x+c)/(cos(d*x+c)*b)^(4/3),x)
```

3.241.5 Fricas [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\, dx = \int \frac{\sec (dx+c)}{(b\cos (dx+c))^{\frac{4}{3}}}\, dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

3.241.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\, dx = \int \frac{\sec (c+dx)}{(b\cos (c+dx))^{\frac{4}{3}}}\, dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)/(b*cos(c + d*x))**(4/3), x)
```

3.241.7 Maxima [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.241.8 Giac [F]

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

3.242 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.242.1 Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/7*b*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{10/3}}$$

input
```
Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b^2*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sqr
t[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))
```

3.242.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^2\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$
$$\Big\downarrow \text{ 2030}$$
$$b^2\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{10/3}}dx$$
$$\Big\downarrow \text{ 3122}$$
$$\frac{3b\sin(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

input
```
Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d
*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

3.242.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.242.4 Maple [F]

$$\int \frac{\sec^2(dx+c)}{(\cos(dx+c)\,b)^{\frac{4}{3}}}dx$$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(sec(d*x+c)^2/(cos(d*x+c)*b)^(4/3),x)
```

3.242.5 Fricas [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

3.242.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(4/3), x)
```

3.242.7 Maxima [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.242.8 Giac [F]

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

3.243 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

3.243.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2 \text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}$$

output
```
3/10*b^2*hypergeom([-5/3, 1/2],[-2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*
x+c))^(10/3)/(sin(d*x+c)^2)^(1/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3b^2\csc(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{10d(b\cos(c+dx))^{10/3}}$$

input
```
Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b^2*Csc[c + d*x]*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sqr
t[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(10/3))
```

3.243.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sin\left(c+dx+\frac{\pi}{2}\right)^3\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{4/3}}dx$$

$$\Big\downarrow \; 2030$$

$$b^3\int \frac{1}{\left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{13/3}}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3b^2\sin(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{10/3}}$$

input
```
Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(4/3),x]
```

output
```
(3*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(1
0*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

3.243.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.243.4 Maple [F]

$$\int \frac{\sec^{3}\left(dx+c\right)}{\left(\cos\left(dx+c\right)b\right)^{\frac{4}{3}}}dx$$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3),x)
```

output
```
int(sec(d*x+c)^3/(cos(d*x+c)*b)^(4/3),x)
```

3.243.5 Fricas [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\sec\left(dx+c\right)^{3}}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)
```

3.243.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx=\int \frac{\sec^{3}\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

output
```
Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(4/3), x)
```

3.243.7 Maxima [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.243.8 Giac [F]

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

input
```
integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

output
```
integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(b\,\cos(c+d\,x))^{4/3}}\,dx$$

input
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
```

output
```
int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

3.244 $\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx$

3.244.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = -\frac{(a\cos(e+fx))^{1+m}(b\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\cos^2(e+fx)\right)}{af(1+m+n)\sqrt{\sin^2(e+fx)}}$$

output
```
-(a*cos(f*x+e))^(1+m)*(b*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3
/2+1/2*m+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(1+m+n)/(sin(f*x+e)^2)^(1/2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = -\frac{(a\cos(e+fx))^m(b\cos(e+fx))^n\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\cos^2\right.}{f(1+m+n)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n,x]
```

output
```
-(((a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n*Cot[e + f*x]*Hypergeometric2F1[1/
2, (1 + m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*
(1 + m + n)))
```

3.244.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$(a\cos(e+fx))^{-n}(b\cos(e+fx))^n\int (a\cos(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cos(e+fx))^{-n}(b\cos(e+fx))^n\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m+n}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\cos(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\cos^2(e+f$$
$$af(m+n+1)\sqrt{\sin^2(e+fx)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n,x]
```

output
```
-(((a*Cos[e + f*x])^(1 + m)*(b*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 +
 m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m + n)*S
qrt[Sin[e + f*x]^2]))
```

3.244.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.244.4 Maple [F]

$$\int (\cos (fx+e) a)^m (b\cos (fx+e))^n\,dx$$

input
```
int((cos(f*x+e)*a)^m*(b*cos(f*x+e))^n,x)
```

output
```
int((cos(f*x+e)*a)^m*(b*cos(f*x+e))^n,x)
```

3.244.5 Fricas [F]

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = \int (a\cos (fx+e))^m (b\cos (fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

3.244.6 Sympy [F]

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = \int (a\cos (e+fx))^m (b\cos (e+fx))^n\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(b*cos(f*x+e))**n,x)
```

output
```
Integral((a*cos(e + f*x))**m*(b*cos(e + f*x))**n, x)
```

3.244.7 Maxima [F]

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = \int (a\cos(fx+e))^m\,(b\cos(fx+e))^n\;dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

3.244.8 Giac [F]

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = \int (a\cos(fx+e))^m\,(b\cos(fx+e))^n\;dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx = \int (a\,\cos(e+f\,x))^m\,(b\,\cos(e+f\,x))^n\,dx$$

input
```
int((a*cos(e + f*x))^m*(b*cos(e + f*x))^n,x)
```

output
```
int((a*cos(e + f*x))^m*(b*cos(e + f*x))^n, x)
```

3.245 $\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx$

3.245.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = -\frac{(b\cos(c+dx))^{3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)
```

3.245.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = -\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(3+n)}$$

input
```
Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n,x]
```

output
```
-((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (
3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(3 + n)))
```

3.245.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 2030}$$

$$\frac{\int (b\cos(c+dx))^{n+2}dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+2}dx}{b^2}$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+3}{2},\frac{n+5}{2},\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n,x]
```

output
```
-(((b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))
```

3.245.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.245.4 Maple [F]

$$\int \left(\cos^2(dx+c)\right)(\cos(dx+c)\,b)^n\,dx$$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^n,x)
```

output
```
int(cos(d*x+c)^2*(cos(d*x+c)*b)^n,x)
```

3.245.5 Fricas [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.245.6 Sympy [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(c+dx))^n\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n,x)
```

output
```
Integral((b*cos(c + d*x))**n*cos(c + d*x)**2, x)
```

3.245.7 Maxima [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.245.8 Giac [F]

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

input
```
integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx = \int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^2*(b*cos(c + d*x))^n, x)
```

3.246 $\int \cos(c+dx)(b\cos(c+dx))^n\,dx$

3.246.1 Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = -\frac{(b\cos(c+dx))^{2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin
(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)
```

3.246.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = -\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(2+n)}$$

input
```
Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n,x]
```

output
```
-((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (2
+ n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(2 + n)))
```

3.246.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2030, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (b\cos(c+dx))^{n+1}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^{n+1}dx}{b}$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+2}{2},\frac{n+4}{2},\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]*(b*Cos[c + d*x])^n,x]
```

output
```
-(((b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2]))
```

3.246.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.246.4 Maple [F]

$$\int \cos(dx+c)\left(\cos(dx+c)\,b\right)^n dx$$

input
```
int(cos(d*x+c)*(cos(d*x+c)*b)^n,x)
```

output
```
int(cos(d*x+c)*(cos(d*x+c)*b)^n,x)
```

3.246.5 Fricas [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.246.6 Sympy [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(c+dx))^n\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))**n,x)
```

output
```
Integral((b*cos(c + d*x))**n*cos(c + d*x), x)
```

3.246.7 Maxima [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n \cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.246.8 Giac [F]

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n \cos(dx+c)\,dx$$

input
```
integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(b\cos(c+dx))^n\,dx = \int \cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^n\,dx$$

input
```
int(cos(c + d*x)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)*(b*cos(c + d*x))^n, x)
```

3.247 $\int (b\cos(c+dx))^n\,dx$

3.247.1 Optimal result

Integrand size = 10, antiderivative size = 69

$$\int (b\cos(c+dx))^n\,dx = -\frac{(b\cos(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

output
```
-(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)
*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

3.247.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int (b\cos(c+dx))^n\,dx = -\frac{(b\cos(c+dx))^n\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(1+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n,x]
```

output
```
-(((b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + n)))
```

3.247.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n,x]
```

output
```
-(((b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

3.247.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.247.4 Maple [F]

$$\int (\cos (dx + c) b)^n dx$$

input
```
int((cos(d*x+c)*b)^n,x)
```

output
```
int((cos(d*x+c)*b)^n,x)
```

3.247.5 Fricas [F]

$$\int (b\cos(c + dx))^n dx = \int (b\cos (dx + c))^n dx$$

input
```
integrate((b*cos(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n, x)
```

3.247.6 Sympy [F]

$$\int (b\cos(c + dx))^n dx = \int (b\cos (c + dx))^n dx$$

input
```
integrate((b*cos(d*x+c))**n,x)
```

output
```
Integral((b*cos(c + d*x))**n, x)
```

3.247.7 Maxima [F]

$$\int (b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n, x)
```

3.247.8 Giac [F]

$$\int (b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\,dx$$

input
```
integrate((b*cos(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n, x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n\,dx = \int (b\cos(c+d\,x))^n\,dx$$

input
```
int((b*cos(c + d*x))^n,x)
```

output
```
int((b*cos(c + d*x))^n, x)
```

3.248 $\int (b\cos(c+dx))^n \sec(c+dx)\,dx$

3.248.1 Optimal result

Integrand size = 17, antiderivative size = 60

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = -\frac{(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

output
```
-(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c
)/d/n/(sin(d*x+c)^2)^(1/2)
```

3.248.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = -\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{dn}$$

input
```
Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x],x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*Hypergeometric2F1[1/2, n/2, (2
 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*n))
```

3.248.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{2030}$$

$$b\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-1}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{n+2}{2},\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*Sec[c + d*x],x]
```

output
```
-(((b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]))
```

3.248.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.248.4 Maple [F]

$$\int (\cos (dx+c) b)^{n} \sec (dx+c) dx$$

input
```
int((cos(d*x+c)*b)^n*sec(d*x+c),x)
```

output
```
int((cos(d*x+c)*b)^n*sec(d*x+c),x)
```

3.248.5 Fricas [F]

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)\ dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.248.6 Sympy [F]

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = \int (b\cos(c+dx))^n \sec(c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**n*sec(d*x+c),x)
```

output
```
Integral((b*cos(c + d*x))**n*sec(c + d*x), x)
```

3.248.7 Maxima [F]

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.248.8 Giac [F]

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \sec(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^n}{\cos(c+d\,x)}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x),x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x), x)
```

3.249 $\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx$

3.249.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx = \frac{b(b\cos(c+dx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

output
```
b*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)
^2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)
```

3.249.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx = -\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(-1+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^2,x]
```

output
```
-((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1 + n
)/2, (1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n)))
```

3.249.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$b^2\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-2}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{b\sin(c+dx)(b\cos(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-1}{2},\frac{n+1}{2},\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^2,x]
```

output
```
(b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

3.249.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.249.4 Maple [F]

$$\int (\cos(dx+c)\, b)^n \left(\sec^2(dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^n*sec(d*x+c)^2,x)
```

output
```
int((cos(d*x+c)*b)^n*sec(d*x+c)^2,x)
```

3.249.5 Fricas [F]

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\, dx = \int (b\cos(dx+c))^n \sec(dx+c)^2\, dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.249.6 Sympy [F]

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\, dx = \int (b\cos(c+dx))^n \sec^2(c+dx)\, dx$$

input
```
integrate((b*cos(d*x+c))**n*sec(d*x+c)**2,x)
```

output
```
Integral((b*cos(c + d*x))**n*sec(c + d*x)**2, x)
```

3.249.7 Maxima [F]

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.249.8 Giac [F]

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)^2\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.249.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^n}{\cos(c+d\,x)^2}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^2,x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^2, x)
```

3.250 $\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx$

3.250.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx = \frac{b^2(b\cos(c+dx))^{-2+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

output
```
b^2*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)
```

3.250.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sec^2(c+dx)\sqrt{\sin^2(c+}}{d(-2+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^3,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2,
 Cos[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 + n)))
```

3.250.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$b^3\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-3}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{b^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-2}{2},\frac{n}{2},\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^3,x]
```

output
```
(b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

3.250.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.250.4 Maple [F]

$$\int (\cos (dx+c)\, b)^n \left(\sec^3 (dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^n*sec(d*x+c)^3,x)
```

output
```
int((cos(d*x+c)*b)^n*sec(d*x+c)^3,x)
```

3.250.5 Fricas [F]

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\, dx = \int (b\cos (dx+c))^n \sec (dx+c)^3\, dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.250.6 Sympy [F]

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\, dx = \int (b\cos (c+dx))^n \sec^3 (c+dx)\, dx$$

input
```
integrate((b*cos(d*x+c))**n*sec(d*x+c)**3,x)
```

output
```
Integral((b*cos(c + d*x))**n*sec(c + d*x)**3, x)
```

3.250.7 Maxima [F]

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.250.8 Giac [F]

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)^3\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.250.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx = \int \frac{(b\cos(c+dx))^n}{\cos(c+dx)^3}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^3,x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^3, x)
```

3.251 $\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx$

3.251.1 Optimal result

Integrand size = 19, antiderivative size = 70

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = \frac{b^3(b\cos(c+dx))^{-3+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

output
```
b^3*(b*cos(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x
+c)^2)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)
```

3.251.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\sec^3(c+dx)\sqrt{\ }}{d(-3+n)}$$

input
```
Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^4,x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-3 + n)/2, (-1
+ n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(d*(-3 + n)))
```

3.251.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 2030, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^n}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$b^4\int \left(b\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^{n-4}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{b^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n-3}{2},\frac{n-1}{2},\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^4,x]
```

output
```
(b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)
/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])
```

3.251.3.1 Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.251.4 Maple [F]

$$\int (\cos (dx+c) b)^n \left(\sec^4 (dx+c)\right) dx$$

input
```
int((cos(d*x+c)*b)^n*sec(d*x+c)^4,x)
```

output
```
int((cos(d*x+c)*b)^n*sec(d*x+c)^4,x)
```

3.251.5 Fricas [F]

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = \int (b\cos (dx+c))^n \sec (dx+c)^4 \,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.251.6 Sympy [F]

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = \int (b\cos (c+dx))^n \sec^4 (c+dx)\,dx$$

input
```
integrate((b*cos(d*x+c))**n*sec(d*x+c)**4,x)
```

output
```
Integral((b*cos(c + d*x))**n*sec(c + d*x)**4, x)
```

3.251.7 Maxima [F]

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.251.8 Giac [F]

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = \int (b\cos(dx+c))^n \sec(dx+c)^4\,dx$$

input
```
integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.251.9 Mupad [F(-1)]

Timed out.

$$\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx = \int \frac{(b\cos(c+d\,x))^n}{\cos(c+d\,x)^4}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^4,x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^4, x)
```

3.252 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx$

3.252.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = -\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*
n],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{7}{2}+n\right),\frac{1}{2}\left(\frac{11}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{7}{2}+n\right)}$$

input
```
Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n,x]
```

output
```
-((Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/
2, (7/2 + n)/2, (11/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(7/
2 + n)))
```

3.252.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{5}{2}}dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+7),\frac{1}{4}(2n+11),\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n,x]
```

output
```
(-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)
/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c +
d*x]^2])
```

3.252.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.252.4 Maple [F]

$$\int \left(\cos^{\frac{5}{2}}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{n}dx$$

input
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n,x)
```

output
```
int(cos(d*x+c)^(5/2)*(cos(d*x+c)*b)^n,x)
```

3.252.5 Fricas [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx=\int \left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx=\text{Timed out}$$

input
```
integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n,x)
```

output
```
Timed out
```

3.252.7 Maxima [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n \cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.252.8 Giac [F]

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n \cos(dx+c)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.252.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \int \cos(c+d\,x)^{5/2}\,(b\,\cos(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n, x)
```

3.253 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx$

3.253.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.253.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{5}{2}+n\right),\frac{1}{2}\left(\frac{9}{2}+n\right),\cos^2(c+dx)\right)\sqrt{}}{d\left(\frac{5}{2}+n\right)}$$

input
```
Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n,x]
```

output
```
-((Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/
2, (5/2 + n)/2, (9/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(5/2
 + n)))
```

3.253.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \ 2034$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \ 3042$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{3}{2}}dx$$

$$\Big\downarrow \ 3122$$

$$-\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+5),\frac{1}{4}(2n+9),\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n,x]
```

output
```
(-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)
/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d
*x]^2])
```

3.253.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.253.4 Maple [F]

$$\int \left(\cos^{\frac{3}{2}}\left(dx+c\right)\right)\left(\cos\left(dx+c\right)b\right)^{n}dx$$

input
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n,x)
```

output
```
int(cos(d*x+c)^(3/2)*(cos(d*x+c)*b)^n,x)
```

3.253.5 Fricas [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{n}\,dx=\int \left(b\cos\left(dx+c\right)\right)^{n}\cos\left(dx+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.253.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{n}\,dx=\int \left(b\cos\left(c+dx\right)\right)^{n}\cos^{\frac{3}{2}}\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n,x)
```

output
```
Integral((b*cos(c + d*x))**n*cos(c + d*x)**(3/2), x)
```

3.253.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx=\int (b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.253.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx=\int (b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx=\int \cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n, x)
```

3.254 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx$

3.254.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = -\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{3}{2}+n\right),\frac{1}{2}\left(\frac{7}{2}+n\right),\cos^2(c+dx)\right)\sqrt{}}{d\left(\frac{3}{2}+n\right)}$$

input
```
Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n,x]
```

output
```
-((Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/
2, (3/2 + n)/2, (7/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(3/2
 + n))
```

3.254.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n+\frac{1}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n+\frac{1}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+3),\frac{1}{4}(2n+7),\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n,x]
```

output
```
(-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)
/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2])
```

3.254.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

3.254.4 Maple [F]

$$\int \left(\sqrt{\cos}(dx+c)\right)\left(\cos\left(dx+c\right)b\right)^{n}dx$$

input
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^n,x)
```

output
```
int(cos(d*x+c)^(1/2)*(cos(d*x+c)*b)^n,x)
```

3.254.5 Fricas [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = \int \left(b\cos\left(dx+c\right)\right)^{n}\sqrt{\cos\left(dx+c\right)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.254.6 Sympy [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = \int \left(b\cos\left(c+dx\right)\right)^{n}\sqrt{\cos\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**n,x)
```

output
```
Integral((b*cos(c + d*x))**n*sqrt(cos(c + d*x)), x)
```

3.254.7 Maxima [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.254.8 Giac [F]

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = \int (b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,dx = \int \sqrt{\cos(c+d\,x)}\,(b\,\cos(c+d\,x))^n\,dx$$

input
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n,x)
```

output
```
int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n, x)
```

3.255 $\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$

3.255.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

output
```
-2*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)*cos(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

3.255.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = -\frac{\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{2}+n\right),\frac{1}{2}\left(\frac{5}{2}+n\right),\cos^2(c+dx)\right)\sqrt{}}{d\left(\frac{1}{2}+n\right)}$$

input
```
Integrate[(b*Cos[c + d*x])^n/Sqrt[Cos[c + d*x]],x]
```

output
```
-((Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/
2, (1/2 + n)/2, (5/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1/2
 + n)))
```

3.255.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{1}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{1}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n+1),\frac{1}{4}(2n+5),\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n/Sqrt[Cos[c + d*x]],x]
```

output
```
(-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)
/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2])
```

3.255.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.255.4 Maple [F]

$$\int \frac{(\cos(dx+c)\, b)^n}{\sqrt{\cos(dx+c)}}dx$$

input
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(1/2),x)
```

output
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(1/2),x)
```

3.255.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\, dx = \int \frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\, dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.255.6 Sympy [F]

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$$

input
```
integrate((b*cos(d*x+c))**n/cos(d*x+c)**(1/2),x)
```

output
```
Integral((b*cos(c + d*x))**n/sqrt(cos(c + d*x)), x)
```

3.255.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.255.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^n}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(1/2),x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(1/2), x)
```

3.256 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

3.256.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

output
```
2*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

3.256.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{2}+n\right),\frac{1}{2}\left(\frac{3}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx}}{d\left(-\frac{1}{2}+n\right)\sqrt{\cos(c+dx)}}$$

input
```
Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(3/2),x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3
/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1/2 + n)*Sqrt[Cos[c
+ d*x]]))
```

3.256.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{3}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{3}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-1),\frac{1}{4}(2n+3),\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

input
```
Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(3/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d
*x]^2])
```

3.256.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.256.4 Maple [F]

$$\int \frac{(\cos (dx+c) b)^n}{\cos (dx+c)^{\frac{3}{2}}} dx$$

input
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(3/2),x)
```

output
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(3/2),x)
```

3.256.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\, dx = \int \frac{(b\cos (dx+c))^n}{\cos (dx+c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.256.6 Sympy [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n/cos(d*x+c)**(3/2),x)
```

output
```
Integral((b*cos(c + d*x))**n/cos(c + d*x)**(3/2), x)
```

3.256.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.256.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^n}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(3/2),x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(3/2), x)
```

3.257 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

3.257.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
2*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

3.257.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{3}{2}+n\right),\frac{1}{2}\left(\frac{1}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{3}{2}+n\right)\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(5/2),x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1
/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-3/2 + n)*Cos[c + d*x
]^(3/2)))
```

3.257.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{5}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{5}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-3),\frac{1}{4}(2n+1),\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

input
```
Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(5/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2])
```

3.257.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.257.4 Maple [F]

$$\int \frac{(\cos(dx+c)\,b)^n}{\cos(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(5/2),x)
```

output
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(5/2),x)
```

3.257.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.257.6 Sympy [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

input
```
integrate((b*cos(d*x+c))**n/cos(d*x+c)**(5/2),x)
```

output
```
Integral((b*cos(c + d*x))**n/cos(c + d*x)**(5/2), x)
```

3.257.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.257.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos\left(c+d\,x\right))^n}{\cos\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(5/2),x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(5/2), x)
```

3.258 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

3.258.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
2*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

3.258.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{5}{2}+n\right),\frac{1}{2}\left(-\frac{1}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{5}{2}+n\right)\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(7/2),x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-
1/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-5/2 + n)*Cos[c + d*
x]^(5/2)))
```

3.258.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{7}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{7}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-5),\frac{1}{4}(2n-1),\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}$$

input
```
Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(7/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c +
d*x]^2])
```

3.258.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.258.4 Maple [F]

$$\int \frac{(\cos(dx+c)\, b)^n}{\cos(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(7/2),x)
```

output
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(7/2),x)
```

3.258.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\, dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\, dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n/cos(d*x+c)**(7/2),x)
```

output
```
Timed out
```

3.258.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.258.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(7/2),x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(7/2), x)
```

3.259 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

3.259.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2(b\cos(c+dx))^n \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

output
```
2*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/d/(7-2*n)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = -\frac{(b\cos(c+dx))^n\csc(c+dx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{7}{2}+n\right),\frac{1}{2}\left(-\frac{3}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{7}{2}+n\right)\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(9/2),x]
```

output
```
-(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-7/2 + n)/2, (-
3/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-7/2 + n)*Cos[c + d*
x]^(7/2)))
```

3.259.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}dx$$

$$\Big\downarrow \text{2034}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \cos^{n-\frac{9}{2}}(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^{-n}(c+dx)(b\cos(c+dx))^n\int \sin\left(c+dx+\frac{\pi}{2}\right)^{n-\frac{9}{2}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2n-7),\frac{1}{4}(2n-3),\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}$$

input
```
Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(9/2),x]
```

output
```
(2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c +
d*x]^2])
```

3.259.3.1 Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&
  !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.259.4 Maple [F]

$$\int \frac{(\cos (dx+c)\, b)^{n}}{\cos (dx+c)^{\frac{9}{2}}}dx$$

input
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(9/2),x)
```

output
```
int((cos(d*x+c)*b)^n/cos(d*x+c)^(9/2),x)
```

3.259.5 Fricas [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^{n}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(d*x+c))**n/cos(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.259.7 Maxima [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.259.8 Giac [F]

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{(b\,\cos{(c+d\,x)})^n}{\cos{(c+d\,x)}^{9/2}}\,dx$$

input
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(9/2),x)
```

output
```
int((b*cos(c + d*x))^n/cos(c + d*x)^(9/2), x)
```

3.260 $\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx$

3.260.1 Optimal result

Integrand size = 21, antiderivative size = 88

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx = -\frac{(a\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\cos^2(e+fx)\right)(b\sec(e+fx))^n}{af(1+m-n)\sqrt{\sin^2(e+fx)}}$$

output
```
-(a*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m-1/2*n],[3/2+1/2*m-1/2*n],c
os(f*x+e)^2)*(b*sec(f*x+e))^n*sin(f*x+e)/a/f/(1+m-n)/(sin(f*x+e)^2)^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx = -\frac{\cos(e+fx)(a\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\cos^2(e+fx)\right)(b\sec}{f(1+m-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Sec[e + f*x])^n,x]
```

output
```
-((Cos[e + f*x]*(a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m - n)/2, (
3 + m - n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + m -
 n)*Sqrt[Sin[e + f*x]^2]))
```

3.260.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3068, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 3068}$$

$$(a\cos(e+fx))^n(b\sec(e+fx))^n\int (a\cos(e+fx))^{m-n}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(a\cos(e+fx))^n(b\sec(e+fx))^n\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^{m-n} dx$$

$$\Big\downarrow \text{ 3122}$$

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\sec(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m-n+1),\frac{1}{2}(m-n+3),\cos^2(e+fx)\right)}{af(m-n+1)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Sec[e + f*x])^n,x]
```

output
```
-(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 + m -
 n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(a*f*(1 + m - n)*S
qrt[Sin[e + f*x]^2]))
```

3.260.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3068
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
[e + f*x])^FracPart[n]   Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a,
b, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

3.260.4 Maple [F]

$$\int (\cos (fx+e)\, a)^{m} (b\sec (fx+e))^{n} \, dx$$

input
```
int((cos(f*x+e)*a)^m*(b*sec(f*x+e))^n,x)
```

output
```
int((cos(f*x+e)*a)^m*(b*sec(f*x+e))^n,x)
```

3.260.5 Fricas [F]

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n \, dx = \int (a\cos (fx+e))^{m} (b\sec (fx+e))^{n} \, dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

3.260.6 Sympy [F]

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx=\int (a\cos{(e+fx)})^m\,(b\sec{(e+fx)})^n\,\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(b*sec(f*x+e))**n,x)
```

output
```
Integral((a*cos(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

3.260.7 Maxima [F]

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx=\int (a\cos{(fx+e)})^m\,(b\sec{(fx+e)})^n\,\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

3.260.8 Giac [F]

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx=\int (a\cos{(fx+e)})^m\,(b\sec{(fx+e)})^n\,\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m (b\sec(e+fx))^n\,dx = \int (a\,\cos\left(e+f\,x\right))^m \left(\frac{b}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a*cos(e + f*x))^m*(b/cos(e + f*x))^n,x)
```

output
```
int((a*cos(e + f*x))^m*(b/cos(e + f*x))^n, x)
```

3.261 $\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

3.261.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2}{b\sqrt{\csc(a+bx)}}$$

output
```
2/b/csc(b*x+a)^(1/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2}{b\sqrt{\csc(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

output
```
2/(b*Sqrt[Csc[a + b*x]])
```

3.261.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc(a+bx)}}{\sec(a+bx)}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

input
```
Int[Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

output
```
2/(b*Sqrt[Csc[a + b*x]])
```

3.261.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.261.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2}{b\sqrt{\csc(bx+a)}}$	14
default	$\frac{2}{b\sqrt{\csc(bx+a)}}$	14
risch	$\frac{2\sqrt{2}\,\sqrt{\frac{i\,\mathrm{e}^{i(bx+a)}}{\mathrm{e}^{2i(bx+a)}-1}}\,\sin(bx+a)}{b}$	42

input
```
int(cos(b*x+a)*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/csc(b*x+a)^(1/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\sqrt{\sin(bx+a)}}{b}$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(sin(b*x + a))/b
```

3.261.6 Sympy [F]

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

output
```
Integral(cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\sqrt{\sin(bx+a)}}{b}$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(sin(b*x + a))/b
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\sqrt{\sin(bx+a)}}{b}$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(sin(b*x + a))/b
```

3.261.9 Mupad [B] (verification not implemented)

Time = 13.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2}{b\sqrt{\frac{1}{\sin(a+b\,x)}}}$$

input
```
int(cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
```

output
```
2/(b*(1/sin(a + b*x))^(1/2))
```

3.262 $\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}} \, dx$

3.262.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}} \, dx = \frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

output
```
2/3/b/csc(b*x+a)^(3/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}} \, dx = \frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

output
```
2/(3*b*Csc[a + b*x]^(3/2))
```

3.262.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\csc(a+bx)}\sec(a+bx)}dx$$
$$\Big\downarrow \text{3101}$$
$$-\frac{\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}d\csc(a+bx)}{b}$$
$$\Big\downarrow \text{15}$$
$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Cos[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

output
```
2/(3*b*Csc[a + b*x]^(3/2))
```

3.262.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.262.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2}{3b\csc\left(bx+a\right)^{\frac{3}{2}}}$	14
default	$\frac{2}{3b\csc\left(bx+a\right)^{\frac{3}{2}}}$	14

input
```
int(cos(b*x+a)/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b/csc(b*x+a)^(3/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2\left(\cos\left(bx+a\right)^2-1\right)}{3\,b\sqrt{\sin\left(bx+a\right)}}$$

input
```
integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(cos(b*x + a)^2 - 1)/(b*sqrt(sin(b*x + a)))
```

3.262.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos{(a+bx)}}{\sqrt{\csc{(a+bx)}}}\,dx$$

input
```
integrate(cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

output
```
Integral(cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\,\sin{(bx+a)}^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sin(b*x + a)^(3/2)/b
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\,\sin{(bx+a)}^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sin(b*x + a)^(3/2)/b
```

3.262.9 Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2}{3\,b\,{\left(\frac{1}{\sin(a+b\,x)}\right)}^{3/2}}$$

input
```
int(cos(a + b*x)/(1/sin(a + b*x))^(1/2),x)
```

output
```
2/(3*b*(1/sin(a + b*x))^(3/2))
```

3.263 $\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$

3.263.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3b}$$

output
```
2/3*cos(b*x+a)/b/csc(b*x+a)^(1/2)-4/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/
sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc
(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

3.263.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{\sqrt{\csc(a+bx)}\left(-4\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}+\sin(2(a+bx))\right)}{3b}$$

input
```
Integrate[Cos[a + b*x]^2*Sqrt[Csc[a + b*x]],x]
```

output
```
(Sqrt[Csc[a + b*x]]*(-4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + Sin[2*(a + b*x)]))/(3*b)
```

3.263.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3108, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{\csc(a+bx)}}{\sec(a+bx)^2}dx$$
$$\Big\downarrow \text{3108}$$
$$\frac{2}{3}\int \sqrt{\csc(a+bx)}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2}{3}\int \sqrt{\csc(a+bx)}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}$$
$$\Big\downarrow \text{4258}$$
$$\frac{2}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}$$
$$\Big\downarrow \text{3120}$$
$$\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}$$

input
```
Int[Cos[a + b*x]^2*Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b*x]]) + (4*Sqrt[Csc[a + b*x]]*Elliptic
F[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b)
```

3.263.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.263.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{\frac{2\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\left(\cos^2(bx+a)\right)\sin(bx+a)}{3}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	88

input
```
int(cos(b*x+a)^2*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elli
pticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b
*x+a)/sin(b*x+a)^(1/2)/b
```

3.263.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$$
$$= \frac{2\left(\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)} - i\sqrt{2i}\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)) + i\sqrt{-2i}\text{wei}\right.}{3\,b}$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(cos(b*x + a)*sqrt(sin(b*x + a)) - I*sqrt(2*I)*weierstrassPInverse(4,
0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(-2*I)*weierstrassPInverse(4, 0,
 cos(b*x + a) - I*sin(b*x + a)))/b
```

3.263.6 Sympy [F]

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos^2\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

input
```
integrate(cos(b*x+a)**2*csc(b*x+a)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**2*sqrt(csc(a + b*x)), x)
```

3.263.7 Maxima [F]

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos\left(bx+a\right)^2\sqrt{\csc\left(bx+a\right)}\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

3.263.8 Giac [F]

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos(bx+a)^2\sqrt{\csc(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos(a+b\,x)^2\sqrt{\frac{1}{\sin(a+b\,x)}}\,dx$$

input
```
int(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2),x)
```

output
```
int(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2), x)
```

3.264 $\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.264.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{5b}$$

output
```
2/5*cos(b*x+a)/b/csc(b*x+a)^(3/2)-4/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/
sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc
(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

3.264.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2\sqrt{\csc(a+bx)}\left(2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}-\cos(a+bx)\sin^2(a+bx)\right)}{5b}$$

input
```
Integrate[Cos[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

output
```
(-2*Sqrt[Csc[a + b*x]]*(2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a +
 b*x]] - Cos[a + b*x]*Sin[a + b*x]^2))/(5*b)
```

3.264.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3108, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc(a+bx)}\sec(a+bx)^2}dx$$

$$\Big\downarrow \text{3108}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}$$

input
```
Int[Cos[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Cos[a + b*x])/(5*b*Csc[a + b*x]^(3/2)) + (4*Sqrt[Csc[a + b*x]]*Elliptic
E[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(5*b)
```

3.264.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.264.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.12

method	result
default	$\frac{-\frac{2\left(\sin^{4}(bx+a)\right)}{5}+\frac{2\left(\sin^{2}(bx+a)\right)}{5}-\frac{4\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{2\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+}}{}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

input
```
int(cos(b*x+a)^2/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5*sin(b*x+a)^4+2/5*sin(b*x+a)^2-4/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a
)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))
+2/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elli
pticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

3.264.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\left(\sqrt{2i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))+\sqrt{-2i}\text{weierstrassZe}\right.}{5b}$$

input
```
integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a
) + I*sin(b*x + a))) + sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(b*x + a) - I*sin(b*x + a))) - (cos(b*x + a)^3 - cos(b*x + a))/
sqrt(sin(b*x + a)))/b
```

3.264.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(cos(b*x+a)**2/csc(b*x+a)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**2/sqrt(csc(a + b*x)), x)
```

3.264.7 Maxima [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos(bx+a)^2}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

3.264.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^2}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int(cos(a + b*x)^2/(1/sin(a + b*x))^(1/2),x)
```

output
```
int(cos(a + b*x)^2/(1/sin(a + b*x))^(1/2), x)
```

3.265 $\int \cos^3(x) \csc^{\frac{9}{2}}(x)\,dx$

3.265.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\,dx = \frac{2}{3}\csc^{\frac{3}{2}}(x) - \frac{2}{7}\csc^{\frac{7}{2}}(x)$$

output
```
2/3*csc(x)^(3/2)-2/7*csc(x)^(7/2)
```

3.265.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\,dx = \frac{2}{21}\csc^{\frac{3}{2}}(x)\left(7 - 3\csc^2(x)\right)$$

input
```
Integrate[Cos[x]^3*Csc[x]^(9/2),x]
```

output
```
(2*Csc[x]^(3/2)*(7 - 3*Csc[x]^2))/21
```

3.265.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3101, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(x)\csc^{\frac{9}{2}}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(x)^{9/2}}{\sec(x)^3}dx$$

$$\Big\downarrow \text{3101}$$

$$-\int -\sqrt{\csc(x)}\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \text{25}$$

$$\int \sqrt{\csc(x)}\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\sqrt{\csc(x)}-\csc^{\frac{5}{2}}(x)\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)$$

input
```
Int[Cos[x]^3*Csc[x]^(9/2),x]
```

output
```
(2*Csc[x]^(3/2))/3 - (2*Csc[x]^(7/2))/7
```

3.265.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.265.4 Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2}{7 \sin \left(x \right)^{\frac{7}{2}}}+\frac{2}{3 \sin \left(x \right)^{\frac{3}{2}}}$	14

input
```
int(cos(x)^3*csc(x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/sin(x)^(7/2)+2/3/sin(x)^(3/2)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\, dx = \frac{2\left(7 \cos(x)^2 - 4\right)}{21\left(\cos(x)^2 - 1\right) \sin(x)^{\frac{3}{2}}}$$

input
```
integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="fricas")
```

output
```
2/21*(7*cos(x)^2 - 4)/((cos(x)^2 - 1)*sin(x)^(3/2))
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\, dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*csc(x)**(9/2),x)
```

output
```
Timed out
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\, dx = \frac{2}{3 \sin(x)^{\frac{3}{2}}} - \frac{2}{7 \sin(x)^{\frac{7}{2}}}$$

input
```
integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="maxima")
```

output
```
2/3/sin(x)^(3/2) - 2/7/sin(x)^(7/2)
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\, dx = \frac{2\left(7 \sin(x)^2 - 3\right)}{21 \sin(x)^{\frac{7}{2}}}$$

input
```
integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="giac")
```

output
```
2/21*(7*sin(x)^2 - 3)/sin(x)^(7/2)
```

3.265.9 Mupad [B] (verification not implemented)

Time = 13.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \cos^3(x) \csc^{\frac{9}{2}}(x)\, dx = \frac{2\left(7 \sin(x)^2 - 3\right) \left(\frac{1}{\sin(x)}\right)^{7/2}}{21}$$

input
```
int(cos(x)^3*(1/sin(x))^(9/2),x)
```

output
```
(2*(7*sin(x)^2 - 3)*(1/sin(x))^(7/2))/21
```

3.266 $\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx$

3.266.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx=-\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{2}{b\sqrt{\csc(a+bx)}}$$

output
```
-2/5/b/csc(b*x+a)^(5/2)+2/b/csc(b*x+a)^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx=\frac{9+\cos(2(a+bx))}{5b\sqrt{\csc(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^3*Sqrt[Csc[a + b*x]],x]
```

output
```
(9 + Cos[2*(a + b*x)])/(5*b*Sqrt[Csc[a + b*x]])
```

3.266.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3101, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc(a+bx)}}{\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{1-\csc^2(a+bx)}{\csc^{\frac{7}{2}}(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1-\csc^2(a+bx)}{\csc^{\frac{7}{2}}(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{1}{\csc^{\frac{7}{2}}(a+bx)}-\frac{1}{\csc^{\frac{3}{2}}(a+bx)}\right)d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2}{5\csc^{\frac{5}{2}}(a+bx)}-\frac{2}{\sqrt{\csc(a+bx)}}}{b}$$

input
```
Int[Cos[a + b*x]^3*Sqrt[Csc[a + b*x]],x]
```

output
```
-((2/(5*Csc[a + b*x]^(5/2)) - 2/Sqrt[Csc[a + b*x]])/b)
```

3.266.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.266.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{-\frac{2\left(\sin^{\frac{5}{2}}(bx+a)\right)}{5}+2\left(\sqrt{\sin}(bx+a)\right)}{b}$	26

input
```
int(cos(b*x+a)^3*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/5*sin(b*x+a)^(5/2)+2*sin(b*x+a)^(1/2))/b
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\left(\cos\left(bx+a\right)^2+4\right)\sqrt{\sin\left(bx+a\right)}}{5\,b}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(cos(b*x + a)^2 + 4)*sqrt(sin(b*x + a))/b
```

3.266.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*csc(b*x+a)**(1/2),x)
```

output
```
Timed out
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\left(\frac{5}{\sin(bx+a)^2}-1\right)\sin\left(bx+a\right)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(5/sin(b*x + a)^2 - 1)*sin(b*x + a)^(5/2)/b
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx = -\frac{2\left(\sin\left(bx+a\right)^{\frac{5}{2}}-5\sqrt{\sin\left(bx+a\right)}\right)}{5\,b}$$

input
```
integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/5*(sin(b*x + a)^(5/2) - 5*sqrt(sin(b*x + a)))/b
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos\left(a+b\,x\right)^3\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

input
```
int(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2),x)
```

output
```
int(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2), x)
```

3.267 $\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.267.1 Optimal result

Integrand size = 19, antiderivative size = 35

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2}{7b\csc^{\frac{7}{2}}(a+bx)}+\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

output
```
-2/7/b/csc(b*x+a)^(7/2)+2/3/b/csc(b*x+a)^(3/2)
```

3.267.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2(-3+7\csc^2(a+bx))}{21b\csc^{\frac{7}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*(-3 + 7*Csc[a + b*x]^2))/(21*b*Csc[a + b*x]^(7/2))
```

3.267.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3101, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc(a+bx)}\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{1-\csc^2(a+bx)}{\csc^{\frac{9}{2}}(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1-\csc^2(a+bx)}{\csc^{\frac{9}{2}}(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\frac{1}{\csc^{\frac{9}{2}}(a+bx)}-\frac{1}{\csc^{\frac{5}{2}}(a+bx)}\right)d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2}{7\csc^{\frac{7}{2}}(a+bx)}-\frac{2}{3\csc^{\frac{3}{2}}(a+bx)}}{b}$$

input
```
Int[Cos[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

output
```
-((2/(7*Csc[a + b*x]^(7/2)) - 2/(3*Csc[a + b*x]^(3/2)))/b)
```

3.267.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.267.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{-\frac{2\left(\sin^{\frac{7}{2}}(bx+a)\right)}{7}+\frac{2\left(\sin^{\frac{3}{2}}(bx+a)\right)}{3}}{b}$	26

input
```
int(cos(b*x+a)^3/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/7*sin(b*x+a)^(7/2)+2/3*sin(b*x+a)^(3/2))/b
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2\left(3\cos\left(bx+a\right)^4+\cos\left(bx+a\right)^2-4\right)}{21\,b\sqrt{\sin\left(bx+a\right)}}$$

input
```
integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(3*cos(b*x + a)^4 + cos(b*x + a)^2 - 4)/(b*sqrt(sin(b*x + a)))
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3/csc(b*x+a)**(1/2),x)
```

output
```
Timed out
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\left(\frac{7}{\sin(bx+a)^2}-3\right)\sin\left(bx+a\right)^{\frac{7}{2}}}{21\,b}$$

input
```
integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(7/sin(b*x + a)^2 - 3)*sin(b*x + a)^(7/2)/b
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2\left(3\sin\left(bx+a\right)^{\frac{7}{2}}-7\sin\left(bx+a\right)^{\frac{3}{2}}\right)}{21\,b}$$

input
```
integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2/21*(3*sin(b*x + a)^(7/2) - 7*sin(b*x + a)^(3/2))/b
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int(cos(a + b*x)^3/(1/sin(a + b*x))^(1/2),x)
```

output
```
int(cos(a + b*x)^3/(1/sin(a + b*x))^(1/2), x)
```

3.268 $\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx$

3.268.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{8\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{7b}$$

output
```
4/7*cos(b*x+a)/b/csc(b*x+a)^(1/2)+2/7*cos(b*x+a)^3/b/csc(b*x+a)^(1/2)-8/7*
(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(co
s(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

3.268.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{\sqrt{\csc(a+bx)}\left(-32\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}+10\sin(2(a+bx))+\sin(4(a+bx))\right)}{28b}$$

input
```
Integrate[Cos[a + b*x]^4*Sqrt[Csc[a + b*x]],x]
```

output
```
(Sqrt[Csc[a + b*x]]*(-32*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a +
b*x]] + 10*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]))/(28*b)
```

3.268.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3108, 3042, 3108, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc(a+bx)}}{\sec(a+bx)^4}dx$$

$$\Big\downarrow \text{3108}$$

$$\frac{6}{7}\int \cos^2(a+bx)\sqrt{\csc(a+bx)}dx+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sqrt{\csc(a+bx)}}{\sec(a+bx)^2}dx+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3108}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \sqrt{\csc(a+bx)}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \sqrt{\csc(a+bx)}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{6}{7}\left(\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}\right)$$

input
```
Int[Cos[a + b*x]^4*Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Cos[a + b*x]^3)/(7*b*Sqrt[Csc[a + b*x]]) + (6*((2*Cos[a + b*x])/(3*b*Sq
rt[Csc[a + b*x]]) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]
*Sqrt[Sin[a + b*x]])/(3*b)))/7
```

3.268.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.268.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\frac{4\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{7}+\frac{2\left(\sin^{5}(bx+a)\right)}{7}-\frac{8\left(\sin^{3}(bx+a)\right)}{7}+\frac{6\sin(bx+a)}{7}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	100

input
```
int(cos(b*x+a)^4*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(4/7*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elli
pticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2/7*sin(b*x+a)^5-8/7*sin(b*x+a)^3+
6/7*sin(b*x+a))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

3.268.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx$$
$$=\frac{2\left(\left(\cos\left(bx+a\right)^3+2\,\cos\left(bx+a\right)\right)\sqrt{\sin\left(bx+a\right)}-2i\,\sqrt{2i}\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(b\right.}{7\,b}$$

input
```
integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/7*((cos(b*x + a)^3 + 2*cos(b*x + a))*sqrt(sin(b*x + a)) - 2*I*sqrt(2*I)*
weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) + 2*I*sqrt(-2*I)*
weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

3.268.6 Sympy [F]

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos^4{(a+bx)}\sqrt{\csc{(a+bx)}}\,dx$$

input
```
integrate(cos(b*x+a)**4*csc(b*x+a)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**4*sqrt(csc(a + b*x)), x)
```

3.268.7 Maxima [F]

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos{(bx+a)}^4\sqrt{\csc{(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

3.268.8 Giac [F]

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos{(bx+a)}^4\sqrt{\csc{(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \cos(a+b\,x)^4\,\sqrt{\frac{1}{\sin(a+b\,x)}}\,dx$$

input
```
int(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2),x)
```

output
```
int(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2), x)
```

3.269 $\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.269.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)}+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}+\frac{8\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{15b}$$

output
```
4/15*cos(b*x+a)/b/csc(b*x+a)^(3/2)+2/9*cos(b*x+a)^3/b/csc(b*x+a)^(3/2)-8/1
5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

3.269.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\frac{39\cos(a+bx)+5\cos(3(a+bx))-\frac{48E\left(\frac{1}{4}(-2a+\pi-2bx)|2\right)}{\sin^{\frac{3}{2}}(a+bx)}}{90b\csc^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

output
```
(39*Cos[a + b*x] + 5*Cos[3*(a + b*x)] - (48*EllipticE[(-2*a + Pi - 2*b*x)/
4, 2])/Sin[a + b*x]^(3/2))/(90*b*Csc[a + b*x]^(3/2))
```

3.269.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3108, 3042, 3108, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{\csc(a+bx)}\sec(a+bx)^4}dx$$

$$\Big\downarrow \ 3108$$

$$\frac{2}{3}\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}dx+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\csc(a+bx)}\sec(a+bx)^2}dx+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3108$$

$$\frac{2}{3}\left(\frac{2}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\left(\frac{2}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 4258$$

$$\frac{2}{3}\left(\frac{2}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\left(\frac{2}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx+\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \ 3119$$

$$\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}+\frac{2}{3}\left(\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}\right)$$

input
```
Int[Cos[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Cos[a + b*x]^3)/(9*b*Csc[a + b*x]^(3/2)) + (2*((2*Cos[a + b*x])/(5*b*Cs
c[a + b*x]^(3/2)) + (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]
*Sqrt[Sin[a + b*x]])/(5*b)))/3
```

3.269.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3108
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Simp[(n + 1)/(b^2*(m + n))   Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -
1] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.269.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

method	result
default	$\frac{-\frac{2\left(\cos^6(bx+a)\right)}{9}-\frac{8\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{15}+\frac{4\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F}{15}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

input
```
int(cos(b*x+a)^4/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/9*cos(b*x+a)^6-8/15*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin
(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+4/15*(sin(b*x+a
)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+
a)+1)^(1/2),1/2*2^(1/2))-2/45*cos(b*x+a)^4+4/15*cos(b*x+a)^2)/cos(b*x+a)/s
in(b*x+a)^(1/2)/b
```

3.269.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\left(6\sqrt{2i}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))+6\sqrt{-2i}\,\text{weierstras}\right.}{45\,b}$$

input
```
integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x
+ a) + I*sin(b*x + a))) + 6*sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassPI
nverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) - (5*cos(b*x + a)^5 + cos(b*x
 + a)^3 - 6*cos(b*x + a))/sqrt(sin(b*x + a)))/b
```

3.269.6 Sympy [F]

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(cos(b*x+a)**4/csc(b*x+a)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**4/sqrt(csc(a + b*x)), x)
```

3.269.7 Maxima [F]

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos(bx+a)^4}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

3.269.8 Giac [F]

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos(bx+a)^4}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\cos(a+b\,x)^4}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int(cos(a + b*x)^4/(1/sin(a + b*x))^(1/2),x)
```

output
```
int(cos(a + b*x)^4/(1/sin(a + b*x))^(1/2), x)
```

3.270 $\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx$

3.270.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

output
```
-3/4*csc(x)^(4/3)
```

3.270.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = -\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

input
```
Integrate[Cos[x]*Csc[x]^(7/3),x]
```

output
```
(-3*Csc[x]^(4/3))/4
```

3.270.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3101, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc(x)^{7/3}}{\sec(x)}dx$$

$$\Big\downarrow \; 3101$$

$$-\int \sqrt[3]{\csc(x)}d\csc(x)$$

$$\Big\downarrow \; 15$$

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

input
```
Int[Cos[x]*Csc[x]^(7/3),x]
```

output
```
(-3*Csc[x]^(4/3))/4
```

3.270.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.270.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{3\left(\csc^{\frac{4}{3}}(x)\right)}{4}$	7
default	$-\frac{3\left(\csc^{\frac{4}{3}}(x)\right)}{4}$	7

input
```
int(cos(x)*csc(x)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*csc(x)^(4/3)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = -\frac{3}{4\sin(x)^{\frac{4}{3}}}$$

input
```
integrate(cos(x)*csc(x)^(7/3),x, algorithm="fricas")
```

output
```
-3/4/sin(x)^(4/3)
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*csc(x)**(7/3),x)
```

output
```
Timed out
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = -\frac{3}{4\sin(x)^{\frac{4}{3}}}$$

input
```
integrate(cos(x)*csc(x)^(7/3),x, algorithm="maxima")
```

output
```
-3/4/sin(x)^(4/3)
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = -\frac{3}{4\sin(x)^{\frac{4}{3}}}$$

input
```
integrate(cos(x)*csc(x)^(7/3),x, algorithm="giac")
```

output
```
-3/4/sin(x)^(4/3)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx = -\frac{3\left(\frac{1}{\sin(x)}\right)^{4/3}}{4}$$

input
```
int(cos(x)*(1/sin(x))^(7/3),x)
```

output
```
-(3*(1/sin(x))^(4/3))/4
```

3.271 $\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx$

3.271.1 Optimal result

Integrand size = 17, antiderivative size = 32

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx=-\frac{\arctan\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\operatorname{arctanh}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

output
```
-arctan(csc(b*x+a)^(1/2))/b+arctanh(csc(b*x+a)^(1/2))/b
```

3.271.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx =\frac{\left(\arctan\left(\sqrt{\sin(a+bx)}\right)+\operatorname{arctanh}\left(\sqrt{\sin(a+bx)}\right)\right)\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}}{b}$$

input
```
Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x],x]
```

output
```
((ArcTan[Sqrt[Sin[a + b*x]]] + ArcTanh[Sqrt[Sin[a + b*x]]])*Sqrt[Csc[a + b
*x]]*Sqrt[Sin[a + b*x]])/b
```

3.271.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3101, 25, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\sqrt{\csc(a+bx)}}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{\csc(a+bx)}}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{\csc(a+bx)}{1-\csc^2(a+bx)}d\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}-\frac{1}{2}\int \frac{1}{\csc(a+bx)+1}d\sqrt{\csc(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}-\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\csc(a+bx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)\right)}{b}$$

input
```
Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x],x]
```

output
```
(2*(-1/2*ArcTan[Sqrt[Csc[a + b*x]]] + ArcTanh[Sqrt[Csc[a + b*x]]]/2))/b
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.271.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{\sin}\left(bx+a\right)\right)+\arctan\left(\sqrt{\sin}\left(bx+a\right)\right)}{b}$	24

input
```
int(csc(b*x+a)^(1/2)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(arctanh(sin(b*x+a)^(1/2))+arctan(sin(b*x+a)^(1/2)))/b
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx = \frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)+\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)}{4b}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a))) + log((cos(b*x +
a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b*
x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)))/b
```

3.271.6 Sympy [F]

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx = \int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**(1/2)*sec(b*x+a),x)
```

output
```
Integral(sqrt(csc(a + b*x))*sec(a + b*x), x)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx = -\frac{2\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right) - \log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right) + \log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*arctan(1/sqrt(sin(b*x + a))) - log(1/sqrt(sin(b*x + a)) + 1) + log
(1/sqrt(sin(b*x + a)) - 1))/b
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx = \frac{2\arctan\left(\sqrt{\sin(bx+a)}\right) + \log\left(\sqrt{\sin(bx+a)}+1\right) - \log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*arctan(sqrt(sin(b*x + a))) + log(sqrt(sin(b*x + a)) + 1) - log(abs(
sqrt(sin(b*x + a)) - 1)))/b
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx = \int \frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos(a+b\,x)}\,dx$$

input
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x),x)
```

output
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x), x)
```

3.272 $\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.272.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\frac{\arctan\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\operatorname{arctanh}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

output
```
arctan(csc(b*x+a)^(1/2))/b+arctanh(csc(b*x+a)^(1/2))/b
```

3.272.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{\left(\arctan\left(\sqrt{\sin(a+bx)}\right)-\operatorname{arctanh}\left(\sqrt{\sin(a+bx)}\right)\right)\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}}{b}$$

input
```
Integrate[Sec[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

output
```
-(((ArcTan[Sqrt[Sin[a + b*x]]] - ArcTanh[Sqrt[Sin[a + b*x]]])*Sqrt[Csc[a + 
b*x]]*Sqrt[Sin[a + b*x]])/b)
```

3.272.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3101, 25, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{1}{\sqrt{\csc(a+bx)}(1-\csc^2(a+bx))}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{\csc(a+bx)}(1-\csc^2(a+bx))}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{1-\csc^2(a+bx)}d\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{756}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}+\frac{1}{2}\int \frac{1}{\csc(a+bx)+1}d\sqrt{\csc(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}+\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\csc(a+bx)}\right)\right)}{b}$$

input
```
Int[Sec[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*(ArcTan[Sqrt[Csc[a + b*x]]]/2 + ArcTanh[Sqrt[Csc[a + b*x]]]/2))/b
```

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.272.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{-\frac{\ln\left(\sqrt{\sin}(bx+a)-1\right)}{2}+\frac{\ln\left(1+\sqrt{\sin}(bx+a)\right)}{2}-\arctan\left(\sqrt{\sin}(bx+a)\right)}{b}$	43

input
```
int(sec(b*x+a)/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*ln(sin(b*x+a)^(1/2)-1)+1/2*ln(1+sin(b*x+a)^(1/2))-arctan(sin(b*x+a)^
(1/2)))/b
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(27) = 54.

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)-\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)}{4b}$$

input
```
integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a))) - log((cos(b*x +
 a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b
*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)))/b
```

3.272.6 Sympy [F]

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\sec\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

input
```
integrate(sec(b*x+a)/csc(b*x+a)**(1/2),x)
```

output
```
Integral(sec(a + b*x)/sqrt(csc(a + b*x)), x)
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right)+\log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right)-\log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{2\,b}$$

input
```
integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*arctan(1/sqrt(sin(b*x + a))) + log(1/sqrt(sin(b*x + a)) + 1) - log(
1/sqrt(sin(b*x + a)) - 1))/b
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{2\arctan\left(\sqrt{\sin\left(bx+a\right)}\right)-\log\left(\sqrt{\sin\left(bx+a\right)}+1\right)+\log\left(\left|\sqrt{\sin\left(bx+a\right)}-1\right|\right)}{2\,b}$$

input
```
integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(2*arctan(sqrt(sin(b*x + a))) - log(sqrt(sin(b*x + a)) + 1) + log(abs
(sqrt(sin(b*x + a)) - 1)))/b
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}}\,dx$$

input
```
int(1/(cos(a + b*x)*(1/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/(cos(a + b*x)*(1/sin(a + b*x))^(1/2)), x)
```

3.273 $\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$

3.273.1 Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}} + \frac{\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{b}$$

output
```
sec(b*x+a)/b/csc(b*x+a)^(1/2)-(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*
a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^
(1/2)*sin(b*x+a)^(1/2)/b
```

3.273.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \frac{\sec(a+bx) + \frac{\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2bx),2\right)}{\sqrt{\sin(a+bx)}}}{b\sqrt{\csc(a+bx)}}$$

input
```
Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^2,x]
```

output
```
(Sec[a + b*x] + EllipticF[(2*a - Pi + 2*b*x)/4, 2]/Sqrt[Sin[a + b*x]])/(b*
Sqrt[Csc[a + b*x]])
```

3.273.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3106, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{1}{2}\int \sqrt{\csc(a+bx)}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{\csc(a+bx)}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b}$$

input
```
Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^2,x]
```

output
```
Sec[a + b*x]/(b*Sqrt[Csc[a + b*x]]) + (Sqrt[Csc[a + b*x]]*EllipticF[(a - P
i/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

3.273.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.273.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\sqrt{(\cos^2(bx+a))\sin(bx+a)}\left(\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)+2\sin(bx+a)\right)}{2\sqrt{-\sin(bx+a)(\sin(bx+a)-1)(\sin(bx+a)+1)}\,\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	123

input
```
int(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2
)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*
sin(b*x+a))/(-sin(b*x+a)*(sin(b*x+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/s
in(b*x+a)^(1/2)/b
```

3.273.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \frac{-i\sqrt{2i}\cos(bx+a)\,\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{-2i}\cos(bx+a)\,\text{weierstr}}{2b\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(-I*sqrt(2*I)*cos(b*x + a)*weierstrassPInverse(4, 0, cos(b*x + a) + I*
sin(b*x + a)) + I*sqrt(-2*I)*cos(b*x + a)*weierstrassPInverse(4, 0, cos(b*
x + a) - I*sin(b*x + a)) + 2*sqrt(sin(b*x + a)))/(b*cos(b*x + a))
```

3.273.6 Sympy [F]

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**2,x)
```

output
```
Integral(sqrt(csc(a + b*x))*sec(a + b*x)**2, x)
```

3.273.7 Maxima [F]

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \int \sqrt{\csc(bx+a)}\sec(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

3.273.8 Giac [F]

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \int \sqrt{\csc(bx+a)}\sec(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \int \frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos(a+b\,x)^2}\,dx$$

input
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^2,x)
```

output
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^2, x)
```

3.274 $\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.274.1 Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}$$

output
```
sec(b*x+a)/b/csc(b*x+a)^(3/2)+(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*
a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^
(1/2)*sin(b*x+a)^(1/2)/b
```

3.274.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\frac{\sqrt{\csc(a+bx)}\left(E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+\sin(a+bx)\tan(a+bx)\right)}{b}$$

input
```
Integrate[Sec[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

output
```
(Sqrt[Csc[a + b*x]]*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]
] + Sin[a + b*x]*Tan[a + b*x]))/b
```

3.274.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3106, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^2}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\int \frac{1}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\int \frac{1}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

input
```
Int[Sec[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

output
```
Sec[a + b*x]/(b*Csc[a + b*x]^(3/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - P
i/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

3.274.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.274.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(85) = 170$.

Time = 1.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.85

method	result
default	$\frac{\sqrt{(\cos^2(bx+a))\sin(bx+a)}\left(2\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(b}{2\sqrt{-\sin(bx+a)(\sin(bx+a)-1)(\sin(bx+a)+1)}\,\cos(bx+a)\sqrt{\sin(bx+}$

input
```
int(sec(b*x+a)^2/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*(2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)
+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-
(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF
((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2+2)/(-sin(b*x+a)*(sin(b*x
+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

3.274.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx =$$

$$-\frac{\sqrt{2i}\cos(bx+a)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a)))+\sqrt{-2i}\,\text{c}}{2\,b\cos}$$

input
```
integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2*I)*cos(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(4,
0, cos(b*x + a) + I*sin(b*x + a))) + sqrt(-2*I)*cos(b*x + a)*weierstrassZe
ta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*(co
s(b*x + a)^2 - 1)/sqrt(sin(b*x + a)))/(b*cos(b*x + a))
```

3.274.6 Sympy [F]

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(sec(b*x+a)**2/csc(b*x+a)**(1/2),x)
```

output
```
Integral(sec(a + b*x)**2/sqrt(csc(a + b*x)), x)
```

3.274.7 Maxima [F]

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\sec(bx+a)^2}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

3.274.8 Giac [F]

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\sec\left(bx+a\right)^2}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

input
```
integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\,\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int(1/(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2)), x)
```

3.275 $\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$

3.275.1 Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx = -\frac{3\arctan\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{3\text{arctanh}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}}$$

output
```
-3/4*arctan(csc(b*x+a)^(1/2))/b+3/4*arctanh(csc(b*x+a)^(1/2))/b+1/2*sec(b*
x+a)^2/b/csc(b*x+a)^(1/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx = \frac{\sqrt{\csc(a+bx)}\left(3\arctan\left(\sqrt{\sin(a+bx)}\right) + 3\text{arctanh}\left(\sqrt{\sin(a+bx)}\right) + 2\sec^2(a+bx)\sqrt{\sin(a+bx)}\right)\sqrt{\text{si}}}{4b}$$

input
```
Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^3,x]
```

output
```
(Sqrt[Csc[a + b*x]]*(3*ArcTan[Sqrt[Sin[a + b*x]]] + 3*ArcTanh[Sqrt[Sin[a +
 b*x]]] + 2*Sec[a + b*x]^2*Sqrt[Sin[a + b*x]])*Sqrt[Sin[a + b*x]])/(4*b)
```

3.275.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3101, 252, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^{\frac{5}{2}}(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^{\frac{3}{2}}(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{4}\int \frac{\sqrt{\csc(a+bx)}}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{\frac{\csc^{\frac{3}{2}}(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc(a+bx)}{1-\csc^2(a+bx)}d\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{\frac{\csc^{\frac{3}{2}}(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}-\frac{1}{2}\int \frac{1}{\csc(a+bx)+1}d\sqrt{\csc(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{\csc^{\frac{3}{2}}(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}-\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\csc^{\frac{3}{2}}(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\csc(a+bx)}\right)-\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)\right)}{b}$$

input
```
Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^3,x]
```

output
```
-(((-3*(-1/2*ArcTan[Sqrt[Csc[a + b*x]]] + ArcTanh[Sqrt[Csc[a + b*x]]]/2))/
2 + Csc[a + b*x]^(3/2)/(2*(1 - Csc[a + b*x]^2)))/b)
```

3.275.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.275.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{-\left(-3\ln\left(1+\sqrt{\sin}(bx+a)\right)+3\ln\left(\sqrt{\sin}(bx+a)-1\right)-6\arctan\left(\sqrt{\sin}(bx+a)\right)\right)\left(\cos^2(bx+a)\right)+4\left(\sqrt{\sin}(bx+a)\right)}{8\cos(bx+a)^2 b}$	73

input
```
int(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-(-3*ln(1+sin(b*x+a)^(1/2))+3*ln(sin(b*x+a)^(1/2)-1)-6*arctan(sin(b*x
+a)^(1/2)))*cos(b*x+a)^2+4*sin(b*x+a)^(1/2))/cos(b*x+a)^2/b
```

3.275.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(50) = 100.

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$$
$$= \frac{6\,\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)\cos\left(bx+a\right)^2+3\,\cos\left(bx+a\right)^2\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\,\sin(bx+a)-2}{\cos(bx+a)^2+2\,\sin(bx+a)-2}\right)}{16\,b\cos\left(bx+a\right)^2}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/16*(6*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a)))*cos(b*x + a)^2 +
 3*cos(b*x + a)^2*log((cos(b*x + a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) -
 1)/sqrt(sin(b*x + a)) - 6*sin(b*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x +
 a) - 2)) + 8*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2)
```

3.275.6 Sympy [F]

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx = \int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**3,x)
```

output
```
Integral(sqrt(csc(a + b*x))*sec(a + b*x)**3, x)
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$$
$$= \frac{\frac{4}{\left(\frac{1}{\sin(bx+a)^2}-1\right)\sin(bx+a)^{\frac{3}{2}}} - 6\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right) + 3\log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right) - 3\log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{8\,b}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4/((1/sin(b*x + a)^2 - 1)*sin(b*x + a)^(3/2)) - 6*arctan(1/sqrt(sin(b
*x + a))) + 3*log(1/sqrt(sin(b*x + a)) + 1) - 3*log(1/sqrt(sin(b*x + a)) -
 1))/b
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx =$$
$$-\frac{\frac{4\sqrt{\sin(bx+a)}}{\sin(bx+a)^2-1} - 6\arctan\left(\sqrt{\sin(bx+a)}\right) - 3\log\left(\sqrt{\sin(bx+a)}+1\right) + 3\log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)}{8\,b}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/8*(4*sqrt(sin(b*x + a))/(sin(b*x + a)^2 - 1) - 6*arctan(sqrt(sin(b*x +
a))) - 3*log(sqrt(sin(b*x + a)) + 1) + 3*log(abs(sqrt(sin(b*x + a)) - 1)))
/b
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx = \int \frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos\left(a+b\,x\right)^3}\,dx$$

input
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^3,x)
```

output
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^3, x)
```

3.276 $\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.276.1 Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{\arctan\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\operatorname{arctanh}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}$$

output
```
1/4*arctan(csc(b*x+a)^(1/2))/b+1/4*arctanh(csc(b*x+a)^(1/2))/b+1/2*sec(b*x
+a)^2/b/csc(b*x+a)^(3/2)
```

3.276.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.53

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},\sin^2(a+bx)\right)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Sec[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Hypergeometric2F1[3/4, 2, 7/4, Sin[a + b*x]^2])/(3*b*Csc[a + b*x]^(3/2)
)
```

3.276.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3101, 252, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^3}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^{\frac{3}{2}}(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\sqrt{\csc(a+bx)}}{2(1-\csc^2(a+bx))}-\frac{1}{4}\int \frac{1}{\sqrt{\csc(a+bx)}(1-\csc^2(a+bx))}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{\frac{\sqrt{\csc(a+bx)}}{2(1-\csc^2(a+bx))}-\frac{1}{2}\int \frac{1}{1-\csc^2(a+bx)}d\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{756}$$

$$-\frac{\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}-\frac{1}{2}\int \frac{1}{\csc(a+bx)+1}d\sqrt{\csc(a+bx)}\right)+\frac{\sqrt{\csc(a+bx)}}{2(1-\csc^2(a+bx))}}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{1-\csc(a+bx)}d\sqrt{\csc(a+bx)}-\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)\right)+\frac{\sqrt{\csc(a+bx)}}{2(1-\csc^2(a+bx))}}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{1}{2}\left(-\frac{1}{2}\arctan\left(\sqrt{\csc(a+bx)}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{\csc(a+bx)}\right)\right)+\frac{\sqrt{\csc(a+bx)}}{2(1-\csc^2(a+bx))}}{b}$$

input
```
Int[Sec[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

output
```
-(((-1/2*ArcTan[Sqrt[Csc[a + b*x]]] - ArcTanh[Sqrt[Csc[a + b*x]]]/2)/2 + S
qrt[Csc[a + b*x]]/(2*(1 - Csc[a + b*x]^2)))/b)
```

3.276.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.276. $\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.276.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\left(-\ln\left(1+\sqrt{\sin}(bx+a)\right)+\ln\left(\sqrt{\sin}(bx+a)-1\right)+2\arctan\left(\sqrt{\sin}(bx+a)\right)\right)\left(\cos^{2}(bx+a)\right)+4\left(\sin^{\frac{3}{2}}(bx+a)\right)}{8\cos(bx+a)^{2}b}$	71

input
```
int(sec(b*x+a)^3/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-(-ln(1+sin(b*x+a)^(1/2))+ln(sin(b*x+a)^(1/2)-1)+2*arctan(sin(b*x+a)^
(1/2)))*cos(b*x+a)^2+4*sin(b*x+a)^(3/2))/cos(b*x+a)^2/b
```

3.276.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(50) = 100.

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx =$$

$$-\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)\cos\left(bx+a\right)^2-\cos\left(bx+a\right)^2\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)}{16\,b\cos\left(bx+a\right)^2}$$

input
```
integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a)))*cos(b*x + a)^2
- cos(b*x + a)^2*log((cos(b*x + a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) -
1)/sqrt(sin(b*x + a)) - 6*sin(b*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x +
a) - 2)) + 8*(cos(b*x + a)^2 - 1)/sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2)
```

3.276.6 Sympy [F]

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(sec(b*x+a)**3/csc(b*x+a)**(1/2),x)
```

output
```
Integral(sec(a + b*x)**3/sqrt(csc(a + b*x)), x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{\frac{4}{\left(\frac{1}{\sin(bx+a)^2}-1\right)\sqrt{\sin(bx+a)}} + 2\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right) + \log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right) - \log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{8\,b}$$

input
```
integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(4/((1/sin(b*x + a)^2 - 1)*sqrt(sin(b*x + a))) + 2*arctan(1/sqrt(sin(b
*x + a))) + log(1/sqrt(sin(b*x + a)) + 1) - log(1/sqrt(sin(b*x + a)) - 1))
/b
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = -\frac{\frac{4\sin(bx+a)^{\frac{3}{2}}}{\sin(bx+a)^2-1} + 2\arctan\left(\sqrt{\sin(bx+a)}\right) - \log\left(\sqrt{\sin(bx+a)}+1\right) + \log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)}{8\,b}$$

input
```
integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(4*sin(b*x + a)^(3/2)/(sin(b*x + a)^2 - 1) + 2*arctan(sqrt(sin(b*x +
a))) - log(sqrt(sin(b*x + a)) + 1) + log(abs(sqrt(sin(b*x + a)) - 1)))/b
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int(1/(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2)), x)
```

3.277 $\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx$

3.277.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx = \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}}+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{5\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{6b}$$

output
```
5/6*sec(b*x+a)/b/csc(b*x+a)^(1/2)+1/3*sec(b*x+a)^3/b/csc(b*x+a)^(1/2)-5/6*
(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(co
s(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

3.277.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx = \frac{\sqrt{\csc(a+bx)}\left(-5\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}+\left(5+2\sec^2(a+bx)\right)\tan(a+bx)\right)}{6b}$$

input
```
Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^4,x]
```

output
```
(Sqrt[Csc[a + b*x]]*(-5*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + (5 + 2*Sec[a + b*x]^2)*Tan[a + b*x]))/(6*b)
```

3.277.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3106, 3042, 3106, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc(a+bx)}\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{5}{6}\int \sqrt{\csc(a+bx)}\sec^2(a+bx)dx+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sqrt{\csc(a+bx)}\sec(a+bx)^2dx+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3106}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \sqrt{\csc(a+bx)}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}\right)+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \sqrt{\csc(a+bx)}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}\right)+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}\right)+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx+\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}\right)+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{5}{6}\left(\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b}\right)$$

input
```
Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^4,x]
```

output
```
Sec[a + b*x]^3/(3*b*Sqrt[Csc[a + b*x]]) + (5*(Sec[a + b*x]/(b*Sqrt[Csc[a +
 b*x]]) + (Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
+ b*x]])/b))/6
```

3.277.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.277.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

method	result
default	$-\frac{\sqrt{(\cos^2(bx+a))\sin(bx+a)}\left(5\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,F\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)(\cos^2(bx+a))+10(\cos^2(b}{12(\sin(bx+a)+1)(\sin(bx+a)-1)\sqrt{-\sin(bx+a)(\sin(bx+a)-1)(\sin(bx+a)+1)}\,\cos(bx+a)\sqrt{\sin(bx+a}$

input
```
int(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*(5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+
a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)
)*cos(b*x+a)^2+10*cos(b*x+a)^2*sin(b*x+a)+4*sin(b*x+a))/(sin(b*x+a)+1)/(si
n(b*x+a)-1)/(-sin(b*x+a)*(sin(b*x+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/s
in(b*x+a)^(1/2)/b
```

3.277.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx$$
$$=\frac{-5i\sqrt{2i}\cos{(bx+a)}^3\,\text{weierstrassPInverse}(4,0,\cos{(bx+a)}+i\,\sin{(bx+a)})+5i\sqrt{-2i}\cos{(bx+a)}^3\,\text{weie}}{12\,b\cos{(bx+a)}}$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/12*(-5*I*sqrt(2*I)*cos(b*x + a)^3*weierstrassPInverse(4, 0, cos(b*x + a)
 + I*sin(b*x + a)) + 5*I*sqrt(-2*I)*cos(b*x + a)^3*weierstrassPInverse(4,
0, cos(b*x + a) - I*sin(b*x + a)) + 2*(5*cos(b*x + a)^2 + 2)*sqrt(sin(b*x
+ a)))/(b*cos(b*x + a)^3)
```

3.277.6 Sympy [F]

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx = \int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**4,x)
```

output
```
Integral(sqrt(csc(a + b*x))*sec(a + b*x)**4, x)
```

3.277.7 Maxima [F]

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx = \int \sqrt{\csc(bx+a)}\sec(bx+a)^4\,dx$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(csc(b*x + a))*sec(b*x + a)^4, x)
```

3.277.8 Giac [F]

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx = \int \sqrt{\csc(bx+a)}\sec(bx+a)^4\,dx$$

input
```
integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(csc(b*x + a))*sec(b*x + a)^4, x)
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx=\int \frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos\left(a+b\,x\right)^4}\,dx$$

input
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^4,x)
```

output
```
int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^4, x)
```

3.278 $\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.278.1 Optimal result

Integrand size = 19, antiderivative size = 92

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{2b}$$

output
```
1/2*sec(b*x+a)/b/csc(b*x+a)^(3/2)+1/3*sec(b*x+a)^3/b/csc(b*x+a)^(3/2)+1/2*
(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(co
s(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

3.278.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{\cos(a+bx)\sqrt{\csc(a+bx)}\left(-3+\sec^2(a+bx)+2\sec^4(a+bx)+3E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sec(a+bx)\sqrt{\right.}{6b}$$

input
```
Integrate[Sec[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

output
```
(Cos[a + b*x]*Sqrt[Csc[a + b*x]]*(-3 + Sec[a + b*x]^2 + 2*Sec[a + b*x]^4 +
 3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sec[a + b*x]*Sqrt[Sin[a + b*x]]))/(
6*b)
```

3.278.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3106, 3042, 3106, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^4}{\sqrt{\csc(a+bx)}}dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{1}{2}\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}dx+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sec(a+bx)^2}{\sqrt{\csc(a+bx)}}dx+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3106}$$

$$\frac{1}{2}\left(\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\int \frac{1}{\sqrt{\csc(a+bx)}}dx\right)+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\int \frac{1}{\sqrt{\csc(a+bx)}}dx\right)+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{2}\left(\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx\right)+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{1}{2}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\sqrt{\sin(a+bx)}dx\right)+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}+\frac{1}{2}\left(\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}\right)$$

input
```
Int[Sec[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

output
```
Sec[a + b*x]^3/(3*b*Csc[a + b*x]^(3/2)) + (Sec[a + b*x]/(b*Csc[a + b*x]^(3
/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/b)/2
```

3.278.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.278.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.74

method	result
default	$\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,E\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)(\cos^2(bx+a))-3\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-}}{12\cos(bx+a)^3\sqrt{\sin(bx+a)}\,b}$

input
```
int(sec(b*x+a)^4/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/cos(b*x+a)^3/sin(b*x+a)^(1/2)*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*c
os(b*x+a)^2-3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(
1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-6*cos(b*x+a)
^4+2*cos(b*x+a)^2+4)/b
```

3.278.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx =$$

$$-\frac{3\sqrt{2i}\cos(bx+a)^3\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))+3\sqrt{-}}{}$$

input
```
integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(3*sqrt(2*I)*cos(b*x + a)^3*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*sqrt(-2*I)*cos(b*x + a)^3*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))
) + 2*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)/sqrt(sin(b*x + a)))/(b*cos(b
*x + a)^3)
```

3.278.6 Sympy [F]

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(sec(b*x+a)**4/csc(b*x+a)**(1/2),x)
```

output
```
Integral(sec(a + b*x)**4/sqrt(csc(a + b*x)), x)
```

3.278.7 Maxima [F]

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\int \frac{\sec(bx+a)^4}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

3.278.8 Giac [F]

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx=\int \frac{\sec(bx+a)^4}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^4\,\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int(1/(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2)),x)
```

output
```
int(1/(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2)), x)
```

3.279 $\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx$

3.279.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx=\frac{d\sqrt{d\cos(a+bx)}\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}$$

output
```
d*csc(b*x+a)^(-1+p)*hypergeom([-1/4, 1/2-1/2*p],[3/2-1/2*p],sin(b*x+a)^2)*
(d*cos(b*x+a))^(1/2)/b/(1-p)/(cos(b*x+a)^2)^(1/4)
```

3.279.2 Mathematica [A] (verified)

Time = 32.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}\csc^{-1+p}(a+bx)\left(9\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{2}(-1+p),\frac{9}{4},\cos^2(a+bx)\right)+5\cos^2(a+bx)\right.}{45bd}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^p,x]
```

output
```
(-2*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^(-1 + p)*(9*Hypergeometric2F1[5/4,
 (-1 + p)/2, 9/4, Cos[a + b*x]^2] + 5*Cos[a + b*x]^2*Hypergeometric2F1[9/4
, (1 + p)/2, 13/4, Cos[a + b*x]^2])*(Sin[a + b*x]^2)^((-1 + p)/2))/(45*b*d
)
```

3.279.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}\left(-\sec\left(a+bx+\frac{\pi}{2}\right)\right)^p dx$$

$$\Big\downarrow \text{3067}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int (d\cos(a+bx))^{3/2}\sin^{-p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int (d\cos(a+bx))^{3/2}\sin(a+bx)^{-p}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\sqrt{d\cos(a+bx)}\csc^{p-1}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^p,x]
```

output
```
(d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(Cos[a + b*x]^2)^(1/4))
```

3.279.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.279.4 Maple [F]

$$\int (d\cos(bx+a))^{\frac{3}{2}} (\csc^p(bx+a))\, dx$$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x)
```

output
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x)
```

3.279.5 Fricas [F]

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\, dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^p\, dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*d*csc(b*x + a)^p*cos(b*x + a), x)
```

3.279.6 Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**p,x)
```

output
```
Timed out
```

3.279.7 Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^p\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^p, x)
```

3.279.8 Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^p\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^p, x)
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx = \int (d\,\cos(a+b\,x))^{3/2}\left(\frac{1}{\sin(a+b\,x)}\right)^p\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)*(1/sin(a + b*x))^p,x)
```

output
```
int((d*cos(a + b*x))^(3/2)*(1/sin(a + b*x))^p, x)
```

3.280 $\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx$

3.280.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}$$

output
```
d*(cos(b*x+a)^2)^(1/4)*csc(b*x+a)^(-1+p)*hypergeom([1/4, 1/2-1/2*p],[3/2-1
/2*p],sin(b*x+a)^2)/b/(1-p)/(d*cos(b*x+a))^(1/2)
```

3.280.2 Mathematica [A] (verified)

Time = 10.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+p}{2},\frac{7}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1}{2}(-1+p)}}{3bd}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^p,x]
```

output
```
(-2*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1
 + p)/2, 7/4, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(3*b*d)
```

3.280.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}\left(-\sec\left(a+bx+\frac{\pi}{2}\right)\right)^p dx$$

$$\Big\downarrow \text{3067}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \sqrt{d\cos(a+bx)}\sin^{-p}(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \sqrt{d\cos(a+bx)}\sin(a+bx)^{-p}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^p,x]
```

output
```
(d*(Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[1/4, (1
- p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

3.280.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.280.4 Maple [F]

$$\int \sqrt{d\cos(bx+a)}\,(\csc^p(bx+a))\,dx$$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x)
```

output
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x)
```

3.280.5 Fricas [F]

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\csc(bx+a)^p\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

3.280.6 Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = \int \sqrt{d\cos{(a+bx)}}\csc^p{(a+bx)}\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**p,x)
```

output
```
Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**p, x)
```

3.280.7 Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = \int \sqrt{d\cos{(bx+a)}}\csc{(bx+a)}^p\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

3.280.8 Giac [F]

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = \int \sqrt{d\cos{(bx+a)}}\csc{(bx+a)}^p\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx = \int \sqrt{d\,\cos\left(a+b\,x\right)}\left(\frac{1}{\sin\left(a+b\,x\right)}\right)^p\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)*(1/sin(a + b*x))^p,x)
```

output
```
int((d*cos(a + b*x))^(1/2)*(1/sin(a + b*x))^p, x)
```

3.281 $\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

3.281.1 Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}$$

output
```
d*(cos(b*x+a)^2)^(3/4)*csc(b*x+a)^(-1+p)*hypergeom([3/4, 1/2-1/2*p],[3/2-1
/2*p],sin(b*x+a)^2)/b/(1-p)/(d*cos(b*x+a))^(3/2)
```

3.281.2 Mathematica [A] (verified)

Time = 10.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}\csc^{1+p}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+p}{2},\frac{5}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1+p}{2}}}{bd}$$

input
```
Integrate[Csc[a + b*x]^p/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(-2*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(1 + p)*Hypergeometric2F1[1/4, (1 +
p)/2, 5/4, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 + p)/2))/(b*d)
```

3.281.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(-\sec\left(a+bx+\frac{\pi}{2}\right)\right)^p}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3067}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \frac{\sin^{-p}(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \frac{\sin(a+bx)^{-p}}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{d\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^p/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(d*(Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1
- p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

3.281.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.281.4 Maple [F]

$$\int \frac{\csc^p(bx+a)}{\sqrt{d\cos(bx+a)}}dx$$

input
```
int(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x)
```

output
```
int(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x)
```

3.281.5 Fricas [F]

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc(bx+a)^p}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d*cos(b*x + a)), x)
```

3.281.6 Sympy [F]

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**p/sqrt(d*cos(a + b*x)), x)
```

3.281.7 Maxima [F]

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc(bx+a)^p}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^p/sqrt(d*cos(b*x + a)), x)
```

3.281.8 Giac [F]

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc(bx+a)^p}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^p/sqrt(d*cos(b*x + a)), x)
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\left(\frac{1}{\sin(a+b\,x)}\right)^p}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(1/2),x)
```

output
```
int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(1/2), x)
```

3.282 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

3.282.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}$$

output
```
(cos(b*x+a)^2)^(1/4)*csc(b*x+a)^(-1+p)*hypergeom([5/4, 1/2-1/2*p],[3/2-1/2
*p],sin(b*x+a)^2)/b/d/(1-p)/(d*cos(b*x+a))^(1/2)
```

3.282.2 Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+p}{2},\frac{3}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1}{2}(-1}}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^p/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 + p)/2, 3/4, Cos[a + b
*x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(b*d*Sqrt[d*Cos[a + b*x]])
```

3.282.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\sec\left(a+bx+\frac{\pi}{2}\right)\right)^p}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3067}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \frac{\sin^{-p}(a+bx)}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \frac{\sin(a+bx)^{-p}}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}$$

input
```
Int[Csc[a + b*x]^p/(d*Cos[a + b*x])^(3/2),x]
```

output
```
((Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[5/4, (1 -
p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

3.282.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.282.4 Maple [F]

$$\int \frac{\csc^p(bx+a)}{(d\cos(bx+a))^{\frac{3}{2}}}dx$$

input
```
int(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x)
```

output
```
int(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x)
```

3.282.5 Fricas [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^p}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d^2*cos(b*x + a)^2), x)
```

3.282.6 Sympy [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc^p{(a+bx)}}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**p/(d*cos(a + b*x))**(3/2), x)
```

3.282.7 Maxima [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}^p}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(3/2), x)
```

3.282.8 Giac [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}^p}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(3/2), x)
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\sin(a+b\,x)}\right)^p}{\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(3/2),x)
```

output
```
int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(3/2), x)
```

3.283 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

3.283.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx=\frac{\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}$$

output
```
(cos(b*x+a)^2)^(3/4)*csc(b*x+a)^(-1+p)*hypergeom([7/4, 1/2-1/2*p],[3/2-1/2
*p],sin(b*x+a)^2)/b/d/(1-p)/(d*cos(b*x+a))^(3/2)
```

3.283.2 Mathematica [A] (verified)

Time = 10.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx=\frac{2\csc^{-1+p}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+p}{2},\frac{1}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1}{2}(-1}}{3bd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^p/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-3/4, (1 + p)/2, 1/4, Cos[a + b
*x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

3.283.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(-\sec\left(a+bx+\frac{\pi}{2}\right)\right)^p}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3067}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \frac{\sin^{-p}(a+bx)}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sin^p(a+bx)\csc^p(a+bx)\int \frac{\sin(a+bx)^{-p}}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^p/(d*Cos[a + b*x])^(5/2),x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[7/4, (1 -
p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

3.283.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.283.4 Maple [F]

$$\int \frac{\csc^{p}\left(bx+a\right)}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x)
```

output
```
int(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x)
```

3.283.5 Fricas [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^{p}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d^3*cos(b*x + a)^3), x)
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

3.283.7 Maxima [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^p}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(5/2), x)
```

3.283.8 Giac [F]

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^p}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(5/2), x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\sin(a+b\,x)}\right)^p}{(d\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(5/2),x)
```

output
```
int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(5/2), x)
```

3.284 $\int \cos^m(e+fx)\csc^n(e+fx)\,dx$

3.284.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \frac{\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

output
```
cos(f*x+e)^(-1+m)*(cos(f*x+e)^2)^(-1/2*m+1/2)*csc(f*x+e)^(-1+n)*hypergeom(
[1/2-1/2*n, -1/2*m+1/2],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

3.284.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.39 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.67

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx =$$

$$-\frac{2(-3+n)\,\text{Appe}}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1}{2}-\frac{n}{2},-m,1+m-n,\frac{3}{2}-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

input
```
Integrate[Cos[e + f*x]^m*Csc[e + f*x]^n,x]
```

output
```
(-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Cos[e + f*x]^m*Csc[e + f*x]^n
*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n
, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 +
 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2
 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

3.284.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(e+fx+\frac{\pi}{2}\right)^m\left(-\sec\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \ 3067$$

$$\sin^n(e+fx)\csc^n(e+fx)\int \cos^m(e+fx)\sin^{-n}(e+fx)dx$$

$$\Big\downarrow \ 3042$$

$$\sin^n(e+fx)\csc^n(e+fx)\int \cos(e+fx)^m\sin(e+fx)^{-n}dx$$

$$\Big\downarrow \ 3057$$

$$\frac{\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Cos[e + f*x]^m*Csc[e + f*x]^n,x]
```

output
```
(Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1 + n)*
Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1
- n))
```

3.284.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.284.4 Maple [F]

$$\int (\cos^m (fx + e)) (\csc^n (fx + e)) \, dx$$

input
```
int(cos(f*x+e)^m*csc(f*x+e)^n,x)
```

output
```
int(cos(f*x+e)^m*csc(f*x+e)^n,x)
```

3.284.5 Fricas [F]

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \int \cos(fx+e)^m \csc(fx+e)^n \,dx$$

input
```
integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

3.284.6 Sympy [F]

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \int \cos^m(e+fx)\csc^n(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**m*csc(f*x+e)**n,x)
```

output
```
Integral(cos(e + f*x)**m*csc(e + f*x)**n, x)
```

3.284.7 Maxima [F]

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \int \cos(fx+e)^m \csc(fx+e)^n \,dx$$

input
```
integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

3.284.8 Giac [F]

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \int \cos(fx+e)^m\csc(fx+e)^n\,dx$$

input
```
integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \int \cos(e+f\,x)^m\left(\frac{1}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int(cos(e + f*x)^m*(1/sin(e + f*x))^n,x)
```

output
```
int(cos(e + f*x)^m*(1/sin(e + f*x))^n, x)
```

3.285 $\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx$

3.285.1 Optimal result

Integrand size = 19, antiderivative size = 88

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = \frac{a(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

output
```
a*(a*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(-1/2*m+1/2)*csc(f*x+e)^(-1+n)*hype
rgeom([1/2-1/2*n, -1/2*m+1/2],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

3.285.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.00 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.57

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = -\frac{2(-3+n)\,\text{Appell}}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1}{2}-\frac{n}{2},-m,1+m-n,\frac{3}{2}-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

input
```
Integrate[(a*Cos[e + f*x])^m*Csc[e + f*x]^n,x]
```

output
```
(-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(a*Cos[e + f*x])^m*Csc[e + f*
x]^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m
 - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]
^2 + 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n,
 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

3.285.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^n(e+fx)(a\cos(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(-\sec\left(e+fx+\frac{\pi}{2}\right)\right)^n\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \; 3067$$

$$\sin^n(e+fx)\csc^n(e+fx)\int (a\cos(e+fx))^m\sin^{-n}(e+fx)dx$$

$$\Big\downarrow \; 3042$$

$$\sin^n(e+fx)\csc^n(e+fx)\int (a\cos(e+fx))^m\sin(e+fx)^{-n}dx$$

$$\Big\downarrow \; 3057$$

$$\frac{a\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)(a\cos(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(a*Cos[e + f*x])^m*Csc[e + f*x]^n,x]
```

output
```
(a*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1
 + n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/
(f*(1 - n))
```

3.285.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.285.4 Maple [F]

$$\int (\cos (fx+e)\, a)^m (\csc^n (fx+e))\, dx$$

input
```
int((cos(f*x+e)*a)^m*csc(f*x+e)^n,x)
```

output
```
int((cos(f*x+e)*a)^m*csc(f*x+e)^n,x)
```

3.285.5 Fricas [F]

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = \int (a\cos(fx+e))^m \csc(fx+e)^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="fricas")
```

output
```
integral((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

3.285.6 Sympy [F]

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = \int (a\cos(e+fx))^m \csc^n(e+fx)\,dx$$

input
```
integrate((a*cos(f*x+e))**m*csc(f*x+e)**n,x)
```

output
```
Integral((a*cos(e + f*x))**m*csc(e + f*x)**n, x)
```

3.285.7 Maxima [F]

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = \int (a\cos(fx+e))^m \csc(fx+e)^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

3.285.8 Giac [F]

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = \int (a\cos(fx+e))^m \csc(fx+e)^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx = \int (a\cos(e+fx))^m \left(\frac{1}{\sin(e+fx)}\right)^n dx$$

input
```
int((a*cos(e + f*x))^m*(1/sin(e + f*x))^n,x)
```

output
```
int((a*cos(e + f*x))^m*(1/sin(e + f*x))^n, x)
```

3.286 $\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx$

3.286.1 Optimal result

Integrand size = 19, antiderivative size = 88

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx$$
$$=\frac{b\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

output
```
b*cos(f*x+e)^(-1+m)*(cos(f*x+e)^2)^(-1/2*m+1/2)*(b*csc(f*x+e))^(-1+n)*hype
rgeom([1/2-1/2*n, -1/2*m+1/2],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

3.286.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.01 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.57

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx =$$
$$-\frac{2(-3+n)\,\text{Appell}}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1}{2}-\frac{n}{2},-m,1+m-n,\frac{3}{2}-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right.}$$

input
```
Integrate[Cos[e + f*x]^m*(b*Csc[e + f*x])^n,x]
```

output
```
(-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Cos[e + f*x]^m*(b*Csc[e + f*x
])^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m
 - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]
^2 + 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n,
 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

3.286.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(e+fx+\frac{\pi}{2}\right)^m\left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos^m(e+fx)(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int \cos(e+fx)^m(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{b\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Cos[e + f*x]^m*(b*Csc[e + f*x])^n,x]
```

output
```
(b*Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x])^(-1
 + n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/
(f*(1 - n))
```

3.286.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.286.4 Maple [F]

$$\int (\cos^m (fx + e)) (b \csc (fx + e))^n \, dx$$

input
```
int(cos(f*x+e)^m*(b*csc(f*x+e))^n,x)
```

output
```
int(cos(f*x+e)^m*(b*csc(f*x+e))^n,x)
```

3.286.5 Fricas [F]

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

3.286.6 Sympy [F]

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(e+fx))^n\cos^m(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**m*(b*csc(f*x+e))**n,x)
```

output
```
Integral((b*csc(e + f*x))**n*cos(e + f*x)**m, x)
```

3.286.7 Maxima [F]

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n\cos(fx+e)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

3.286.8 Giac [F]

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \int (b\csc(fx+e))^n \cos(fx+e)^m\,dx$$

input
```
integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \int \cos(e+f\,x)^m \left(\frac{b}{\sin(e+f\,x)}\right)^n\,dx$$

input
```
int(cos(e + f*x)^m*(b/sin(e + f*x))^n,x)
```

output
```
int(cos(e + f*x)^m*(b/sin(e + f*x))^n, x)
```

3.287 $\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx$

3.287.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx$$
$$=\frac{ab(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+\ldots\right.}{f(1-n)}$$

output
```
a*b*(a*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(-1/2*m+1/2)*(b*csc(f*x+e))^(-1+n
)*hypergeom([1/2-1/2*n, -1/2*m+1/2],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

3.287.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.47

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx =$$
$$-\frac{2(-3+n)\,\text{AppellF}\ldots}{f(-1+n)\left((-3+n)\,\text{AppellF1}\left(\frac{1}{2}-\frac{n}{2},-m,1+m-n,\frac{3}{2}-\frac{n}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\ldots\right.}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^n,x]
```

output
```
(-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(a*Cos[e + f*x])^m*(b*Csc[e +
 f*x])^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1
 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x
)/2]^2 + 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m
- n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]
^2))
```

3.287.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m\left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int (a\cos(e+fx))^m(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{n-1}(b\csc(e+fx))^{n-1}\int (a\cos(e+fx))^m(b\sin(e+fx))^{-n}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{ab\cos^2(e+fx)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}(b\csc(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\sin^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^n,x]
```

output
```
(a*b*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x
])^(-1 + n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x
]^2])/(f*(1 - n))
```

3.287.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.287.4 Maple [F]

$$\int (\cos (fx+e) a)^m (b \csc (fx+e))^n \, dx$$

input
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^n,x)
```

output
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^n,x)
```

3.287.5 Fricas [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx = \int (a\cos(fx+e))^m(b\csc(fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

3.287.6 Sympy [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx = \int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**n,x)
```

output
```
Integral((a*cos(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

3.287.7 Maxima [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx = \int (a\cos(fx+e))^m(b\csc(fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

3.287.8 Giac [F]

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^n\,dx = \int (a\cos(fx+e))^m (b\csc(fx+e))^n\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^n\,dx = \int (a\,\cos(e+f\,x))^m \left(\frac{b}{\sin(e+f\,x)}\right)^n dx$$

input
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^n,x)
```

output
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

3.288 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx$

3.288.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx = -\frac{b^3(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{af(1+m)}$$

output
```
-b^3*(a*cos(f*x+e))^(1+m)*hypergeom([9/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e
)^2)*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/f/(1+m)
```

3.288.2 Mathematica [A] (verified)

Time = 16.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx = \frac{2ab(a\cos(e+fx))^{-1+m}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(b\csc(e+fx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4}(7-2m\right.}{f(-7+2m)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(7/2),x]
```

output
```
(2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*(b*Csc[e +
f*x])^(5/2)*Hypergeometric2F1[(7 - 2*m)/4, (1 - m)/2, (11 - 2*m)/4, Csc[e
+ f*x]^2])/(f*(-7 + 2*m))
```

3.288.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\csc(e+fx))^{7/2}(a\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^{7/2}\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\sin(e+fx))^{5/2}(b\csc(e+fx))^{5/2}\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{5/2}(b\csc(e+fx))^{5/2}\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin^2(e+fx)^{5/4}(b\csc(e+fx))^{5/2}(a\cos(e+fx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{af(m+1)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(7/2),x]
```

output
```
-((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(5/2)*Hypergeometric2F1[9/4
, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(5/4))/(a*f*(1 +
m)))
```

3.288.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.288.4 Maple [F]

$$\int (\cos (fx+e)\, a)^m\, (b\csc (fx+e))^{\frac{7}{2}}\, dx$$

input
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(7/2),x)
```

output
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(7/2),x)
```

3.288.5 Fricas [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{7/2}\, dx = \int (b\csc (fx+e))^{\frac{7}{2}}\, (a\cos (fx+e))^m\, dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b^3*csc(f*x + e)^3, x)
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(7/2),x)
```

output
```
Timed out
```

3.288.7 Maxima [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{7/2}\,dx = \int (b\csc(fx+e))^{\frac{7}{2}}(a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^(7/2)*(a*cos(f*x + e))^m, x)
```

3.288.8 Giac [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{7/2}\,dx = \int (b\csc(fx+e))^{\frac{7}{2}}(a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^(7/2)*(a*cos(f*x + e))^m, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx = \int (a\,\cos(e+f\,x))^m\left(\frac{b}{\sin(e+f\,x)}\right)^{7/2}\,dx$$

input
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(7/2),x)
```

output
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(7/2), x)
```

3.289 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx$

3.289.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx = -\frac{b(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{3/4}}{af(1+m)}$$

output
```
-b*(a*cos(f*x+e))^(1+m)*(b*csc(f*x+e))^(3/2)*hypergeom([7/4, 1/2+1/2*m],[3
/2+1/2*m],cos(f*x+e)^2)*(sin(f*x+e)^2)^(3/4)/a/f/(1+m)
```

3.289.2 Mathematica [A] (verified)

Time = 10.76 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx = \frac{2ab(a\cos(e+fx))^{-1+m}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(b\csc(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{4}(5-2m),\dots\right)}{f(-5+2m)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(5/2),x]
```

output
```
(2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*(b*Csc[e +
f*x])^(3/2)*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Csc[e +
 f*x]^2])/(f*(-5 + 2*m))
```

3.289.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\csc(e+fx))^{5/2}(a\cos(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\sin(e+fx))^{3/2}(b\csc(e+fx))^{3/2}\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\sin(e+fx))^{3/2}(b\csc(e+fx))^{3/2}\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{af(m+1)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(5/2),x]
```

output
```
-((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[7/4
, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*f*(1 +
m)))
```

3.289.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

3.289.4 Maple [F]

$$\int (\cos (fx+e)\,a)^m\,(b\csc (fx+e))^{\frac{5}{2}}\,dx$$

input
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(5/2),x)
```

output
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(5/2),x)
```

3.289.5 Fricas [F]

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{5/2}\,dx = \int (b\csc (fx+e))^{\frac{5}{2}}\,(a\cos (fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b^2*csc(f*x + e)^2, x)
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.289.7 Maxima [F]

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx = \int (b\csc(fx+e))^{\frac{5}{2}} (a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^(5/2)*(a*cos(f*x + e))^m, x)
```

3.289.8 Giac [F]

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx = \int (b\csc(fx+e))^{\frac{5}{2}} (a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^(5/2)*(a*cos(f*x + e))^m, x)
```

3.289.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{5/2}\,dx=\int (a\,\cos\left(e+f\,x\right))^m\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^{5/2}\,dx$$

input
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(5/2),x)
```

output
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(5/2), x)
```

3.290 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx$

3.290.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = -\frac{b(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{af(1+m)}$$

output
```
-b*(a*cos(f*x+e))^(1+m)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^
2)*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/f/(1+m)
```

3.290.2 Mathematica [A] (verified)

Time = 9.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = \frac{2ab(a\cos(e+fx))^{-1+m}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\sqrt{b\csc(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4}(3-2m)\right.}{f(-3+2m)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(3/2),x]
```

output
```
(2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*Sqrt[b*Csc[
e + f*x]]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Csc[e + f
*x]^2])/(f*(-3 + 2*m))
```

3.290.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\csc(e+fx))^{3/2}(a\cos(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \ 3067$$

$$b^2\sqrt{b\sin(e+fx)}\sqrt{b\csc(e+fx)}\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$b^2\sqrt{b\sin(e+fx)}\sqrt{b\csc(e+fx)}\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3056$$

$$-\frac{b\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{af(m+1)}$$

input
```
Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(3/2),x]
```

output
```
-((b*(a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[5/4,
(1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*f*(1 + m)
))
```

3.290.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

3.290.4 Maple [F]

$$\int (\cos (fx+e) a)^m (b\csc (fx+e))^{\frac{3}{2}} dx$$

input
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(3/2),x)
```

output
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(3/2),x)
```

3.290.5 Fricas [F]

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2} dx = \int (b\csc (fx+e))^{\frac{3}{2}} (a\cos (fx+e))^m dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b*csc(f*x + e), x)
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.290.7 Maxima [F]

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = \int (b\csc(fx+e))^{\frac{3}{2}} (a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^(3/2)*(a*cos(f*x + e))^m, x)
```

3.290.8 Giac [F]

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = \int (b\csc(fx+e))^{\frac{3}{2}} (a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^(3/2)*(a*cos(f*x + e))^m, x)
```

3.290.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = \int (a\,\cos\left(e+f\,x\right))^m \left(\frac{b}{\sin\left(e+f\,x\right)}\right)^{3/2}\,dx$$

input
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(3/2),x)
```

output
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(3/2), x)
```

3.291 $\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx$

3.291.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = -\frac{(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin^2(e+fx)^{3/4}}{abf(1+m)}$$

output
```
-(a*cos(f*x+e))^(1+m)*(b*csc(f*x+e))^(3/2)*hypergeom([3/4, 1/2+1/2*m],[3/2
+1/2*m],cos(f*x+e)^2)*(sin(f*x+e)^2)^(3/4)/a/b/f/(1+m)
```

3.291.2 Mathematica [A] (verified)

Time = 8.90 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = \frac{2(a\cos(e+fx))^m\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\sqrt{b\csc(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4}(1-2m),\frac{1-m}{2},\frac{1}{4}(5-2m),\right.}{f(-1+2m)}$$

input
```
Integrate[(a*Cos[e + f*x])^m*Sqrt[b*Csc[e + f*x]],x]
```

output
```
(2*(a*Cos[e + f*x])^m*(-Cot[e + f*x]^2)^((1 - m)/2)*Sqrt[b*Csc[e + f*x]]*H
ypergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, Csc[e + f*x]^2]*Tan[
e + f*x])/(f*(-1 + 2*m))
```

3.291.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3066, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\csc(e+fx)}(a\cos(e+fx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{-b\sec\left(e+fx+\frac{\pi}{2}\right)}\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m dx$$

$$\Big\downarrow \ 3066$$

$$\frac{(b\sin(e+fx))^{3/2}(b\csc(e+fx))^{3/2}\int \frac{(a\cos(e+fx))^m}{\sqrt{b\sin(e+fx)}}dx}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(b\sin(e+fx))^{3/2}(b\csc(e+fx))^{3/2}\int \frac{(a\cos(e+fx))^m}{\sqrt{b\sin(e+fx)}}dx}{b^2}$$

$$\Big\downarrow \ 3056$$

$$-\frac{\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{abf(m+1)}$$

input
```
Int[(a*Cos[e + f*x])^m*Sqrt[b*Csc[e + f*x]],x]
```

output
```
-(((a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[3/4,
(1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*b*f*(1 +
m)))
```

3.291.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

3.291.4 Maple [F]

$$\int (\cos (fx+e) a)^m \sqrt{b \csc (fx+e)}dx$$

input
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(1/2),x)
```

output
```
int((cos(f*x+e)*a)^m*(b*csc(f*x+e))^(1/2),x)
```

3.291.5 Fricas [F]

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = \int \sqrt{b \csc (fx+e)}(a \cos (fx+e))^m \, dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

3.291.6 Sympy [F]

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = \int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx$$

input
```
integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(1/2),x)
```

output
```
Integral((a*cos(e + f*x))**m*sqrt(b*csc(e + f*x)), x)
```

3.291.7 Maxima [F]

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = \int \sqrt{b\csc(fx+e)}(a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

3.291.8 Giac [F]

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = \int \sqrt{b\csc(fx+e)}(a\cos(fx+e))^m\,dx$$

input
```
integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

3.291.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx = \int (a\,\cos(e+f\,x))^m\,\sqrt{\frac{b}{\sin(e+f\,x)}}\,dx$$

input
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(1/2),x)
```

output
```
int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(1/2), x)
```

3.292 $\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx$

3.292.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = -\frac{(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{abf(1+m)}$$

output
```
-(a*cos(f*x+e))^(1+m)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)
*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/b/f/(1+m)
```

3.292.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 28.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.88

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \frac{14b\,\text{AppellF1}\,(}{3f(b\csc(e+fx))^{3/2}\left(7\,\text{AppellF1}\left(\frac{3}{4},-m,\frac{3}{2}+m,\frac{7}{4},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left(2m\,\text{Appe}\right.\right.}$$

input
```
Integrate[(a*Cos[e + f*x])^m/Sqrt[b*Csc[e + f*x]],x]
```

output
```
(14*b*AppellF1[3/4, -m, 3/2 + m, 7/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(a*Cos[e + f*x])^m)/(3*f*(b*Csc[e + f*x])^(3/2)*(7*AppellF1[3/4, -m,
3/2 + m, 7/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(2*m*AppellF1[7
/4, 1 - m, 3/2 + m, 11/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 +
2*m)*AppellF1[7/4, -m, 5/2 + m, 11/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Tan[(e + f*x)/2]^2))
```

3.292.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3066, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\sqrt{-b\sec\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \ 3066$$

$$\frac{\sqrt{b\sin(e+fx)}\sqrt{b\csc(e+fx)}\int(a\cos(e+fx))^m\sqrt{b\sin(e+fx)}dx}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{b\sin(e+fx)}\sqrt{b\csc(e+fx)}\int(a\cos(e+fx))^m\sqrt{b\sin(e+fx)}dx}{b^2}$$

$$\Big\downarrow \ 3056$$

$$-\frac{\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}(a\cos(e+fx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{abf(m+1)}$$

input
```
Int[(a*Cos[e + f*x])^m/Sqrt[b*Csc[e + f*x]],x]
```

output
```
-(((a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[1/4, (1
 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*b*f*(1 + m)
))
```

3.292.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

3.292.4 Maple [F]

$$\int \frac{(\cos(fx+e)\,a)^m}{\sqrt{b\csc(fx+e)}}dx$$

input
```
int((cos(f*x+e)*a)^m/(b*csc(f*x+e))^(1/2),x)
```

output
```
int((cos(f*x+e)*a)^m/(b*csc(f*x+e))^(1/2),x)
```

3.292.5 Fricas [F]

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \int \frac{(a\cos(fx+e))^m}{\sqrt{b\csc(fx+e)}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b*csc(f*x + e)), x)
```

3.292.6 Sympy [F]

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx$$

input
```
integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(1/2),x)
```

output
```
Integral((a*cos(e + f*x))**m/sqrt(b*csc(e + f*x)), x)
```

3.292.7 Maxima [F]

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \int \frac{(a\cos(fx+e))^m}{\sqrt{b\csc(fx+e)}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m/sqrt(b*csc(f*x + e)), x)
```

3.292.8 Giac [F]

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \int \frac{(a\cos(fx+e))^m}{\sqrt{b\csc(fx+e)}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m/sqrt(b*csc(f*x + e)), x)
```

3.292.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \int \frac{(a\cos(e+fx))^m}{\sqrt{\frac{b}{\sin(e+fx)}}}\,dx$$

input
```
int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(1/2),x)
```

output
```
int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(1/2), x)
```

3.293 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx$

3.293.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = -\frac{(a\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)}{abf(1+m)\sqrt{b\csc(e+fx)}\sqrt[4]{\sin^2(e+fx)}}$$

output
```
-(a*cos(f*x+e))^(1+m)*hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2
)/a/b/f/(1+m)/(sin(f*x+e)^2)^(1/4)/(b*csc(f*x+e))^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \frac{2a(a\cos(e+fx))^{-1+m}\cos(2(e+fx))\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\text{Hypergeometric2F1}\left(\right.}{bf(3+2m)\sqrt{b\csc(e+fx)}\left(-2+\csc^2(e+\right.}$$

input
```
Integrate[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(3/2),x]
```

output
```
(2*a*(a*Cos[e + f*x])^(-1 + m)*Cos[2*(e + f*x)]*(-Cot[e + f*x]^2)^((1 - m)
/2)*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)/4, Csc[e + f*x]^2
])/(b*f*(3 + 2*m)*Sqrt[b*Csc[e + f*x]]*(-2 + Csc[e + f*x]^2))
```

3.293.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3066, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3066}$$

$$\frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{3/2}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{b\csc(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{3/2}dx}{b^2\sqrt{b\sin(e+fx)}\sqrt{b\csc(e+fx)}}$$

$$\Big\downarrow \text{3056}$$

$$-\frac{(a\cos(e+fx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{abf(m+1)\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}}$$

input
```
Int[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(3/2),x]
```

output
```
-(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2,
Cos[e + f*x]^2])/(a*b*f*(1 + m)*Sqrt[b*Csc[e + f*x]]*(Sin[e + f*x]^2)^(1/4
)))
```

3.293.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

3.293.4 Maple [F]

$$\int \frac{(\cos(fx+e)\,a)^m}{(b\csc(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((cos(f*x+e)*a)^m/(b*csc(f*x+e))^(3/2),x)
```

output
```
int((cos(f*x+e)*a)^m/(b*csc(f*x+e))^(3/2),x)
```

3.293.5 Fricas [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b^2*csc(f*x + e)^2), x)
```

3.293.6 Sympy [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \int \frac{(a\cos{(e+fx)})^m}{(b\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(3/2),x)
```

output
```
Integral((a*cos(e + f*x))**m/(b*csc(e + f*x))**(3/2), x)
```

3.293.7 Maxima [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \int \frac{(a\cos{(fx+e)})^m}{(b\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(3/2), x)
```

3.293.8 Giac [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \int \frac{(a\cos{(fx+e)})^m}{(b\csc{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(3/2), x)
```

3.293.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \int \frac{(a\ \cos(e+f\,x))^m}{\left(\frac{b}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(3/2),x)
```

output
```
int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(3/2), x)
```

3.294 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx$

3.294.1 Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = -\frac{(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{ab^3f(1+m)}$$

output
```
-(a*cos(f*x+e))^(1+m)*hypergeom([-3/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2
)*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/b^3/f/(1+m)
```

3.294.2 Mathematica [A] (verified)

Time = 11.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \frac{2(a\cos(e+fx))^m(1+2\cos(2(e+fx)))\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\text{Hypergeometric2F}}{b^2f(5+2m)\sqrt{b\csc(e+fx)}\,(-4+}$$

input
```
Integrate[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(5/2),x]
```

output
```
(2*(a*Cos[e + f*x])^m*(1 + 2*Cos[2*(e + f*x)])*(-Cot[e + f*x]^2)^((1 - m)/
2)*Hypergeometric2F1[(-5 - 2*m)/4, (1 - m)/2, (-1 - 2*m)/4, Csc[e + f*x]^2
]*Tan[e + f*x])/(b^2*f*(5 + 2*m)*Sqrt[b*Csc[e + f*x]]*(-4 + 3*Csc[e + f*x]
^2))
```

3.294.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3066, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)\right)^m}{\left(-b\sec\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \ 3066$$

$$\frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{5/2}dx}{b^2(b\sin(e+fx))^{3/2}(b\csc(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{5/2}dx}{b^2(b\sin(e+fx))^{3/2}(b\csc(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3056$$

$$-\frac{(a\cos(e+fx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{abf(m+1)\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}}$$

input
```
Int[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(5/2),x]
```

output
```
-(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-3/4, (1 + m)/2, (3 + m)/2,
Cos[e + f*x]^2])/(a*b*f*(1 + m)*(b*Csc[e + f*x])^(3/2)*(Sin[e + f*x]^2)^(3
/4)))
```

3.294.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

3.294.4 Maple [F]

$$\int \frac{(\cos(fx+e)\,a)^m}{(b\csc(fx+e))^{\frac{5}{2}}}dx$$

input
```
int((cos(f*x+e)*a)^m/(b*csc(f*x+e))^(5/2),x)
```

output
```
int((cos(f*x+e)*a)^m/(b*csc(f*x+e))^(5/2),x)
```

3.294.5 Fricas [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b^3*csc(f*x + e)^3), x)
```

3.294.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.294.7 Maxima [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(5/2), x)
```

3.294.8 Giac [F]

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(5/2), x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \int \frac{(a\ \cos(e+f\,x))^m}{\left(\frac{b}{\sin(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(5/2),x)
```

output
```
int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(5/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```